Modifying Lines-of-Flight in the Planning Process for Improved Maintenance Robustness

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Abstract

As part of their planning process, airlines construct lines-of-flight (LOFs) — daily repeating sequences of flights, each of which will be flown by a single aircraft. In the week leading up to the actual day-of-operations, these LOFs are then assigned to specific aircraft (tails), forming multi-day aircraft routings that in turn enable the scheduling of routine maintenance checks. Operational disruptions, however, can lead to deviations from these routings, which in turn disrupt the maintenance plan. The goal of our research is to improve the construction of LOFs so as to increase the likelihood of being able to recover from maintenance disruptions without costly over-the-day aircraft swaps. We present a new metric, maintenance reachability (MR), which measures the robustness of a planned set of LOFs, and develop a mathematical programming approach to improving the MR of a given set of LOFs. We provide computational results based on data from a major U.S. carrier demonstrating that significant improvements in MR can be achieved with only a small number of changes to the original set of LOFs. Finally, we conclude by showing that even under imperfect input data, MR can be improved relative to a planned set of LOFs.

1 Introduction

As part of their planning process for a new schedule, domestic U.S. airlines typically construct lines-of-flight (LOFs), day-long sequences of flights that will each be flown by a single aircraft. These LOFs are then repeated on a daily basis throughout the duration of the schedule. Such repetition provides operational stability, defines opportunities to keep crews and aircraft together, and generates a consistent set of passenger itineraries that do not require changing aircraft.

Once the planning process has been completed and operation of the schedule begins, specific aircraft (tails) must be assigned to the LOFs. The tail assignment problem is often solved on a rolling n-day horizon (e.g. one week), with specific tails being assigned to a sequence of n consecutive and connecting LOFs. Such aircraft rotations not only ensure coverage for each of the LOFs, but also provide scheduled times at which the specific tails will remain overnight at a maintenance station where routine maintenance checks can take place. [For the carrier with which we worked, aircraft require routine maintenance checks approximately every seven days. For the sake of exposition, we will focus on this case. We use the term day-seven aircraft to refer to an aircraft that is beginning its seventh day and thus must terminate the day at a maintenance station.] These rotations are updated on a daily basis, in part to add a new nth day to the end of the rotation, but also to correct for any over-the-day operational swaps that occurred throughout the day.

During day-of-operations, it is not uncommon for an operations controller to perform a tail swap in order to mitigate a variety of unexpected disruptions. Such swaps can be beneficial in the immediate future by
providing coverage for a disrupted flight. Downstream problems can arise, however. For example, although operations controller will typically not swap a day-seven aircraft with another tail whose current LOF does not end the day at a maintenance station, it is not uncommon for a day-six aircraft to be swapped onto a rotation that does not terminate at a maintenance station the following day. Thus, when the rotations are repaired at the end of the day, this aircraft must be assigned to a revised rotation that terminates that following day at a maintenance station.

We define a maintenance line-of-flight (MLOF) as a LOF that terminates at a maintenance station, as seen in Figure (1). Here, a LOF directs an aircraft to fly DTW→BOS→FLL→DFW over the course of a day. If the number of day-six aircraft terminating the day at a given station is larger than the number of MLOFs that originate at that station, then additional costly over-the-day changes to the planned LOFs will be required the following day (when these day-six aircraft become day-seven aircraft) to ensure that all of these tails can get to a maintenance station. We refer to the difference between the number of day-seven aircraft starting the day at a station and the number of MLOFs originating at that station as its number of maintenance misalignments.

The more MLOFs a station has, the less likely it is to experience maintenance misalignments, and incur the associated costs and complexities of modifying the day’s LOFs to recover and re-establish maintenance feasibility. Moreover, the more outgoing LOFs a station has, the more benefit it receives from an additional MLOF (in terms of being able to easily recover from over the day swaps), because the expected number of day-seven tails at the start of each day is larger. The total number of possible MLOFs that can be built in the planning process is limited, however, by the number of end-of-day flights in the flight schedule that terminate at a maintenance station.

The focus of our research is on improving the robustness of a planned schedule by redistributing these maintenance opportunities across all stations, while only making limited changes to the originally-planned LOFs. Our goal is to minimize the overall expected number of maintenance misalignments. Operationally, this provides a higher likelihood of being able to re-assign disrupted tails to new rotations that ensure maintenance feasibility without further disruption to the planned LOFs.

The contributions of our work are two-fold. First, we introduce a new robustness metric called maintenance reachability (MR), with which we are able to analyze a set of LOFs and quantify its ability to withstand unplanned changes from a maintenance feasibility standpoint. Second, we develop an optimization model to construct improved LOFs so as to maximize maintenance reachability by minimizing the total number of maintenance misalignments.

In the next section, we review the relevant literature. In §3, we provide an introduction to the airline planning process, define maintenance reachability, and discuss how the maintenance reachability of a given schedule can be improved with only minor changes to the original set of LOFs. In §4, we present a model for optimizing the maintenance reachability of a planned set of LOFs. In §5, we provide computational results based on U.S. carrier data. Finally, in §6 we offer concluding thoughts and motivate future work to be done in the area of maintenance robustness.
2 Literature Review

Airline operations have benefited greatly from the advent of applied optimization techniques. Such techniques have been used to address various processes of the airline operations portfolio, including schedule planning [Gopalan and Talluri, 1998b], fleet assignment [Hane et al., 1995], crew scheduling [Barnhart et al., 2003] and aircraft routing [Clarke et al., 1997].

These planning problems are typically solved once per schedule, often several months in advance of the execution of a particular schedule, and set the framework for daily operations. It is during the aircraft routing phase that the lines-of-flight are created. Desaulniers et al. [1997] use a column-generation approach to generate feasible aircraft routings with maintenance events. Gopalan and Talluri [1998a] introduce the concept of a LOF within an optimization framework and provide a polynomial-time algorithm to solve a three-day routing problem for a fixed set of lines-of-flight.

Once the execution of the schedule begins, shorter-term decisions must be made on an ongoing basis leading up to day-of-operations. Such decisions include the assigning of specific aircraft to multi-day rotations consisting of consecutive and connected LOFs. This process, known as the tail-assignment problem, is detailed by Grönkvist [2004]. The tail assignment process achieves two primary goals: to cover each LOF with an aircraft and to build rotations that include planned maintenance events for each tail.

During the actual day-of-operation, disruptions may occur, such as mechanical problems or weather delays. To recover from these disruptions, carriers often swap aircraft between two flights. Thus, an aircraft originally on a rotation that included a planned maintenance event may be swapped to a rotation that does not enable required maintenance checks. As a result, additional modifications must be made to ensure overall feasibility of the aircraft with respect to maintenance. Recovery of the tail assignment under uncertainty is explored by Teodorovic and Stojkovic [1995], in which the authors provide a model that alerts the dispatcher to inconsistencies in a particular maintenance routing plan. It is then the responsibility of the dispatcher to find a line-splicing opportunity and route aircraft accordingly to receive maintenance. Filar et al. [2001] provide a survey of various models that provide recovery methods under uncertainty.

An alternative approach to dealing with uncertainty is through building a more robust schedule to begin, where robustness includes both reducing the likelihood of a disruption and increasing the number of opportunities to recover from a disruption easily. There has been significant research in building more robust airline plans as a way to reduce the potential impact of real-time disruptions. For example, Lan et al. [2006] explores a gain in schedule robustness through re-timing of flights. This idea is further extended by Dickson and Boland [2009] and Ahmadbeygi et al. [2010] by re-timing individual flights based on the underlying delay distribution of individual stations. Borndörfer et al. [2010] use a column-generation approach to generate tail assignments, while minimizing the expected delay. Recently Eggenberg et al. [2010] solve the airline recovery problem with considerations for maintenance events using the concept of a recovery network.

The focus of our work falls into this category -- improving the robustness of an airline plan to reduce the impact of daily disruptions. Our approach consists of taking the initial set of planned lines-of-flight and making limited changes to this plan to enhance maintenance robustness.

3 Maintenance Reachability

3.1 Airline Planning Process

Before presenting our approach to developing planned lines-of-flight that are more robust in responding to operational disruption, we first describe the planning process at the major U.S. carrier with whom we conducted this research.

To begin the planning process, the airline sets its schedule of daily-repeating flights for a fixed time period, for example, a quarterly schedule. Given the set of flights, the fleet assignment problem is then solved,
assigning each flight a specific aircraft type. Once fleeting is completed, the schedule can be decomposed by fleet type and, for each fleet type, the crew scheduling problem and maintenance routing problems are solved.

Figure 2: Typical Airline Planning Process

Within maintenance routing, the lines-of-flight are constructed. These lines-of-flight are then used to construct feasible aircraft rotations. The rotations are not assigned to specific aircraft (and, in fact, may be modified on a daily basis once the operation of the schedule begins), but they provide a mechanism for ensuring that a maintenance-feasible set of rotations exists before the schedule is set. This process is illustrated in Figure (2).

On any given morning, each station in the flight network will have a number of outgoing LOFs, some of which will terminate in a maintenance station. If a station has at least as many MLOFs as it does day-7 aircraft (aircraft starting the day at the station which require maintenance at the end of the day), then the schedule can be executed as planned. However, should the station hold more day-7 aircraft than MLOFs, then it may be necessary to perform costly, over-the-day swaps with other lines-of-flight to ensure that all aircraft reach their required maintenance events. From a planning perspective, it is of interest to airlines to ensure that whenever an aircraft begins its seventh day of operation, there is a high likelihood that there is an available MLOF to which it can be assigned, ensuring that it ends the day at a maintenance station without the need for costly over-the-day swaps.

3.2 Disruption Management

Although the planned lines-of-flight and corresponding aircraft rotations are maintenance feasible, these plans are often disrupted in practice during day of operations. In such cases tail assignments, rotations, and even the LOFs themselves may need to be modified to regain maintenance feasibility. These disruptions tend to manifest themselves in the form of unexpected equipment changes and other measures to counter delay propagation. For example, an operation controller may choose to alter the tail assignment to mitigate a delay propagation effect by performing an aircraft substitution. Such a scenario is presented as a numerical example next.

Table (1) depicts a small portion of a flight schedule, as well as corresponding lines-of-flight. In this example, flight sequences \{1, 4\}, \{2, 3\}, \{5\}, \{6\} and \{7\} each form a LOF. Table (1) also indicates the aircraft, A, B and C, and the rotations to which they have been assigned.

Supposed that Flight 2 from DTW to TUS is delayed by 15 minutes. Incorporating a 30 minute turn-time, the earliest the subsequent flight, Flight 3, can depart is at 11:20, 15 minutes past its scheduled departure time. Suppose also that Flight 1 has landed on time and thus, even with a 30 minute turn-time, could satisfy an 11:05 departure. The operation controllers now have the choice of performing an aircraft swap (assuming the aircraft are of the same fleet type). If an aircraft swap is performed, then the planned schedule is changed as illustrated in Table (2). The result of this aircraft swap is that the delay is absorbed, and both Flights 3 and 4 are able to depart on time. On the other hand, the maintenance plan for each aircraft has now been disrupted, because the swap caused the aircraft to exchange their respective routings.
Table 1: Example Aircraft Schedule

<table>
<thead>
<tr>
<th>Flight</th>
<th>Day</th>
<th>Origin</th>
<th>Dest.</th>
<th>STD</th>
<th>STA</th>
<th>Tail</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>SEA</td>
<td>TUS</td>
<td>08:10</td>
<td>10:30</td>
<td>A</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>DTW</td>
<td>TUS</td>
<td>09:05</td>
<td>10:35</td>
<td>B</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>TUS</td>
<td>BOS</td>
<td>11:05</td>
<td>15:00</td>
<td>B</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>TUS</td>
<td>ORD</td>
<td>11:20</td>
<td>15:30</td>
<td>A</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>BOS</td>
<td>DAL*</td>
<td>09:00</td>
<td>14:30</td>
<td>B</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>ORD</td>
<td>DFW</td>
<td>09:30</td>
<td>10:45</td>
<td>A</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>DFW</td>
<td>LAX</td>
<td>11:15</td>
<td>12:45</td>
<td>A</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>LAX</td>
<td>SEA</td>
<td>14:10</td>
<td>16:50</td>
<td>A</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>BOS</td>
<td>MCO*</td>
<td>10:30</td>
<td>13:45</td>
<td>C</td>
</tr>
</tbody>
</table>

*MCO and DAL are maintenance stations.

Table 2: Example Aircraft Schedule After Performing Swap

<table>
<thead>
<tr>
<th>Flight</th>
<th>Day</th>
<th>Origin</th>
<th>Dest.</th>
<th>STD</th>
<th>STA</th>
<th>Tail</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>SEA</td>
<td>TUS</td>
<td>08:10</td>
<td>10:30</td>
<td>A</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>DTW</td>
<td>TUS</td>
<td>09:05</td>
<td>10:50</td>
<td>B</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>TUS</td>
<td>BOS</td>
<td>11:05</td>
<td>15:00</td>
<td>B</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>TUS</td>
<td>ORD</td>
<td>11:20</td>
<td>15:30</td>
<td>B</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>BOS</td>
<td>DAL*</td>
<td>09:00</td>
<td>14:30</td>
<td>A</td>
</tr>
<tr>
<td>6</td>
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<td>09:30</td>
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<td>BOS</td>
<td>MCO*</td>
<td>10:30</td>
<td>13:45</td>
<td>C</td>
</tr>
</tbody>
</table>

*MCO and DAL are maintenance stations.

We now extend this example to add maintenance implications. Suppose that tail B, which is swapped to mitigate delay propagation, is a day-6 aircraft, referring to an aircraft that is on day six of its maintenance rotation. Given that aircraft must be maintained every seven days, the aircraft in this example must receive maintenance at the end of the day tomorrow.

Performing a swap in this situation may have costly consequences for scheduled maintenance. Tail B, which started flying the first rotation, now ends the day in ORD. It turns out that the following day, day seven, the only outgoing line from ORD is destined for SEA which is not a maintenance base, i.e. no MLOF exists. This example is seen graphically in Figure (3). In this situation, we have no possible maintenance outlets for the aircraft, and thus we must find a swap opportunity over the course of the day to ensure this aircraft enters required maintenance. In other words, the intermediate stations indicated in Figure (3), as DFW and LAX, must intersect with another line that does end in a maintenance station. Furthermore, if an intersection with a maintenance line is found, then this MLOF must not already be required to bring another tail to a maintenance station, such that we can perform a swap and allow the aircraft departing ORD on the beginning of day seven to enter maintenance at the end of the day.

By using swap opportunities at either DFW or LAX to bring the aircraft to its required maintenance station, we are changing our operational schedule again, i.e. forcing another aircraft to change its route, causing further network effects. Additionally, we may be breaking a through-flight, causing connecting passengers and crew to be disrupted. In either case, we end with a situation that may be sub-optimal and costly.

At the carrier with whom we worked, over the day swaps are pronounced. On an average day at least 20% of all LOFs incur the type of swap illustrated in the previous examples to mitigate an operational disruption. Such swaps result in aircraft ending their day-of-operations at random stations in the network with various amounts of flight-time left before maintenance must be performed. It is these significant network impacts that motivate our search for a schedule that is resistant (robust) to such unplanned changes.
3.3 Maintenance Reachability - A New Metric

In this section, we describe how to compute the expected number of maintenance misalignments for a given station and its set of MLOFs. This input parameter will later feed into the optimization models presented in §4.1.

An aircraft that ferries passengers on a daily basis is required to receive a routine “A-Check” based on the block or flying time [Sriram and Haghani, 2009]. For the major U.S. carrier considered in this analysis, the maintenance requirement can be translated into an aircraft requiring an A-Check every seven days. To model this recurring maintenance event, we initially assign all aircraft in the fleet a \( \frac{1}{7} \) probability (\( p_r = \frac{1}{7} \)) of being a day-7 aircraft, i.e. an aircraft that requires maintenance at the end of the day. [We later relax the assumption that \( p_r = \frac{1}{7} \) for all aircraft.]

Given the individual probability of requiring maintenance for each aircraft, we use the binomial distribution to derive the expected number of maintenance misalignments at station \( s \) given \( n \) MLOFs using Equation (1).

\[
p^n_s = \sum_{i=n+1}^{L_s} \binom{L_s}{i} (p_r)^i (1 - p_r)^{L_s-i} (i - n)
\]

In Equation (1), the parameter \( L_s \) indicates the total number of LOFs exiting from station \( s \). To determine the expected number of maintenance misalignments, we iterate through the number of LOFs beyond the availability of MLOFs and compute the probability that aircraft will require such a line of maintenance. We then multiply this probability by the number of aircraft that are misaligned, i.e. require maintenance, but do not have access to an MLOF, to compute the expected value. To illustrate this concept more clearly, we present a numerical example next.

Consider two stations, BOS and ORD, each with ten outgoing lines-of-flight. Out of the ten LOFs exiting station ORD, two end in a maintenance station, i.e. are MLOFs. On the other hand, of the ten stations leaving BOS, zero are MLOFs. Of the ten aircraft starting the day in BOS, on average \( \frac{1}{7} \) or \( \approx 1.43 \) will require maintenance. However, these aircraft will not have the opportunity to be routed to a maintenance station directly due to the fact that none of the LOFs leaving BOS are scheduled to end the day in a maintenance station. Hence, for BOS, the expected maintenance misalignment is 1.43. This lack of access...
to MLOFs forces manual intervention (precisely the scenario we aim to avoid) over the course of the day to force a swap with another line, such that a maintenance opportunity is created.

On the other hand, applying a binomial approximation to the ten LOFs out of ORD, of which two are MLOFs, we see that on average only 0.21 day-7 aircraft will be unable to reach a required maintenance station. Alternatively, it would be beneficial to reallocate one of the maintenance opportunities out of ORD to BOS so as to decrease the overall expected number of maintenance misalignments. For this example, using the binomial approximation, moving one of the two maintenance opportunities at ORD to BOS would reduce the expected number of maintenance misalignments from 1.64 (1.43 + 0.21) to 1.28 (0.64 + 0.64).

3.4 Achieving Maintenance Reachability

In our research, we aim to combat the effects of unplanned operational schedule changes, such that day-seven aircraft that end up starting the day at an unplanned station can still be assigned to an MLOF. To do so, we examine the schedule from a planning perspective and attempt to “pre-plan” for recovery from operational, over-the-day swaps, as illustrated in Figure (4).

In our approach, we seek changes to the original plan that can improve maintenance reachability without substantially altering the initial lines-of-flight. We do so by identifying candidate LOFs for splicing opportunities. That is, when two lines-of-flight departing from differing stations intersect at some point in space and time, these lines become candidates for a line-splice. In Figure (5), two lines are candidates for splicing during an intersection in TUS. Such a splice would send Rotation 2 to PHX, while Rotation 1 would now end at maintenance station MCO. In effect, at the splice, the lines-of-flight of the respective aircraft are changed such that each aircraft now flies the remaining route of the other. As a result, if one line-of-flight ends in maintenance, while the other does not, a re-distribution of maintenance opportunities has occurred. It should be noted that these swaps are fleet-restricted. That is, we only consider line-swaps that involve the same aircraft type to ensure complete downstream compatibility.

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This approach of performing line splices ignores the underlying network structure and its possible effect of changing the probability of the having a day-7 aircraft. From our experience, it is not clear whether re-routing, by changing the assignment of MLOFs, will impact the probability of having a day-7 aircraft. However, as we will show in §5, changes in the day-7 aircraft probabilities still allow our model to outperform the original schedule.

Returning to the example in §3.3, if one of the ten LOFs that depart BOS and and one of the two MLOFs leaving ORD intersect at some point in time and space, we can perform a line-splice. In this example, such a line-splice results in two changes: the line leaving BOS gains a maintenance opportunity, while the line leaving ORD loses one. In other words, we have improved maintenance reachability, illustrated by the reduced number of maintenance misalignments from the numerical example in §3.3.

This example demonstrates an important property about the problem situation. A more balanced allocation of maintenance opportunities can produce an overall reduction in the expected number of aircraft requiring additional, disruptive LOF swaps. In effect, each additional maintenance opportunity has a smaller probability of being needed and thus provides incrementally less value. Therefore, for two stations with the same number of outgoing lines, the improvement of going from \(n-1\) to \(n\) at one station outweighs the loss of going from \(n+1\) to \(n\) at the other.

If we have \(m\) stations in the flight network with \(n\) flights ending the day in a maintenance station, then maintenance reachability is optimized by assigning exactly \(\frac{n}{m}\) maintenance opportunities to each station. This optimal allocation is formally proven in Theorem (6.1) and the subsequent Corollary (6.2) in the appendix. It should be noted that this result only holds in the case where each station features the same number of outgoing lines, a scenario unlikely to be true for any major airline.

Of course not all stations will have the same number of total outgoing LOFs, meaning that an equal allocation will most likely not be optimal. The goal of our research is to determine the optimal allocation of maintenance opportunities in such cases. This re-allocation is performed by identifying a set of line-splices, each of which re-distributes a maintenance opportunity from one station to another, such that the optimal balance is achieved.

4 Optimization Model

We now present an optimization model to improve maintenance reachability. This approach takes an existing flight plan, designed to ensure maintenance feasibility, provide desirable through-flight connections and keep crew with aircraft. We then identify limited changes that improve maintenance reachability without major deviations from this original plan. First, in §4.1, we present a mathematical model that solves a relaxation of the maintenance lines-of-flight assignment problem, providing a lower-bound on the optimal solution. That is, we solve a simplified optimization problem to determine the band of the optimal solution space. In §4.2 we provide an overview as to how we determine line-splicing opportunities. In §4.3, we present a more restrictive version of the model from §4.1 in which we ensure that viable line-splicing opportunities exist in order to achieve the reconfiguration of maintenance opportunities. Finally, §4.4 features a second-stage optimization model that achieves the maximum amount of maintenance reachability, while minimizing the number of changes to the flight plan.

4.1 MLOF Assignment Problem

We begin by finding a lower bound on the expected number of maintenance misalignments. We do so by optimizing the distribution of maintenance opportunities, without regard for whether this distribution can actually be feasibly achieved given the existing schedule.

We do so by making the decision to assign a particular number of maintenance opportunities to each station in the flight network. That is, we minimize the overall expected number of maintenance misalignments
across all stations by assigning a number of MLOFs to each station. The number of MLOFs assigned to each station is constrained by the total (finite) number of MLOFs that exist in a single day-of-operations.

Sets

\[ S \]  
Set of stations in the flight network.

Parameters

\[ p^n_s \]  
Expected number of maintenance misalignments at station \( s \) given \( n \) MLOFs, \( \forall s \in S, \forall n \in \{1 \ldots T\} \). [The derivation of this parameter was detailed in §3.3].

\[ \tau \]  
Total number of flights that terminate at a maintenance station at the end of a day.

Variables

\[ x^n_s \]  
Binary variable that is 1 if station \( s \) has \( n \) MLOFs assigned and 0 otherwise, \( \forall s \in S, \forall n \in \{1 \ldots T\} \). For example, \( x^5_1 \) = 1 refers to station 1 having 5 MLOFs assigned.

\[ q_s \]  
Integer variable that represents the number of MLOFs assigned to station \( s \). For example, \( q_1 = 5 \) refers to station 1 having 5 MLOFs assigned.

Objective:

\[
\min \sum_{s \in S} \sum_{n=0}^\tau p^n_s x^n_s \tag{2}
\]

Subject to the following constraints:

\[
\sum_{n=0}^\tau x^n_s = 1 \quad \forall s \in S \tag{3}
\]

\[
\sum_{n=0}^\tau nx^n_s = q_s \quad \forall s \in S \tag{4}
\]

\[
\sum_{s \in S} q_s = \tau \tag{5}
\]

\[
x^n_s \in \{0, 1\} \quad \forall n \in \{1 \ldots \tau\}, \forall s \in S \tag{6}
\]

\[
q_s \geq 0 \quad \forall s \in S \tag{7}
\]

In the above formulation, the objective, seen in Equation (2), minimizes the expected number of maintenance misalignments across all stations in the flight network. Constraint (3) ensures that only one numeric value of maintenance opportunities is assigned to a particular station. Constraint (4) provides a link between the number of maintenance opportunities assigned to station \( s \) and the numeric equivalent \( q_s \). Finally, constraint (5) accounts for all maintenance opportunities available for assignment in the network.

4.2 Determining Splice Opportunities

The previous section provided a lower-bound formulation on the allocation of maintenance opportunities to stations, based on the total number of outgoing lines-of-flight. This is done without considering the feasibility of such an allocation. In this section, we identify an optimal allocation based on modifying the existing schedule.

We begin by finding line-splice opportunities. That is, given two lines exiting from differing stations, we determine if these lines can be spliced, such that at the point of interchange, the aircraft routing may be changed, if so desired by the optimization model. A re-assignment of aircraft at a line-splice opportunity effectively re-distributes the maintenance opportunity from the line that ends in a maintenance opportunity to the one that does not.
In this model, we restrict ourselves to simple line-splices. That is, at any point, only two lines are considered candidates for splicing. [In §6, we discuss more complex line-splices]. In order for a line-splice to occur, several conditions must be met. These conditions include:

- **Space**: The lines must intersect at a common station.
- **Time**: The lines must intersect at this station at the same time.
- **Aircraft**: The aircraft type must match (assuming crews are not interchangeable).
- **Benefit**: A splice is only worth considering if one of the two lines gains a maintenance opportunity while the other loses one. (Splicing two MLOF's or two non-MLOF's provides no operational benefit).
- **Line-Locks**: Certain connections within a line may be “locked”, e.g. they represent a high-value one-stop itinerary that should not be altered.

![Overlap Diagram illustrating Swap Opportunities](image)

Figure 6: Overlap Diagram illustrating Swap Opportunities

It is the overlap in the above-mentioned factors, and as illustrated in Figure (6), that provide a splice-opportunity during the pre-planning process. Thus, our solution approach is now two-fold. First, we determine all splice-opportunities that exist in a schedule. This information then serves as input to the optimization model (presented next) to determine which splicing opportunities should be implemented to minimize the expected number of maintenance misalignments.

### 4.3 Restricted MLOF Assignment Problem

Given a set of candidate line-splice opportunities, derived from the criteria as introduced in the previous section, we now pose an optimization model that selects the optimal set of line-splices to perform so as to minimize the total number of expected maintenance misalignments. While the objective function remains the same as in the previous model, we now account for a line-splice that takes place as part of the optimization process. In essence, we attempt to match the previous model, except we now limit the decisions to include only valid line-splices.
Sets

\( P \)  
Set of all possible line-splices.

\( L \)  
Set of all lines-of-flight in the flight network.

\( S \)  
Number of stations in the system.

Parameters

\( m_s \)  
Number of MLOFs associated with station \( s \) in the original schedule, \( \forall s \in S \)

\( p^n_s \)  
Expected number of maintenance misalignments at station \( s \) given \( n \) MLOFs, \( \forall s \in S, \forall n \in \{1 \ldots T\} \). [The derivation of this parameter is detailed in §5].

\( \delta^+_{sp} \)  
Binary parameter that is 1 if splice \( p \) increases the number of maintenance opportunities at station \( s \) by 1 and 0 otherwise, \( \forall s \in S, \forall p \in P \)

\( \delta^-_{sp} \)  
Binary parameter that is 1 if splice \( p \) decreases the number of maintenance opportunities at station \( s \) by 1 and 0 otherwise, \( \forall s \in S, \forall p \in P \)

\( \omega_{pl} \)  
Binary parameter that is 1 if line \( l \) is part of splice \( p \), and 0 otherwise, \( \forall p \in P, \forall l \in L \)

\( \tau \)  
Total number of flights that terminate at a maintenance station at the end of a day.

Variables

\( x^n_s \)  
Binary variable that is 1 if station \( s \) has \( n \) MLOFs assigned and 0 otherwise, \( \forall s \in S, \forall n \in \{1 \ldots T\} \).

\( y_p \)  
Binary variable that is 1 if splice \( p \) is performed and 0 otherwise, \( \forall p \in P \)

Objective:

\[
\min \sum_{s \in S} \sum_{n=0}^{\tau} p^n_s x^n_s \tag{8}
\]

Subject to the following constraints:

\[
\sum_{n=0}^{\tau} x^n_s = 1 \quad \forall s \in S \tag{9}
\]

\[
\sum_{n=0}^{\tau} nx^n_s = m_s + \sum_{p \in P} \delta^+_{sp} y_p - \sum_{p \in P} \delta^-_{sp} y_p \quad \forall s \in S \tag{10}
\]

\[
\sum_{p \in P} \omega_{pl} y_p \leq 1 \quad \forall l \in L \tag{11}
\]

\[
x^n_s \in \{0, 1\} \quad \forall n \in \{1 \ldots \tau\}, \forall s \in S \tag{12}
\]

\[
y_p \in \{0, 1\} \quad \forall p \in P \tag{13}
\]

As before, the objective as seen in Equation (8) is to minimize the total number of expected maintenance misalignments. Constraint (9) ensures that only one number of maintenance opportunities is assigned to each station. Constraint (10) performs the numerical assignment based on the splice choices \( (y_p) \) made. That is, for every splice that adds an MLOF to station \( s \), we increase the total count by 1. On the contrary, when a splice is made that removes an MLOF from station \( s \), we decrease the total number of MLOFs for station \( s \) by 1. Finally, constraint (11) requires that all lines are spliced at most once.

4.4 Minimization of Schedule Changes

There may be multiple equivalent solutions to the mathematical model presented in §4.3. For example, suppose we have two stations, BOS and SEA, both with two lines-of-flight. For each station, one of the LOFs ends in a maintenance station while the other does not. Further suppose that both LOFs from BOS can splice with both LOFs from SEA. The optimization approach presented thus far could splice the MLOF from BOS with the LOF from SEA and the LOF from BOS with the MLOF from SEA. The net-effect of
these splices is zero with respect to maintenance reachability, unnecessarily changing the original planned LOFs.

To minimize the impact in terms of the number of changes to the schedule, we formulate a second-stage optimization model. This optimization model minimizes the total number of line-splices required to achieve the objective function value as derived during the first-stage optimization problem presented in §4.3. We denote the objective function value of the previous optimization model by $C$, an input parameter to this model.

Objective:

$$\min \sum_{p \in P} y_p \quad (14)$$

Subject to the following constraints:

$$\sum_{n=0}^{T} x^n_s = 1 \quad \forall s \in S \quad (15)$$

$$\sum_{s=1}^{S} \sum_{n=0}^{T} p^n_s x^n_s = C \quad (16)$$

$$\sum_{n=0}^{T} n x^n_s = m_s + \sum_{p \in P} \delta^{sp}_{p} y_p - \sum_{p \in P} \delta^{sp}_{p} y_p \quad \forall s \in S \quad (17)$$

$$x^n_s \in \{0, 1\} \quad \forall n \in \{1 \ldots T\}, \forall s \in S \quad (18)$$

$$y_p \in \{0, 1\} \quad \forall p \in P \quad (19)$$

In the above formulation, objective (14) seeks to minimize the total number of line-splices that are made. We are re-using the notation from the previous formulation, such that $y_p$ is a $\{0, 1\}$ variable allowing for a summation of the total number of splices performed. Constraints (15) ensure that we still only assign a single number of maintenance opportunities to each station. Constraint (16) requires that our new assignment must yield the same objective value ($C$) as the value obtained during the previous optimization step. Finally, constraints (17) perform the required accounting of maintenance stations, as in the first-stage optimization model.

5 Computational Results

To validate the optimization model introduced in §4, we provide results from computational experiments that: demonstrate the tractability of our approach, provide a detailed summary of the quality of the solution and its impacts, and assess the sensitivity of the model to the probabilistic objective coefficients.

To do so, we first use the approximation of the expected number of day-7 aircraft at a station requiring maintenance at the end of the day-of-operations, as developed in §3.3. We then refine this approximation using carrier-based data, showing the impact of more precise coefficients on the overall solution quality. Finally, we conduct a sensitivity analysis to determine how much impact errors in the objective coefficients have on the quality of the solution that is found.

The input data for our model was sourced from a major U.S. carrier. The carrier provided two weeks’ worth of tail assignments from which LOFs were built with maintenance and non-maintenance stations indicated. Additional details regarding the input plan can be found in Table (3).

Using the criteria provided in §4.2, Table (4) illustrates the number of line-splicing opportunities in one-day worth of data. It should be noted that the last two rows of this table are indicative of the size of the problem that must be solved by the optimization models.
Number of Flights: 3353
Number of LOFs: 512
Number of Stations: 64
Number of MLOFs: 206

Table 3: Characteristics of Input Data for Optimization Model

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airport</td>
<td>126,507</td>
</tr>
<tr>
<td>Airport and Time</td>
<td>8,815</td>
</tr>
<tr>
<td>Airport, Time and Benefit</td>
<td>4,076</td>
</tr>
<tr>
<td>Airport, Time, Benefit, Aircraft</td>
<td>2,074</td>
</tr>
<tr>
<td>Airport, Time, Benefit, Aircraft and Unlocked</td>
<td>1,728</td>
</tr>
</tbody>
</table>

Table 4: Line-Splicing Opportunity Count for various Conditions

5.1 Experimental Results

We begin by first solving the lower-bound formulation presented in §4.1, followed by the schedule modification model presented in §4.3. Using the seven-day maintenance assumption with \( p_r \) approximated using the binomial distribution presented in Equation (1) with \( p = 1/7 \), we obtain the following results in terms of the expected number of maintenance misalignments.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schedule without Optimization</td>
<td>7.33 misalignments</td>
</tr>
<tr>
<td>Lower-bound Optimization</td>
<td>0.84 misalignments (-88%)</td>
</tr>
<tr>
<td>Optimized Schedule (with ( p_r = 1/7 ))</td>
<td>0.84 misalignments (-88%)</td>
</tr>
<tr>
<td>Runtime</td>
<td>&lt; 5 seconds*</td>
</tr>
</tbody>
</table>

*Computation performed on an Intel Duo-Core 2.2GHz processor with 4GB of memory using CPLEX 11.0 C++ API

Table 5: Experimental results, including runtime using \( (p_r = 1/7) \)

The optimization model is able to reduce the expected number of maintenance misalignments by 88%. This means that when the newly formed LOFs are operated, on average, our plan will incur 0.84 day-7 aircraft (as opposed to 7.33) that will require manual re-routing to end their day in a maintenance station. Note that the restricted optimization model is able to achieve the same objective value as the lower-bound. Further detail as to the implications of these results on the actual aircraft plan and the airline are detailed in §6.

Figure (7(a)) and Figure (7(b)) illustrate the allocation of maintenance opportunities in the original and optimized sets of LOFs. Overall, we notice a general re-balancing of maintenance opportunities, with much lower peaks but nonetheless higher numbers of maintenance opportunities at stations with larger numbers of LOFs. Evident from these figures, the variability in terms of the number of maintenance opportunities is lower than pre-optimization. Correspondingly, Figure (8) shows an overall decrease in the expected number of maintenance misalignments at every station. In this figure, we provide the original expected number of maintenance misalignments (dashed-line), as well as the expected number of maintenance misalignments under the new LOF configurations (solid-line).

5.2 Probability Scenarios

Having first considered the simple case where all aircraft at all stations are equally likely to be day-seven aircraft (with a probability of 1/7), we next conduct experiments using empirical data from a major U.S. carrier to provide insights into whether using more finely-tuned values for the probability of a day-7 aircraft will change the results of the optimization. To evaluate the effectiveness of our optimization model under
In this data set, we evaluate five probability vectors, as provided in Table (6). Each vector provides empirical estimates, from different periods of time, of the probability of a day-7 aircraft appearing at a maintenance station ($m$), a large station ($l$) and a small ($s$) station. Our primary data set consists of 10, 12 and 42 of these respective stations. A “large” station differentiates itself from a “small” station by the number of LOFs that depart each day. Following the convention provided by our partner airline, a large station is defined as one which has at least seven outgoing LOF. It should be noted that such an aggregation of stations by type is not required by the model, which takes as input a potentially-different value of $p_r$ for each station. In addition, Table (6) provides the required number of MLOFs, determined by the number of day-7 aircraft at each station in the network. The remaining capacity refers to the difference between the total MLOFs available and the number of day-7 aircraft at each station.

<table>
<thead>
<tr>
<th>Probability Vector</th>
<th>MX</th>
<th>Large</th>
<th>Small</th>
<th>Required MLOFs</th>
<th>Remaining MLOF Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_0$</td>
<td>0.1429</td>
<td>0.1429</td>
<td>0.1429</td>
<td>73</td>
<td>64.4%</td>
</tr>
<tr>
<td>$P_1$</td>
<td>0.1162</td>
<td>0.1926</td>
<td>0.1689</td>
<td>84</td>
<td>58.7%</td>
</tr>
<tr>
<td>$P_2$</td>
<td>0.0462</td>
<td>0.1824</td>
<td>0.1181</td>
<td>61</td>
<td>69.9%</td>
</tr>
<tr>
<td>$P_3$</td>
<td>0.0829</td>
<td>0.1585</td>
<td>0.1048</td>
<td>58</td>
<td>72.8%</td>
</tr>
<tr>
<td>$P_4$</td>
<td>0.1632</td>
<td>0.1722</td>
<td>0.1571</td>
<td>82</td>
<td>59.8%</td>
</tr>
<tr>
<td>$P_5$</td>
<td>0.0653</td>
<td>0.1216</td>
<td>0.0685</td>
<td>41</td>
<td>80.2%</td>
</tr>
</tbody>
</table>

Table 6: Probability vectors based on Actual U.S. Carrier Data

Using the parameters shown in Table (6) along with the data set described in Table (3), we solve six different instances of the problem, i.e. we generate six optimal sets of LOFs, one for each probability vector. Probability vector $P_0$ denotes the case where all station types have probability $p_r = 1/7$ as assumed in §3.3. We then evaluate how each of these sets of LOFs performs under all of the other probability vectors. In other words, we ask the question: If we assume one probability vector to represent the true parameters and optimize accordingly, but another probability vector is in fact the true data, how will our “optimized” LOFs perform? This question is answered in Table (7). Each row corresponds to a given set of LOFs — the original set of LOFs and the results of optimizing over each of the six probability vectors, as shown given in the left-most column. The remainder of each row then gives the objective value for that set of LOFs as evaluated under each of the six probability vectors. For example, if probability vector $P_1$ is used to optimize the LOFs but then $P_2$ is realized, the expected number of maintenance misalignments will be 0.45.
In Table (7), the optimal values are highlighted for each set of LOFs and its corresponding probability vector. Of course, by definition, this value is the lowest value in each column. These values illustrate the impact of the optimization model under perfect information with each optimized solution offering a significant improvement in terms of maintenance reachability. In addition, the remainder of the columns show that even without perfect information, the optimization model is able to reduce the total number of maintenance misalignments relative to the original schedule. This improvement is due the relationship between the total number of outgoing LOFs and MLOFs available for assignment. Irrespective of the day-7 probability, stations with a higher number of outgoing LOFs generally receive a greater share of the available MLOFs.

Furthermore, we graphically compare the effects of the various probability scenarios on the final MLOF assignment in Figure (9). Even under uncertain probabilities, the optimization changes the maintenance opportunity assignment typically by +1 or -1 MLOF per station. As noted, regardless of which probability scenario is used, the impact of the optimization is noticeable when compared to the original schedule; however, between probability scenarios, the change to the final maintenance opportunities assigned to each station is minimal.

As noted in Table (8), under various probability scenarios, the impact of the optimization model is still significant. That is, even under imperfect information regarding the location of day-7 aircraft, the link-splices suggested by the optimization yield a reduction in the expected maintenance misalignments across
Figure 9: Maintenance opportunity assignment comparing each of the probability scenarios
the flight network. In addition, we note a relationship between the optimization model’s performance in terms of the improvement as noted in Table (8) and the available MLOF capacity in Table (6). As the available capacity of additional MLOFs increases (MLOFs that are not required by day-7 aircraft), so does the possible improvement. We attribute this result to the impact of assigning available MLOF capacity to stations that reap the greatest incremental benefit from such an additional MLOF.

### 5.3 Sensitivity of Expectation Coefficients

In the previous section we considered a variety of possible values for the probability vector \( P \). We showed that even if the incorrect value of \( P \) was used to optimize the lines-of-flight, the resulting schedule (when evaluated under the true value of \( P \)) yields higher maintenance reachability than the original schedule. In this section, we further explore the relationship between \( P \), the “optimized” LOFs, and the realized maintenance reachability. We show that in most cases, even with large deviations from the actual probability, the “optimized” LOFs still gain relative to the original LOFs with respect to maintenance reachability.

As introduced in the previous section, we divide stations into three different types: maintenance (\( m \)), large (\( l \)) and small (\( s \)). We use a probability vector \((p_m,p_l,p_s)\) that indicates the probability that an aircraft at each of the respective station types is a day-7 aircraft. In this analysis, we vary the probability of a day-7 aircraft from 0.04 up to 0.2 incrementing by 0.04 for each of the station types, resulting in a total of 125 scenarios. We optimize with respect to each of these probability vectors and then evaluate each of these 125 optimized plans relative to the other 124 probability vectors. We quantify the results from this analysis next.

To do so, we first define the notion of perfect information. By this, we consider the case where the probability vector under which the LOFs were optimized is in fact the realized probability vector. Clearly, this is the best case scenario, whereas the worst case scenario is when the LOFs are optimized relative to a probability vector that is very different from the realized probability vector. For the case of perfect information, in all 125 cases, the optimized LOFs performed strictly better than the original schedule. Table (9) shows the average improvement across all 125 probability vectors.

<table>
<thead>
<tr>
<th>Probability Vectors</th>
<th>( P_0 )</th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( P_3 )</th>
<th>( P_4 )</th>
<th>( P_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>7.33</td>
<td>8.98</td>
<td>5.64</td>
<td>5.01</td>
<td>8.69</td>
<td>3.02</td>
</tr>
<tr>
<td>Worst-Case Scenario</td>
<td>3.17</td>
<td>1.93</td>
<td>0.95</td>
<td>0.62</td>
<td>4.9</td>
<td>0.24</td>
</tr>
<tr>
<td>Best-Case Scenario</td>
<td>0.84</td>
<td>1.10</td>
<td>0.31</td>
<td>0.34</td>
<td>1.35</td>
<td>0.13</td>
</tr>
<tr>
<td>Max Improvement</td>
<td>6.49</td>
<td>7.88</td>
<td>5.33</td>
<td>4.67</td>
<td>7.34</td>
<td>2.89</td>
</tr>
<tr>
<td></td>
<td>88.5%</td>
<td>87.8%</td>
<td>94.5%</td>
<td>93.2%</td>
<td>84.5%</td>
<td>95.7%</td>
</tr>
<tr>
<td>Min Improvement</td>
<td>4.16</td>
<td>7.05</td>
<td>4.69</td>
<td>4.40</td>
<td>3.79</td>
<td>2.78</td>
</tr>
<tr>
<td></td>
<td>56.8%</td>
<td>78.5%</td>
<td>83.2%</td>
<td>87.6%</td>
<td>43.61%</td>
<td>92.1%</td>
</tr>
</tbody>
</table>

Table 8: Objective function value (maintenance misalignments) improvement

As alluded to before, our optimization model also performs well in situations where the LOFs are optimized under imperfect information, i.e. relative to the wrong probability vector. Consider an arbitrary probability vector. Suppose that we optimize relative to this probability vector and then evaluate this set of LOFs under all other possible vectors. If we take the maximum of each of these objective function values and this maximum is lower than the original plan, then it must be the case that even if the probabilities of a
day-7 aircraft are unknown, the optimization still outperforms the original plan. In our analysis, there are 71 out of 125 cases where the optimized LOFs out-perform the original LOFs, independent of what probability vector is actually realized. That is, if the input data is erroneous and thus deviates far from the actual day-7 aircraft probability, the optimization model presented here still outperforms the original assignment, illustrating the stability of our approach.

In the remaining 54 scenarios the optimized LOFs are not guaranteed to perform as well as the original schedule under every scenario, but only when the optimized plan is based on a probability vector that deviates by least 0.66 in total error from the actual realized probability vector is the optimized schedule worse than the original. Even in these 54 scenarios, the optimized LOFs out-perform the original LOFs on average.

6 Impact & Conclusions

Aircraft routings are designed to meet strict FAA mandates for aircraft maintenance intervals. As emphasized in this paper, on any given day, these plans are routinely changed to mitigate unforeseen events, such as delay propagation, which disrupt airline schedules. To evaluate these day-of-operation changes and their impact on maintenance routing, we provide a metric, known as maintenance reachability. This metric captures a schedule’s ability to satisfy maintenance constraints. More specifically, for each station in the flight network, maintenance reachability quantifies the ability of an aircraft to reach a maintenance opportunity on its last day-of-operation.

When maximizing maintenance reachability, the goal is to minimize the expected number of aircraft that are unable to reach a maintenance station on their seventh day of operation using existing lines-of-flight. As we have shown, this is done by correlating the number of maintenance opportunities with the total number of lines-of-flight for each station. In special cases, where each station has the same number of LOFs, the optimal solution is one where each station has the same number of maintenance opportunities. However, since airlines rarely have the same number of LOFs leaving each station, an optimization model is required to perform the assignment of maintenance opportunities to stations, so as to minimize the expected number of day-7 aircraft without a maintenance opportunity.

The optimization model presented in this paper provides the necessary network balance of maintenance opportunities to maximize maintenance reachability. That is, we strategically splice lines-of-flight to change the allocation of maintenance opportunities from stations with less need to stations with greater need. Using a sequential optimization approach, we are able to show significant improvements with respect to maintenance reachability by making no-cost changes to the flight plan. As a result, the new schedule is more robust to unexpected changes that may occur on the day-of-operation, reducing the need for costly interventions of manual re-routing aircraft to ensure maintenance feasibility. While our mathematical model depends on probability parameter estimates, we show through a sensitivity analysis that those parameter estimates can often deviate from the actual probability while still producing an improved schedule.

6.1 Future Work

To develop this model further, several avenues of extension exist.

6.1.1 Maintenance Capacity Constraints

While the optimization model presented in this paper does assign maintenance opportunities to stations, it does so without constraints on the actual maintenance station. Since maintenance stations have a finite capacity of the number of aircraft that can be serviced on any given evening, a further extension of this model could incorporate a maintenance balancing decision when making maintenance opportunity assignments. More specifically, the maintenance opportunities and their respective maintenance station should be balanced across each of the stations. Such an assignment would allow for a more equal work-balance in terms of the number of aircraft that will actually require maintenance at the end of a day-of-operations. Alternatively,
capacity constraints could be assigned to each maintenance station, restricting the number of maintenance tasks at each facility in accordance with its capabilities.

6.1.2 Complex Line-Splicing

As mentioned in §1, this model was developed with input from a major U.S. carrier. Within this carrier’s network, we were able to achieve the lower bound on maintenance reachability by employing single line splices. This may not be the case for other carriers’ networks. For example, one can imagine that if a required maintenance opportunity cannot be obtained through single line splices, another level of splicing could provide additional maintenance access. That is, line A is spliced with line B for no gain in maintenance reachability. However, further down the line-of-flight, line B is spliced with line C, creating a maintenance opportunity for the aircraft flying A. Future research would investigate alternative levels of splicing, and the corresponding modification of the optimization model, to achieve higher levels of maintenance reachability.

In addition to more complex line-splicing, existing maintenance robustness can also be evaluated and improved through “like-swaps”. Specifically, having two MLOFs that intersect with an opportunity for an over-the-day swap can enable a carrier to make an operational recovery decision that does not disrupt a day-seven aircraft en route to a maintenance station at the end of the day. Increasing the number of such swap opportunities in the planning process could further improvement maintenance robustness.

6.1.3 Through-Flight Connection Cost

Finally, in our model, we assume that line-splices come at virtually no cost. In fact, there may be a revenue impact associated with breaking an existing through-flight connection, and/or an operating cost associated with changing crew/aircraft pairings. Conversely, new LOFs may create new revenue opportunities (by introducing new through itineraries) and new opportunities for operational savings. Thus, it may be worth extending the existing model to incorporate the costs and benefits of LOF modification.

Acknowledgments: We gratefully acknowledge the invaluable feedback provided by the anonymous reviewers.
References


Appendix

Discrete Convexity Proof

**Theorem 6.1.** The day-6 aircraft expectation coefficients used in the objective function are discrete-convex when the number of LOFs are equal for each station.

**Proof.** To show convexity of the objective function coefficients, a discrete function, we must show that:

\[ f(n-1) + f(n+1) \geq 2f(n) \]  
(20)

We begin with the probability coefficient as seen in equation 21.

\[ f(n) = \sum_{i=n+1}^{L_s} \binom{L_s}{i} \left( \frac{1}{7} \right)^i \left( \frac{6}{7} \right)^{L_s-i} (i-n) \]  
(21)

Replacing the left-hand side of equation 20, we obtain the following expression:

\[ \left( \frac{L_s}{i-1} \right) \cdots \times (1) + \left( \frac{L_s}{i} \right) \cdots \times (2) + \left( \frac{L_s}{i+1} \right) \cdots \times (3) + \cdots + \left( \frac{L_s}{L_s} \right) \cdots \times (L_s - n - 1) \]

Combining terms from the equation above yields equation 22:

\[ \left( \frac{L_s}{i-1} \right) \cdots \times (1) + \left( \frac{L_s}{i} \right) \cdots \times (2) + \left( \frac{L_s}{i+1} \right) \cdots \times (3 + 1) + \cdots + \left( \frac{L_s}{L_s} \right) \cdots \times ((L_s - n + 1) + (L_s - n - 1)) \]  
(22)

Now, replacing the right-hand side of equation 20, we obtain the following expression:

\[ 2 \left[ \left( \frac{L_s}{i} \right) \cdots \times (1) + \left( \frac{L_s}{i+1} \right) \cdots \times (2) + \cdots + \left( \frac{L_s}{L_s} \right) \cdots \times (L_s - n) \right] \]  
(23)

Combining equations 22 and 23, eliminating common terms and re-writing in the form of equation 20, we obtain the following expression

\[ \left( \frac{L_s}{i-1} \right) \left( \frac{1}{7} \right)^{i-1} \left( \frac{6}{7} \right)^{L_s-i-1} \times (1) \geq 0 \]  
(24)

which is trivially true. From this, we have shown that our probability coefficient function is indeed discrete-convex.

Equal MLOF Assignment Proof

**Corollary 6.2.** Given a situation where \( n \) MLOFs are to be assigned to \( m \) stations with the fact that \( n \) is evenly divisible by \( m \), then the optimal allocation is one where an equal number of MLOFs is assigned to each station.

**Proof.** We will prove this corollary by contradiction. Given an assignment whereby all MLOFs outlets are equally distributed, then changing this assignment will result in a lower number of day-6 aircraft that will be unable to reach maintenance. In other words, changing a station by increasing its number of MLOFs by one and therefore decreasing another station by one will result in a lower number of expected day-6 aircraft requiring maintenance.

In mathematical terms:

\[ f(n + 1) + f(n - 1) \leq f(n) + f(n) \]  
(25)
This result is clearly false as it contradicts the convexity argument in the previous proof. Therefore, it must be the case that the optimal allocation is one where an equal number of MLOFs are assigned to each station.