Network Yield Management Models with Asymptotic Property

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Outline

- Introduction
- A Nonlinear YM Model
- A Series of Stochastic Models
- A series of Deterministic Models
- Curry (1990)’s Model
- Conclusion

Introduction

- Single-leg vs. Network Based YM Models
- Literature on Static Network YM Models
  - Curry (1990)
  - Gallego and Van Ryzin (1997)
  - Ciancimino *et al* (1999)
  - Cooper (2002)
- Conclusion: A Need to Explore Alternative Models with Asymptotic Property

A Nonlinear YM Model (I)

$$\max p_j \left[\int_0^{x_j} u f_j^t(u)\,du + \int_0^{x_j} x_j f_j^t(u)\,du \right]$$

Subject to

$$Ax \le C$$

Asymptotic Property of Model (I)

Proposition

Model (I) Possesses Asymptotic Property under the Following Assumption

Assumption

For any limited value U, we assume

$$\lim_{t \to \infty} \int_0^U u f_j^t(u) du = 0, \forall j$$

A Note on the Result

Under the Same Assumption, Models with Additional Side Constraints are Asymptotically Optimal as Well.

Example:

Ciancimino *et al* (1999)
(Side-Constraints: $l \leq x$)

Stochastic Models (II)

Max

$$\sum_j \left(p_j x_j - \alpha_j \int_{x_j}^{+\infty} p_j \left(u - x_j\right) f_j^t(u)\,du \right)$$

Subject to

$$Ax \le C$$

Explanation

$$\sum_j \left\{ \alpha_j \left[\int_0^{x_j} p_j u f_j^t(u)\,du + \int_{x_j}^{+\infty} p_j x_j f_j^t(u)\,du \right] + (1-\alpha_j) p_j x_j \right\}$$

Practical Implications of Model (II)

- Flexibility to Account for the Errors in the Objective Function
- Nesting Effect Might be Better Approximated by Fine Tuning the Objective Function
- Note: it is not required $\alpha_i = \alpha_j, i \neq j,$

Another Vertex of the Space for Asymptotic Models

$$\max_x \left\{ \sum_j \left\{ 2p_j x_j - \left[\int_0^{x_j} p_j u f_j^t(u)\,du + \int_{x_j}^{+\infty} p_j x_j f_j^t(u)\,du \right] \right\} : Ax \le C \right\}$$

Deterministic Models (III)

max $\quad p \cdot x$

Subject to

$$Ax \leq C$$

Deterministic Models (IV)

max $\quad p \cdot x$

Subject to

$$Ax \le C$$

$$0 \le x \le \mu$$

Deterministic Models (V)

max $\quad p \cdot x$

Subject to

$$Ax \le C$$

$$0 \le x \le \mu'$$

Where

$$\mu' = (\mu_i') \quad \mu_i' = \beta_i \mu_i \quad \beta_i \ge 1, \forall i$$

Example for Model (V)

$$A = (1,1)$$
$$x = \left(x_1, x_2\right)'$$

$$C = 2$$

$$\mu_1 = \mu_2 = 1$$

$$p_1 = 10, p_2 = 2$$

- Model (IV) → Suboptimal Allocation
- Model (V) → Optimal Allocation with

$$\beta_1 = 2, \beta_2 = 1$$

Curry (1990)'s Model

$$\max \sum_k R^k(x^k)$$

subject to $Ax^k \leq C$

Where x^k is the seat allocation to the k class nest

Curry (1990)'s Model (Con't)

Proposition

When Demand Arrival Order is Relaxed, Curry (1990)'s Model is Asymptotically Optimal under Assumption 1

Conclusion

- Static Network YM Models Show Applicability
- Two Classes of Asymptotically Optimal Models are Identified
- Curry (1990)'s Model is Proved to Possess Asymptotic Property