Conflict Avoidance: 0-1 linear models for Conflict Detection & Resolution

A. Alonso-Ayuso, L. F. Escudero, P. Olaso and C. Pizarro

Abstract—**The Conflict Avoidance Problem for Air Traffic Flow Management consists of deciding the best strategy for the airborne aircrafts such that it is guaranteed that no conflict takes place, i.e., all aircrafts maintain the minimum safety distance at every moment. Two integer linear optimization models for conflict avoidance between any number of aircrafts in the airspace are proposed, the first one a pure 0–1 LP which avoid conflicts by means of altitude changes, and the second one is a mixed 0–1 LP whose strategy is based on velocity and altitude changes. Several objective functions are established. Due to the small computational time for solving both problems, the approach can be used in real time by using standard optimization software.**

Index Terms—**Air Traffic Flow Management, Conflict Avoidance, Mixed Integer Linear Optimization, Pure 0–1 Linear Optimization, Conflict Detection and Resolution.**

I. INTRODUCTION AND A BRIEF STATE OF THE ART REVIEW

AIR traffic in Europe and the USA has experienced a shocking growth during the last years, and still a 50% increase is expected by 2018. Over this scenario, the objective for Air Traffic Flow Managers is extending the airspace allowing the so called "Free Flight", where the pilots and the airlines are able to decide freely the flight plan, keeping in touch with the air traffic controller. To preserve safety in the air flow, the Conflict Resolution Problem (CR) or Conflict Avoidance Problem (CA) is currently attracting the interest of many air transportation service providers and is been studied deeply from different points of view.

A conflict is an event in which two or more aircrafts are within an unsafe distance from one another. The minimum safety distance is typically 5 nm (nautical miles) of horizontal distance between aircrafts (outside the TRACON - Terminal Radar Approach Control - and 3 nautical miles inside the TRACON) or at least 1000 ft of vertical separation (the current en-route separation standard at lower altitudes). The result is a protected zone (PZ) or volume of airspace surrounding each aircraft that should not be infringed upon by any other vehicle. The PZ could also be defined as a much smaller region (e.g., a sphere 500 ft in diameter) in the case of tactical collision alerting systems, or even in terms of parameters other than distance (e.g., time).

A dynamic trajectory model is required to project the states into the future in order to predict whether a conflict will occur. This projection may be based solely on current state information (e.g., a straight-line extrapolation of the current velocity vector) or may be based on additional, procedural information such as a flight plan. In both situations there is generally some uncertainty in the estimate of the future trajectory. Let a waypoint be a reference point in physical space used for purposes of navigation, it consists of a tupla with latitudinal and longitudinal coordinates, plus altitude with respect to a reference geoid. A flight plan is assumed to consist of a sequence of waypoints in the airspace and a sequence of speeds for moving between them.

Unfortunately, the CA has shown to be a hard problem to solve. To give an idea, the way to represent the actual trajectory of an airplane is by means of a dynamic model, that has to take into account, as an example, the following relations: the speed of the aircraft will depend on the direction, the speed of the wind and the altitude (as higher a plane flies, less air is around it and thus it needs go faster to keep sustentation); acceleration depends too on the speed (e.g. at lower speeds, a plane can reach higher acceleration ratios) and altitude; throughout the flight, the aircraft is losing mass as fuel burns, and this influences the speed and acceleration of the aircraft (and, viceversa, the value of the speed influences the consumption of fuel and thus the mass lose); and so on. Good introductions to flight dynamics modelization can be found on [1], [2], [3]. Finally, CA has to deal with the simultaneous trajectories of (possibly) many aircrafts. Moreover, we must bear in mind that given the intended trajectories, captured in the flight plans, some uncertainty on the actual trajectories of the aircrafts is unavoidable, what makes the Conflict Avoidance Problem harder to solve. Trying to comprise all these issues within a mathematical optimization model would lead to an unmanageable problem (in terms of computational time).

Nevertheless, as we said this problem is been studied deeply and different methods have been proposed by various researchers to address airborne conflict detection and resolution (CDR). These methods have been developed not only for aerospace, but also for ground vehicle, robotics, and maritime applications because the fundamental conflict avoidance issues are similar across transportation modes. What follows is a brief summary of these works.

On a recent paper by EUROCONTROL [4], aimed to specify the required capabilities of Medium-Term Conflict Detection (MTCD) for Air Traffic Management Systems, the MTCD system is required to detect and notify to the controller a probable loss of the required separation between two aircrafts, or an aircraft penetrating a restricted airspace, or an aircraft blocking airspace that might have been used by any other. That paper considers that, although flight data and trajectories are provided to the MTCD, some uncertainty is

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likely on the trajectories. It distinguishes too about tactical and planned trajectories.

Magister in [5] presents two different models: The first one is applied to conflict detection in which a closed interval of conflicting time is calculated but it does not automatically mean that the aircrafts are in conflict, however it is necessary to obtain the final conflict relations. The second one is related to conflict resolution by solving the conflict by descending one of the two aircrafts in conflict. In this resolution, the case in which there are more than two aircrafts is not taken into account. However, Magister in [6] describes the conflict resolution problem in great detail and makes a quantitative analysis of avoidance procedures.

One of the most recent works, due to Jardin (2005) [7], presents some algorithms for strategic conflict detection, based on using a 4-dimensional space and time grid to represent the airspace. This approach to compute conflict detection was previously introduced by Jardin in his PhD thesis (2003) [8], see also [9], where he uses a 3-dimensional grid (two horizontal spatial dimensions and time). Prandini and Hu present [10] a stochastic approximation scheme to estimate the probability that a single aircraft will enter a forbidden area of the airspace within a finite time horizon. A weak approximation of the switching diffusion through a Markov chain is presented. A numerical algorithm is also proposed for computing an estimate of the probability that the aircraft enters an unsafe region of the airspace or comes too close to another aircraft. Hu et al. introduce [11] a model of a two-aircraft encounter with a random field term to address correlation of the wind perturbations to the aircraft motions. Based on this model, they estimate the probability of conflict by using a Markov chain approximation scheme. The same authors study [12] the problem of evaluating if the flight plan assigned to an aircraft is safe. They introduce a kinematic model of the aircraft motion in a three dimensional wind field with spatially correlated random perturbations. They propose as well an iterative algorithm based on a Markov chain approximation scheme.

Kuchar and Yang (2000) [13] present a survey of Conflict Detection and Resolution (CDR) modeling methods with their own classification. The Traffic alert and Collision Avoidance System (TCAS, which is an implementation of the Airborne Collision Avoidance System mandated by the International Civil Aviation Organization) searches through a set of potential climb or descent maneuvers and selects the least-aggressive maneuver that still provides adequate protection; see [14]. Pannequin et al. (2007) [15] present an approach to the problem with severe weather using Nonlinear Model Predictive Control (NMPC). The Mixed Integer Nonlinear Programing (MINLP) model proposed by Christodoulou and Kodaxakis (2006) [16] with linear objective function and nonlinear constraints, only allows velocity changes as maneuvers. Christodoulou and Costoulakis (2004) [17] propose too a MINLP algorithm to solve the conflict problem. Their method allows velocity changes and heading angle control to solve all potential conflicts, only using standard optimization software, but it may require, once again, more computational effort than what it could be affordable. Treleaven (2007) [18] assumes that aircrafts travel at the same altitude and with the same speed, using only horizontal maneuvers for conflict resolution; two, three and multiple intersecting flows are considered.

Obstacle avoidance using the linearized constrained Uninhabited Aerial Vehicle (UAV) dynamic has been modeled by Richards and How in [19]. Centralized Model Predictive Control has been widely developed for constrained systems with many results concerning robustness and has been applied to the co-operative control of multiple vehicles. By augmenting the system with a binary "target state", that indicates whether the target set is reached or not, the approach ends up with a hybrid system at hand. Task completion is then guaranteed by imposing a hard terminal equality constraint on the target state. We have inspired part of our contribution in this research. See also [20].

Pallottino, Feron and Bicchi (2002) [21] propose an approach based on mixed integer optimization to detect and solve conflict problems. Two different maneuvers in two different models are considered, namely, velocity change problem and heading angle change problem in the same plane, i.e., this model does not consider that the aircrafts can climb or descend. These models are based on a geometric approach for studying nonconflict conditions and obtaining the constraints. The first model does not always obtain a solution for the conflict problem, because in the case of head to head conflict, a change of velocity is not sufficient to avoid the conflict. The second model assumes that the velocity is the same for all the aircrafts, and each aircraft can maneuver only once with an instantaneous heading angle deviation that can be positive (left turn), negative (right turn) or null (no deviation). It does not consider returning to the original route, nor it explains how the aircraft, after a maneuver, reaches its destination. Finally, they consider airborne aircrafts are flying on a plane. This could be problematic since their approach relies on direction angles, and projecting the trajectories onto a plane could change noticeably those angles to the point of suppressing real conflicts.

The mixed integer linear model described in [22] is based on a geometric construction and is inspired in the paper from Pallotino et al., whose first model (velocity changing, VC) is extended to permit aircrafts to change both their velocities and altitude levels (altitude changes permit, so, to avoid infeasible situations in the VC problem caused by the velocity bounds, or “head to head” conflict situations). Moreover, all aircrafts will be forced to return to the initial configuration when the conflict situations are avoided, and finally a pathological case of the original VC is avoided. The proposed model assumes the trajectories to be linear over the horizontal plane, and it is intended to be executed repeatedly, each execution within a short time horizon.

Hu has devoted a series of papers and two theses, to this problem. Particularly, [23] can be the closest to the problem we are facing (multi-aircraft 3D CA), and hence the most interesting for us. The problem of designing optimal conflict-free maneuvers (a maneuver is defined to be a continuous and piecewise C^1 map) is studied for multi-aircraft encounters in a three dimensional environment, proposing an algorithm for solving the resultant constrained optimization problem in the two aircraft case. When more than two aircrafts are involved, they consider what they call two-legged maneuvers (a maneuver consisting of two stages, moving at constant velocity and through a straight line in both stages). The original optimization problem is then reduced to a finite dimensional convex optimization problem with linearly approximated conflict-free constraints on the waypoints (with quadratic objective function). Path flightability is taken into account by introducing maximum velocity and turning angle constraints, what can be expressed using second order cone constraints. Therefore, the optimization problem together with the velocity and the simplified turning angle constraints becomes a Second Order Cone Programming (SOCP) problem. However, the assumptions on which the proposal is based on (namely, every aircraft departures and arrives at the same time, all aircrafts move linearly except for one heading angle change in the two-legged maneuver, etc) make it necessary to apply the model recursively, what could make it unaffordable to resort to it in most of the practical cases, due to the nonlinearity of its constraints and objective function. In [24], Hu et al. study the same problem as above, although constrained to the plane, proposing for the general multi-agent case, a randomized convex optimization algorithm to find numerically the optimal multi-legged maneuvers (with an arbitrary number of stages).

Mao et al. (2001) [25] set out geometric constructions to solve the problem, including one by one aircrafts until representing the total number of aircrafts, taking the previous aircrafts as obstacles and making a sequential process. They analyze problems involving aircraft flows, since interactions occurring within a finite set of aircrafts can only have a finite duration. Mao et al. (2005) [26] tackle the problem using instantaneous heading changes as maneuvers between two aircrafts, but this action does not generate cascaded diverging conflict-avoidance behavior in neighboring aircraft. The paper extends the results of Mao et al. [25] in which the maneuvers that are considered are not physically realistic. In [26], the authors prove the closed-loop stability of two intersecting flows of aircrafts under decentralized sequential conflict-resolution schemes, but the cost of the computational calculations is much higher than the cost reported in the original paper, [25].

The main contributions of our paper are as follows:

1) On the contrary to the preceding works, our approach modifies (as less as possible) a given flight plan. It could be the rigid flight plan with fixed beacon points, or the future freely decided flight plan in the context of "Free Flight", as well as the straight-line extrapolation of the current velocity vector, as in Pallottino et al. (2002) [21].
2) Then, we propose first a pure 0-1 linear model that solves the problem in a quite small computing time, changing only flight levels (i.e. forcing the aircrafts to climb or descend in order to avoid conflicts).
3) We also propose a mixed 0-1 linear model that solves the problem changing not only flight levels, but also the velocity of the aircrafts.

The extension of the aerial space to consider in our paper can vary from a simple aerial sector to a wide region of the airspace, since we are not restricting us to linear trajectories nor constant velocities like many of the approaches in the literature. Even if an aircraft had a wide margin to maneuver, in most practical cases it will be enough to be concerned with a reduced one (e.g., 4 or 5 flight levels for aircraft at every route segment will be more than sufficient, even in the first model, where only flight level changes are allowed). With relation to the interval of feasible velocities to consider, it will not only be enough to consider a narrow one, but it will be almost mandatory, in order to guarantee softness in velocity changes from one segment to the next one. We remark that the model will not consider the physical feasibility of accelerating from the slowest speed at a given segment to the fastest one in the next segment.

Our first model, besides been pure 0–1, is stronger than the second one, and will allow us to consider wider aerial zones with higher number of aircrafts and longer time horizon than the second one. Nevertheless, the second model is quite efficient, according to the computational experience that we will report below. On the other hand, the first model has the drawback of only allowing flight level changes, a maneuver that may not be the preferred one for many pilots and airlines since these changes could annoy passengers. Nevertheless, as we said above, in most actual cases it will not be necessary to accumulate many flight level changes and, so, this model will be useful and applicable in most practical situations. Even more, it can be the preferred maneuver, against velocity changes, since the latter may imply more fuel consumption and more risks than the former. Another disadvantage of the first model against the second one is that it considers the velocities of the aircrafts as fixed, so, if an aircraft is forced by the program to ascend instead of maintaining the level, it will need to do so at the same velocity (with respect to the referencial geoid) as it was supposed to fly without ascending. By contrast, the second model could incorporate constraints to consider this issue, we will deal with them in further works. In any case, this is not taken into account in most papers in the literature. To sum up, the models we propose here are both efficient and useful in most real situations, been the second one more comprehensive.

The remainder of the paper is organized as follows: Section 2 technically introduces the problem and the main definitions. Section 3 presents the first model, its preprocessing and its pure 0-1 formulation. Section 4 presents the second model, with some new elements, its preprocessing and its mixed 0–1 formulation. Section 5 reports the computational results for a set of realistic CDR cases. And, finally, section 6 concludes and outlines future work.

II. PROBLEM STATEMENT AND DEFINITIONS

We assume given aircrafts flight plans with the route paths cut in segments (not necessarily equal, but bounded by a max size, as we will see later) and altitude (flight level) and mean velocity through everyone of these segments as well as some bounds to delimitate the possible velocities and altitudes to consider (we will go deeper on this later). Finally, our models suggest some changes (as fewer as possible) in altitude and mean velocity for these flight plans in order to avoid conflicts. Even though the aircrafts have a wide margin to maneuver, in practice it will suffice for us to consider a narrow interval of options (e.g., 3 or 4 flight levels around the originally assigned one, and a narrow interval of velocities too, as we said above).

Let the following notation to be used throughout our first model:

Sets

$\mathcal{F}$, set of aircrafts that potentially can collide.

Θ_f, set of ordered route points for aircraft f, for $f \in \mathcal{F}$.
$\Theta^{-}_{f} = \Theta_f \setminus \{|\Theta_f|\}$ (all the points but the last one)
$\Theta^{'}_{f} = \Theta_f \setminus \{1\}$ (all the points but the first one)
$\Theta^{'-}_{f} = \Theta_f \setminus \{1, |\Theta_f|\}$ (all the points but the first and the last ones)
Note: The points should belong to a so-called *alert zone* (or a zone of potential collision).

$\mathfrak{CF}^f$, set of aircrafts that potentially can collide with aircraft $f \in \mathcal{F}$.

$\mathfrak{CP}^{f,k}$, set of pairs of points $(i,j) \in \Theta_f \times \Theta_k$ where aircrafts f and k can collide $f,k \in \mathcal{F}, k \in \mathfrak{CF}^f$.
Note: The composition of these sets will depend on the model, see later for further explanation.

$\mathfrak{CH}^f_i$, set of allowed flight levels for aircraft f and point $i \in \Theta_f$.

Parameters

t^f_i, planned time while traversing the *ith* waypoint, for aircraft $f \in \mathcal{F}$, $i \in \Theta_f$.

z^f_i, planned altitude while traversing the *ith* waypoint in the route path of aircraft f, for $f \in \mathcal{F}$, $i \in \Theta_f$.

$\underline{z}^f_i, \overline{z}^f_i$, lower and upper bounds that constraint the allowable altitude to traverse the *ith* waypoint in the route path of aircraft f, for $f \in \mathcal{F}$, $i \in \Theta_f$.
Notice that $\mathfrak{CH}^f_i = [\underline{z}^f_i, \overline{z}^f_i]$.

$\underline{V}^f_i, \overline{V}^f_i$, lower and upper bounds that constraint the allowable altitude to traverse the *ith* waypoint in the route path with respect to the altitude arranged for the preceding one, say the *(i-1)th*, for aircraft $f \in \mathcal{F}$, $i \in \Theta^{'}_{f}$.
Note: $\underline{V}^f_i \in \mathcal{Z}^-$, $\overline{V}^f_i \in \mathcal{Z}^+$. See below

$\mathcal{T}_{A}{}^{f,k}_{i,j}$, minimum time needed for aircrafts f and k to reach their next waypoint ($i+1$ and $j+1$, respectively). This parameter can be easily calculated while preprocessing and is used to make sure that no conflict can occur.
$f \in \mathcal{F}, k \in \mathfrak{CF}^f, (i,j) \in \mathfrak{CP}^{f,k}$.

c^f_i, h^f_i, the costs for changing the scheduled flight level for aircraft f and waypoint i, and for climbing or descending, respectively, $\forall f \in \mathcal{F}, i \in \Theta_f$.
Note: (c^f_i is a negative cost so it will be rewarded that $\phi^f_{i,h} = 1$ for $h = z^f_i$, see below)

$(x,y,z)_{f,i}$, Coordinates of the *ith* waypoint in the route of flight f, $\forall f \in \mathcal{F}, i \in \Theta_f$ (see below).

Variables

$\phi^f_{i,h}$, 0-1 variable such that its value is 1 if the altitude (flight level) of aircraft f while traversing the *ith* waypoint in its route path, is h, and otherwise, it is zero $\forall f \in \mathcal{F}$, $i \in \Theta_f$, $h \in \mathfrak{CH}^f_i$. Notice that $\phi^f_{i,h} = 1 \Leftrightarrow$ flight f traverses waypoint i at flight level h.

ν^f_i, 0-1 variable such that its value is 1 if aircraft f changes its flight level at waypoint i to another flight level at the next waypoint and, otherwise, it is zero, for $f \in \mathcal{F}$, $i \in \Theta^{-}_{f}$.
Given aircraft f and waypoint $i \in \Theta^{-}_{f}$, there is only one level $h \in \mathfrak{CH}^f_i$ such that $\phi^f_{i,h} = 1$. Then, if $\phi^f_{i+1,h} = 0$, ν^f_i will be forced to take on value 1.

With these elements, the problem consists of avoiding all conflicts in the *alert zone* (been this one an aerial sector or even the whole airspace) and optimizing an objective function by using a very tight 0-1 linear optimization model and solving it by using standard optimization software.

The output of the problem will be the flight levels at which the aircrafts should fly in order to guarantee that there will be no conflict, been a conflict a situation in which two aircrafts $f,k \in \mathcal{F}, k \in \mathfrak{CF}^f$ traverse, at the same flight level, some points $(i,j) \in \mathfrak{CP}^{f,k}$.

III. MODEL 1

A. Preprocessing

On obtaining the sets $\mathfrak{CF}^f \forall f \in \mathcal{F}, \mathfrak{CP}^{f,k} \forall k \in \mathfrak{CF}^f$: We need to introduce a conflict avoidance constraint for every pair of aircrafts and everywhere a conflict could happen, so if we tried to consider every pair of waypoints (i,j) (where $i \in \Theta_f, j \in \Theta_k$) for every pair of aircrafts $f,k \in \mathcal{F}$ the problem dimensions would become unmanageable. Then, we need to reduce the number of such points to consider, doing some preprocessing. Thus the first data preprocessing will determine the following sets: Aircrafts that potentially can collide with aircraft $f \in \mathcal{F}, \forall f \in \mathcal{F}$, which we have notated as $\mathfrak{CF}^f$, and the set of pairs of points $(i,j) \in \Theta_f \times \Theta_k$ where aircrafts f and k can collide $f,k \in \mathcal{F}, k \in \mathfrak{CF}^f$, which we have notated as $\mathfrak{CP}^{f,k}$.

To do so we first deal with the coordinates of the waypoints given in order to determine when, for a pair of aircrafts $(f,k) \in \mathcal{F} \times \mathcal{F}$, there is a pair of waypoints $(i,j) \in \Theta_f \times \Theta_k$ too close the one to the other, i.e. at a lower distance than the minimum allowed. Here we can remark two observations: the first one is that the coordinates can be given in any system (e.g., longitude, latitude and altitude over a referential geoid) since we will work with them only to calculate distances while preprocessing. The second observation that we need to bear in mind is that we are guaranteeing there are no collisions at a finite number of waypoints for every aircraft, and we must be able to extend this assertion to every single point of those routes, so as a corollary the distance between two given consecutive waypoints (i.e., the length of a segment) must not exceed a given maximum. It is clear that this maximum length must be lower than 5 nm (according to the current en-route separation standard at lower altitudes). Besides, we could increase the radius of the safety disc around the aircrafts in order to guarantee that the desired safety distance is observed at all the intermediate waypoints. It is easy to see that this radius does not need to be increased more than the maximum length of the route segments.

When we have determined all those pairs of waypoints too close the one to the other for two aircrafts f and k, we can reduce even more the number of these pairs by deleting all those ones where both aircrafts do not coincide on time. This last elimination will depend on the model. For this first model, as velocity is fixed, we will delete all the pairs such that $\|t_i^f - t_j^k\| > \mathcal{T}_{A}{}_{i,j}^{f,k}$. Remind that $\mathcal{T}_{A}{}_{i,j}^{f,k}$ is the smallest time needed for aircrafts f and k to reach the next waypoint ($i+1$ and $j+1$, respectively). So, suppose that $t_j^k < t_i^f$, then if the previous inequality holds it happens that when aircraft k reaches waypoint j, aircraft f is at least at waypoint $i+1$, then we do only need to ensure that no conflict takes place for the pairs of waypoints (i', j'), where $i' > i, j' < j$. Finally, $\mathfrak{CF}^f$ will be defined as the set of all aircrafts $k \in \mathcal{F}$ where $\mathfrak{CP}^{f,k} \neq \emptyset$.

On further reducing the dimensions of the problem: To reduce a little more the dimensions of the model, we have implemented a second preprocessing that eliminates all the variables for the points $i \in \Theta_f$ that are not included in any set $\mathfrak{CP}^{f,k}, \forall f \in \mathcal{F}$. And, finally, a third preprocessing divides the problem in subproblems in the following way: We define a partition of the set of aircrafts $\mathcal{F} = \bigcup_{i \in I} \mathcal{F}_i, \mathcal{F}_i \bigcap \mathcal{F}_j = \emptyset, \forall i, j \in I$ where given an aircraft $f \in \mathcal{F}_i$, $\mathfrak{CF}^f \subset \mathcal{F}_i$. Then we solve the problem for every subset $\mathcal{F}_i$.

On ascending or descending flight levels: A hypothesis we are assuming is that at every waypoint of a given route, the aircraft has to fly at a unique flight level, so if it climbs one level from one waypoint to the next one, the ascension must be executed completely while flying through the segment. As an example for a particular segment 2 miles long, the aircraft must be able to ascend or descend 1000 ft. (the standard vertical distance between flight levels) i.e., fly with a climbing angle α such that $tg(\alpha) = 0.09/2 \Leftrightarrow \alpha = 0.045$ radians. But there might be situations in which, for a particular aircraft, it takes more than one "step" to ascend or descend a flight level, (e.g., if we decided to use much more small sized segments), as the aircraft could not be able to ascend 1000 ft. in half a mile, as an example. And there is another point: what if two aircrafts hold the conflict avoidance constraints flying at different levels in the points we are controlling, but they come into a conflict while one is descending and the other one is ascending? We will discuss this issue and show how to solve the problem and how to relax the hypothesis in a future work.

B. Model Formulation

The objective function has two terms. The first one penalizes the number of changes of flight level with respect to the scheduled level, and second one penalizes the number of "jumps" (climbing or descending), for all the aircrafts taken under consideration. In some zones (waypoints) we will be interested in avoiding an aircraft flying at a different flight level than the scheduled one (e.g., at some special beacon waypoints, or where the aircraft is ascending up to cruise flight level). On the contrary, in other zones (waypoints) we will be interested in avoiding "jumpings", that could annoy the passengers and crew.

Remind that c_i^f and h_i^f are the costs for changing the scheduled flight level and for climbing or descending, respectively (notice that c_i^f will be a negative cost so it will be rewarding that $\phi_{i,h}^f = 1$, for $h = z_i^f$).

The model with the composite objective function is as follows:

$$\min \sum_{f \in \mathcal{F}, i \in \Theta_f^-, h = z_i^f} c_i^f \cdot \phi_{i,h}^f + \sum_{f \in \mathcal{F}, i \in \Theta_f} h_i^f \cdot \nu_i^f \tag{1}$$

subject to:

$$\sum_{h \in \mathfrak{CH}_i^f} \phi_{i,h}^f = 1 \quad \forall f \in \mathcal{F}, i \in \Theta_f \tag{2}$$

$$\phi_{i,h}^f \leqslant \sum_{\ell = \underline{V}_i^f}^{\overline{V}_i^f} \phi_{i+1,h+\ell}^f \quad \forall f \in \mathcal{F}, i \in \Theta_f^-, h \in \mathfrak{CH}_i^f \tag{3}$$

$$\phi_{i,h}^f \leqslant \sum_{\ell = \underline{V}_{i-1}^f}^{\overline{V}_{i-1}^f} \phi_{i-1,h-\ell}^f \quad \forall f \in \mathcal{F}, i \in \Theta_f^{'}, h \in \mathfrak{CH}_i^f \tag{4}$$

$$\phi_{i,h}^f - \phi_{i+1,h}^f \leqslant \nu_i^f \quad \forall f \in \mathcal{F}, i \in \Theta_f^-, h \in \mathfrak{CH}_i^f \tag{5}$$

$$\begin{aligned} \phi_{i,h}^f + \phi_{j,h}^k \leqslant 1 \quad & \forall f \in \mathcal{F}, k \in \mathfrak{CF}^f, (i,j) \in \mathfrak{CP}^{f,k} \\ & h \in \mathfrak{CH}_{\Theta_f}^f \cap \mathfrak{CH}_{\Theta_k}^k \end{aligned} \tag{6}$$

$$\phi_{i,h}^f, \nu_i^f \in \{0,1\} \quad \forall f \in \mathcal{F}, i \in \Theta_f, h \in \mathfrak{CH}_i^f \tag{7}$$

Constraints (2) guarantee that all flights traverse every waypoint at only one flight level. Constraints (3)-(4) ensure "soft" flight level changes. Constraints (5) give the number of flight level variations from one waypoint with respect to the next one. Notice that for an aircraft f and a waypoint $i \in \Theta_f^-$, there is only one level $h \in \mathfrak{CH}_i^f$ such that $\phi_{i,h}^f = 1$. Then, if $\phi_{i+1,h}^f = 0$ the corresponding constraint (5) will force $\nu_i^f = 1$. Constraints (6) avoid all conflicts. Notice that it is guaranteed for $\left|t^f + \sum_{m<i} t_m^f - t^k - \sum_{n<j} t_n^k\right| / \mathcal{T}_{A}{}_{i,j}^{f,k} \geqslant 1$ that there is no conflict at that particular instance (f, i, k, j), so the corresponding constraint (6) could be deleted for that situation. Finally, constraints (7) define the integrality character of the 0-1 variables.

Note: We relax the integrality condition of variable ν_i^f (i.e., let $\nu_i^f \in \Re^+$) since it only appears in one set of constraints (5), where it is forced to be $\nu_i^f \geqslant 1$ if and only if $\phi_{i,h}^f = 1$ and $\phi_{i+1,h}^f = 0$, and since it will be penalized in the objective function with a positive cost, it will take the smallest possible value.

IV. Model 2

A. Some new elements

In addition to the previous variables, we need the following ones:

τ_i^f,	variable that represents the time elapsed at the moment of traversing the waypoint *i*, for aircraft $f \in \mathcal{F}$, $i \in \Theta_f$ ($\tau_i^f \in \Re^+$)
$\gamma_{i,j}^{f,k}, \beta_{i,j}^{f,k}$,	0-1 auxiliar variables, used in conflict avoidance constraints, for aircrafts $f \in \mathcal{F}, k \in \mathfrak{CF}^f, (i,j) \in \mathfrak{CP}^{f,k}$. Note: We will only need these variables for aircrafts

and waypoints where a conflict is previously determined as possible.

And we will need additionally the following parameters:

$\underline{t}_i^f, \overline{t}_i^f$, lower and upper bounds that constraint the feasible time that will take aircraft f to traverse the route segment $i \rightarrow (i+1)$, $f \in \mathcal{F}$, $i \in \Theta_f^-$.

B. Model formulation

As in the pure 0-1 model, the first term in the objective function minimizes the number of changes of flight level with respect to the scheduled level, and the second one minimizes the number of "jumps" (climbing or descending), for all the aircrafts taken under consideration. In addition, a third term has been added, which maximizes the number of conflict resolutions by means of velocity changes

The model is as follows:

$$\min \sum_{f \in \mathcal{F}, i \in \Theta_f^-, h = z_i^f} c_i^f \cdot \phi_{i,h}^f + \sum_{f \in \mathcal{F}, i \in \Theta_f} h_i^f \cdot \nu_i^f + \sum_{\forall f \in \mathcal{F}, k \in \mathfrak{CF}^f, (i,j) \in \mathfrak{CP}^{f,k}} s_{i,j}^{f,k} \cdot \gamma_{i,j}^{f,k} \tag{8}$$

subject to:

$$\sum_{h \in \mathfrak{CH}_i^f} \phi_{i,h}^f = 1 \quad \forall f \in \mathcal{F}, i \in \Theta_f \tag{9}$$

$$\phi_{i,h}^f \leqslant \sum_{\ell = \underline{V}_i^f}^{\overline{V}_i^f} \phi_{i+1,h+\ell}^f \quad \forall f \in \mathcal{F}, i \in \Theta_f^-, h \in \mathfrak{CH}_i^f \tag{10}$$

$$\phi_{i,h}^f \leqslant \sum_{\ell = \underline{V}_{i-1}^f}^{\overline{V}_{i-1}^f} \phi_{i-1,h-\ell}^f \quad \forall f \in \mathcal{F}, i \in \Theta_f^{'}, h \in \mathfrak{CH}_i^f \tag{11}$$

$$\phi_{i,h}^f - \phi_{i+1,h}^f \leqslant \nu_i^f \quad \forall f \in \mathcal{F}, i \in \Theta_f^-, h \in \mathfrak{CH}_i^f \tag{12}$$

$$\tau_1^f - t^f \leqslant \mu \quad \forall f \in \mathcal{F} \tag{13}$$

$$t^f - \tau_1^f \leqslant \mu \quad \forall f \in \mathcal{F} \tag{14}$$

$$\tau_{i+1}^f - \tau_i^f \leqslant \overline{t}_i^f \quad \forall f \in \mathcal{F}, i \in \Theta_f^- \tag{15}$$

$$\tau_{i+1}^f - \tau_i^f \geqslant \underline{t}_i^f \quad \forall f \in \mathcal{F}, i \in \Theta_f^- \tag{16}$$

$$\tau_{|\Theta_f|}^f - t_{|\Theta_f|}^f \leqslant \epsilon \quad \forall f \in \mathcal{F} \tag{17}$$

$$t_{|\Theta_f|}^f - \tau_{|\Theta_f|}^f \leqslant \epsilon \quad \forall f \in \mathcal{F} \tag{18}$$

$$\gamma_{i,j}^{f,k} \leqslant \frac{(\tau_i^f - \tau_j^k)}{\mathcal{T}_{A}{}_{i,j}^{f,k}} + \beta_{i,j}^{f,k} \cdot \mathcal{T}_{i,j}^{f,k} \quad \forall f \in \mathcal{F}, k \in \mathfrak{CF}^f, (i,j) \in \mathfrak{CP}^{f,k} \tag{19}$$

$$\gamma_{i,j}^{f,k} \leqslant \frac{(\tau_j^k - \tau_i^f)}{\mathcal{T}_{A}{}_{i,j}^{f,k}} + (1 - \beta_{i,j}^{f,k}) \cdot \mathcal{T}_{i,j}^{f,k} \quad \forall f \in \mathcal{F}, k \in \mathfrak{CF}^f, (i,j) \in \mathfrak{CP}^{f,k} \tag{20}$$

$$\phi_{i,h}^f + \phi_{j,h}^k \leqslant 1 + \gamma_{i,j}^{f,k} \quad \forall f \in \mathcal{F}, k \in \mathfrak{CF}^f, (i,j) \in \mathfrak{CP}^{f,k}, h \in \mathfrak{CH}_{\Theta_f}^f \cap \mathfrak{CH}_{\Theta_k}^k \tag{21}$$

$$\tau_i^f \in \Re^+ \quad \forall f \in \mathcal{F}, i \in \Theta_f^- \tag{22}$$

$$\phi_{i,h}^f, \nu_i^f \in \{0,1\} \quad \forall f \in \mathcal{F}, i \in \Theta_f, h \in \mathfrak{CH}_i^f \tag{23}$$

$$\gamma_{i,j}^{f,k}, \beta_{i,j}^{f,k} \in \{0,1\} \quad \forall f \in \mathcal{F}, k \in \mathfrak{CF}^f, (i,j) \in \mathfrak{CP}^{f,k} \tag{24}$$

where the parameter ϵ in (17) and (18) is a constant, whose value is half the length of a time interval around the scheduled arrival time. Its purpose is to not restrict the aircrafts arrival time to an isolated value (for an alternative to these constraints, see below). The parameter μ in (13) and (14) is half the length of a time interval around the scheduled "departure" time, that will give us a small margin to decide when the aircrafts fly into the conflict zone. The parameter $\mathcal{T}_{i,j}^{f,k}$ in (19) and (20) is the smallest possible value, big enough to guarantee the existence of feasible assignments to variables $\gamma_{i,j}^{f,k}$ and $\beta_{i,j}^{f,k}$, i.e., it must guarantee that the right side of both constraints is positive (because the left side, $\gamma_{i,j}^{f,k} \in \{0,1\}$). The easiest candidate for this parameter would be the total amount of time considered in the problem, but this is clearly improbable (for a detailed explanation on how we have calculated it, see below).

Constraints (9)-(12), are just as (2)-(5) in the first model. Constraints (13) and (14) set the initial time for the aircrafts to come into the conflict zone, as we explained above, while (15) and (16) ensure "soft" speed changes. Constraints (17) and (18), as explained above, force the aircrafts to arrive to their destination point at (almost) their previously assigned time.

Constraints (21) avoid all conflicts, together with the auxiliary constraints (19) and (20), whose purpose is to force the variables $\gamma_{i,j}^{f,k}$ to be zero if aircrafts f and k traverse the waypoints i and j too close (i.e., if the difference between the time at which aircraft f traverses waypoint i and the time at which aircraft k traverses waypoint j is lower than $\mathcal{T}_{A}{}_{i,j}^{f,k}$). In this way, if this difference is greater than $\mathcal{T}_{A}{}_{i,j}^{f,k}$, the variable $\gamma_{i,j}^{f,k}$ will be allowed to take value 1 and so constraints (21) can be relaxed (we will not bother about if both aircrafts fly through both waypoints at the same flight level).

Finally, constraints (22)-(24), define the character of the variables.

Note: As in the first model, we can relax the integrality condition of variable ν_i^f (i.e., let $\nu_i^f \in \Re^+$), as well as we can do with variable $\gamma_{i,j}^{f,k}$, for similar reasons.

C. Preprocessing

On dynamically changing the velocity: As we said above, part of the input to the problem is the assigned time to traverse every route segment. This time will be calculated from the mean velocity (with respect to the referential geoid, not with respect to the wind).

Since the solution to the program can provide new times which would imply a change of the mean velocity in one segment with respect to the previous one, there must be no doubt that this change is physically feasible, i.e., given the wind, the aircraft features and its velocity at a given segment, it is able to accelerate (or decelerate) up to reach the desired mean velocity. In other words, the velocity changes must obey certain soft conditions. Our model contains constraints thought for this purpose, but it all depends on the parameters that specify the bounds for the allowed time to traverse a particular segment, and the model washes its hands on how they are obtained (these are the drawbacks of implementing a linear model for a dynamic aspect of physics, as the movement in the space). But this is not a big problem.

On the one hand, when this model is applied to a reasonably wide time horizon, we might (and should) be conservatives on assigning these bounds, allowing a small margin to maneuver at every segment. Usually this will not be a problem, on the contrary, in most cases a small delay or advance will suffice to avoid conflicts. Moreover, the lower the bounds for changing times are, the stronger the model will be, because we will be able to reduce the $\mathcal{T}_{i,j}^{f,k}$ parameter, as we will see below.

As an example, assigning upper and lower bounds $\bar{t}_i^f = (t_{i+1}^f - t_i^f)+1$ and $\underline{t}_i^f = (t_{i+1}^f - t_i^f)-1$ for $i = 1, \ldots, 10, |\Theta_f| - 10, \ldots, |\Theta_f| - 1$, and $\bar{t}_i^f = \underline{t}_i^f = (t_{i+1}^f - t_i^f)$ for the remaining waypoints $i \in \Theta_f$ can give us better times and at the same time it could be enough to avoid all conflicts avoidable by speed changes in most cases.

On the other hand, we want to point out that most approaches in the literature on this problem consider fixed the velocity of the aircrafts (allowing only an initial decision on which value this constant will take on). Of course, this approach can be applied to our model and besides it would reduce the number of variables. This could be done if we wanted to apply the model within a shorter time horizon (e.g., less than 5 minutes).

On obtaining $\mathcal{T}_{i,j}^{f,k}$: As we said above, the easiest candidate for $\mathcal{T}_{i,j}^{f,k}$ would be the total amount of time considered in the problem, but a tighter candidate can be calculated as follows:

$$\mathcal{T}_{i,j}^{f,k} = \frac{\max\{|\sum_{s<i} \bar{t}_s^f - \sum_{t<j} \underline{t}_t^k|, |\sum_{s<i} \underline{t}_s^f - \sum_{t<j} \bar{t}_t^k|\}}{\mathcal{T}_{A_{i,j}^{f,k}}} + 1$$

Again, we can even reduce $\mathcal{T}_{i,j}^{f,k}$ by taking into account that so far, the aircrafts are forced to arrive to their destination points at their assigned arriving time. Then let us use, instead of $\sum_{s<i} \bar{t}_s^f$, the following formulae: $\min\left\{\sum_{s<i} \bar{t}_s^f, t_{|\Theta_f|}^f - \sum_{s \geqslant i} \underline{t}_s^f\right\}$, and instead of $\sum_{s<i} \underline{t}_s^f$, the following formulae: $\max\left\{\sum_{s<i} \underline{t}_s^f, t_{|\Theta_f|}^f - \sum_{s \geqslant i} \bar{t}_s^f\right\}$. And the same for $\sum_{t<j} \bar{t}_t^k$ and $\sum_{t<j} \underline{t}_t^k$.

On improving the model

The above model just as it is, does not behave well in terms of computational time. The main problem is in constraints (19)-(20), where a relatively high value for $\mathcal{T}_{i,j}^{f,k}$ gives a great chance to the LP to obtain non integer values for $\beta_{i,j}^{f,k}$, allowing $\gamma_{i,j}^{f,k} = 1$ and thus relaxing constraints (21) even if the aircrafts f and k are really too close in time. We have faced several approaches to solve this problem and to reinforce the model.

Adding the following constraints gives good results:

$$\gamma_{i,j}^{f,k} = \gamma_{i+1,j}^{f,k} \quad \forall f \in \mathcal{F}, k \in \mathfrak{CF}^f, (i,j) \in \mathfrak{CP}^{f,k} \tag{25}$$

$$\gamma_{i,j}^{f,k} = \gamma_{i,j+1}^{f,k} \quad \forall f \in \mathcal{F}, k \in \mathfrak{CF}^f, (i,j) \in \mathfrak{CP}^{f,k} \tag{26}$$

$$\gamma_{i,j}^{f,k} = \gamma_{i-1,j}^{f,k} \quad \forall f \in \mathcal{F}, k \in \mathfrak{CF}^f, (i,j) \in \mathfrak{CP}^{f,k} \tag{27}$$

$$\gamma_{i,j}^{f,k} = \gamma_{i,j-1}^{f,k} \quad \forall f \in \mathcal{F}, k \in \mathfrak{CF}^f, (i,j) \in \mathfrak{CP}^{f,k} \tag{28}$$

Constraints (25)-(28) are not mathematically needed by the model, but they tighten it up, obtaining thus a quite better computing time for solving the problem. Actually, they reduce the MIP feasible space, but in a rather reasonable way. To understand their "meaning", let us explain first how the variables $\gamma_{i,j}^{f,k}$ work. If waypoints i and j are too close, but conflict between aircrafts f and k is avoided because the times at which each aircraft traverse the respective waypoint are sufficiently distant, then $\gamma_{i,j}^{f,k} = 1$, and otherwise, it is zero.

Now we can see easily that constraints (25)-(28) force solving a particular group of possible conflicts between two aircrafts (conflicts in consecutive waypoints), by one and only one of the possible maneuvers, e.g., solve them all by changing flight level.

Other approaches could be followed and we will deal with them on further works. However, these constraints have proved to be quite efficient in all the instances we have experimented with.

V. COMPUTATIONAL EXPERIENCE

We report the results of the computational experience obtained while optimizing the pure 0-1 model (1)-(7) and the mixed 0-1 model (8)-(24). The models have been implemented in a C++ experimental code and have been optimized by using CPLEX v12.1. The computations were carried out in a PC Intel Core 2 Duo 4, 2 GHz and 2 Gbytes of RAM. 10 random simulations have been performed for each dimensional case, and the resulting averages are presented. The simulations represent a conflict zone with square shape, the aircrafts come in at any moment within the time horizon (chosen at random throughout a uniform distribution), through any of the four

TABLE I
PROBLEM DIMENSIONS FOR THE PURE 0–1 MODEL

Case	# aircrafts	CZ side	time	atoa	atoap	ptop	ptopp
C0250500300	25	50	300	15	43	36	270
C0250500600	25	50	600	27	70	79	691
C0251000300	25	100	300	8	20	29	177
C0251000600	25	100	600	12	34	40	345
C0252000600	25	200	600	5	12	18	145
C0502000900	50	200	900	22	45	100	908
C0502001800	50	200	1800	20	67	68	1295
C0502003600	50	200	3600	18	77	65	1650
C0504001800	50	400	1800	10	25	50	681
C0504003600	50	400	3600	12	49	52	1301
C0652000900	65	200	900	36	80	138	1338
C0652001800	65	200	1800	37	125	132	2361
C0652003600	65	200	3600	31	124	107	2588
C0654001800	65	400	1800	20	49	89	1208
C0654003600	65	400	3600	18	69	79	1861
C0752000900	75	200	900	49	100	200	1826
C0752001800	75	200	1800	46	168	187	3026
C0752003600	75	200	3600	39	171	122	3398
C0754001800	75	400	1800	26	58	125	1458
C0754003600	75	400	3600	25	98	98	2471
C1004003600	100	400	3600	43	177	173	4433
C1006003600	100	600	3600	30	93	146	2682
C2004001800	200	400	1800	195	463	868	11610
C2004003600	200	400	3600	163	673	693	17665

TABLE II

PROBLEM DIMENSIONS FOR THE MIXED 0–1 MODEL

Case	# aircrafts	CZ side	time	atoa	atoap	ptop	ptopp
C0100500300	10	50	300	2	7	6	48
C0100500600	10	50	600	3	9	8	75
C0101000300	10	100	300	1	3	3	21
C0101000600	10	100	600	1	4	5	51
C0102000600	10	200	600	1	2	4	40
C0200500300	20	50	300	9	27	20	162
C0200500600	20	50	600	17	46	48	406
C0201000300	20	100	300	6	13	19	109
C0201000600	20	100	600	6	15	24	203
C0202000600	20	200	600	4	7	16	104
C0250500300	25	50	300	15	43	36	270
C0250500600	25	50	600	27	70	79	691
C0251000300	25	100	300	8	20	29	177
C0251000600	25	100	600	12	34	40	345
C0252000600	25	200	600	5	12	18	145
C0502000900	50	200	900	22	45	100	908
C0502001800	50	200	1800	20	67	68	1295
C0502003600	50	200	3600	18	77	65	1650
C0504001800	50	400	1800	10	25	50	681
C0504003600	50	400	3600	12	49	52	1301
C0752000900	75	200	900	49	100	200	1826
C0752001800	75	200	1800	46	168	187	3026
C0752003600	75	200	3600	39	171	122	3398
C0754001800	75	400	1800	26	58	125	1458
C0754003600	75	400	3600	25	98	98	2471

sides of the conflict zone (all of them with equal probability) and through any point of that side (we have used here a normal distribution with a standard deviation equal to 1). A random number of flight levels ranges between 1 and 8 per aircraft.

For the pure 0-1 model, the second term in the objective function has been used (i.e. minimizing the number of flight level changes). As far as the mixed 0-1 model is concerned, constraints (25)-(28) have been added, the number of conflict resolutions by changing velocity is maximized and the number of flight level changes is minimized (in particular, the following objective function has been used for this model: $\min \sum_{f\in\mathcal{F},i\in\Theta_f,h\in\mathfrak{CH}_i^f} \nu_i^f + \sum_{\forall f\in\mathcal{F},k\in\mathfrak{CF}^f,(i,j)\in\mathfrak{CP}^{f,k}} (-10)\cdot\gamma_{i,j}^{f,k}$). For both models, the last part of preprocessing, where the set of aircrafts $\mathcal{F}$ is partitioned, has not been carried out.

Tables I and II show the problem dimensions in the 25 cases for the pure 0-1 model and the 18 cases for the mixed 0-1 model testbed that we have experimented with. We can see that the number of aircrafts, conflict zone side length and time horizon have realistic dimensions. In addition, the number of conflicts that took place in the simulation for each case has been measured in 4 different ways. The number of aircraft to aircraft conflicts, namely, the conflicts between aircrafts, not regarding in how many points they take place, counted for the original flight plans; the number of aircraft to aircraft possible conflicts if we modify altitude or velocity for any aircraft; the number of point to point conflicts, namely the number of conflicts between two aircrafts at a single pair of points; and the number of point to point possible conflicts if we modify altitude or velocity for any aircraft. Tables III and IV present the dimensions of the first and second models, and tables V and VI show the computational results.

The headings are as follows: *Case*: Gives the case nomenclature; *# aircrafts*: Number of aircrafts; *CZ side*: Conflict zone side length (in nautical miles); *time*: Time horizon (in seconds); *atoa*: Number of conflicts aircraft to aircraft counted for the original flight plans; *ptop*: Total number of point to point conflicts; *atoap*: Number of aircraft to aircraft possible conflicts counted; *ptopp*: Total number of point to point possible conflicts; *m*: Number of constraints; *n*: Number of variables; *d*: Density of the constraint matrix; m^*: Number of constraints after CPLEX preprocessing; n^*: Number of variables after CPLEX preprocessing; d^*: The constraint matrix density after CPLEX preprocessing; z_{lp}, objective function value of the LP relaxation; z_s, objective function value of the strong relaxation (i.e., the value after appending the cuts identified by CPLEX); z_{ip}, optimal integer solution of the original problem; GAP_{lp} : $\frac{z_{ip}-z_{lp}}{z_{ip}}\%$; GAP_s : $\frac{z_{ip}-z_s}{z_{ip}}\%$ *nn*: Number of CPLEX branch-and-cut nodes; t_{lp}: Time (secs.) to obtain the z_{lp} value; t_s: Time (secs.) to obtain the z_s value; t_{ip}: Time (secs.) to obtain the z_{ip} value; t_t: Total time (secs.); *nc*: Total number of cuts identified and appended by CPLEX.

As far as the pure 0-1 model is concerned, some values have not been included in the tables because in all cases they have shown to be zero, namely z_{lp}, z_s, z_{ip}, GAP_{lp}, GAP_s and *nn*.

We can observe the impressive t_t (total time in seconds) that has been obtained for providing the optimal solution in both the pure 0-1 and the mixed 0-1 models.

VI. CONCLUSIONS AND FUTURE WORK

Two integer linear optimization models for conflict avoidance between any number of aircrafts in the airspace have been proposed for the resolution of the Conflict Avoidance Problem, the first one a pure 0-1 LP which avoid conflicts by means of altitude changes, and the second one a Mixed 0-1 LP whose strategy is based on velocity and altitude changes. The small elapsed time for both problems shows that they can be used in real time, specially for the medium term.

Several extensions to improve both models will be proposed in further works, particularly the possibility of selecting alternative routes, among others (e.g. aircraft climbing or descending to the next flight level in more than one step; relating flight level changes to velocity; and some improvements to the mixed 0–1 model). Besides, heuristics could be applied in order to obtain feasible solutions within smaller computational times (e.g., a Fix and Relax scheme).

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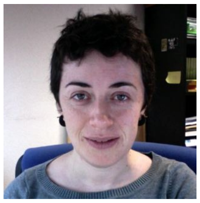

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TABLE III
Model dimensions for the pure 0–1 model

Case	n	m	d	n*	m*	d*
C0250500300	1081	3052	0.0031	537	1493	0.00
C0250500600	1735	5222	0.0020	1264	3758	0.00
C0251000300	844	2265	0.0041	714	1906	0.00
C0251000600	1231	3275	0.0029	909	2385	0.00
C0252000600	787	2007	0.0046	562	1428	0.00
C0502000900	3115	8634	0.0011	2675	7431	0.00
C0502001800	2631	6876	0.0014	2509	6596	0.00
C0502003600	2564	6668	0.0015	2211	5771	0.00
C0504001800	2095	5558	0.0017	2035	5425	0.00
C0504003600	1941	5071	0.0021	1883	4941	0.00
C0652000900	4164	11648	0.0008	4048	11376	0.00
C0652001800	4777	13082	0.0007	4648	12788	0.00
C0652003600	3963	10396	0.0009	3799	10024	0.00
C0654001800	3427	9240	0.0010	3037	8242	0.00
C0654003600	2595	6867	0.0011	2520	6698	0.00
C0752000900	5296	15032	0.0006	5140	14668	0.00
C0752001800	6007	16359	0.0006	5828	15942	0.00
C0752003600	5178	13847	0.0007	5064	13592	0.00
C0754001800	4400	11848	0.0008	4262	11529	0.00
C0754003600	4054	10838	0.0009	3524	9440	0.00
C1004003600	6406	17117	0.0006	6179	16583	0.00
C1006003600	5519	14893	0.0006	5353	14519	0.00
C2004001800	22654	67260	0.0001	22058	65831	0.00
C2004003600	11681	33092	0.0001	11547	32789	0.00

TABLE IV
Model dimensions for the mixed 0–1 model

CASO	n	m	d	n*	m*	d*
C0100500300	404	960	0.0100	313	746	0.00
C0100500600	574	1440	0.0069	424	1064	0.00
C0101000300	255	562	0.0323	208	453	0.00
C0101000600	420	1056	0.0135	319	796	0.00
C0102000600	384	957	0.0110	318	795	0.00
C0200500300	1138	2783	0.0030	839	2085	0.00
C0200500600	2248	6039	0.0015	1508	4215	0.00
C0201000300	1006	2347	0.0040	812	1907	0.00
C0201000600	1556	3971	0.0023	1189	3064	0.00
C0202000600	910	2192	0.0040	699	1665	0.00
C0250500300	1826	4570	0.0018	1361	3475	0.00
C0250500600	3569	9918	0.0010	2412	7012	0.00
C0251000300	1378	3295	0.0025	1054	2572	0.00
C0251000600	2327	5976	0.0015	1697	4381	0.00
C0252000600	1359	3265	0.0026	1084	2600	0.00
C0502000900	6044	15897	0.0005	4539	12094	0.00
C0502001800	7786	21321	0.0004	5289	14073	0.00
C0502003600	9641	27032	0.0003	6472	17451	0.00
C0504001800	4842	12690	0.0007	3614	9302	0.00
C0504003600	7898	21820	0.0004	5450	13947	0.00
C0752000900	10940	29244	0.0003	7891	21485	0.00
C0752001800	17174	47939	0.0002	11780	32576	0.00
C0752003600	19587	55906	0.0002	13376	36855	0.00
C0754001800	10033	26502	0.0003	7465	19513	0.00
C0754003600	14845	41860	0.0002	10268	27504	0.00

TABLE V
Computational results for the pure 0–1 model

Case	t_{lp}	t_s	t_{ip}	t_t	nc
C0250500300	0	0	0	0	0
C0250500600	0	0	0	0	15
C0251000300	0	0	0	0	1
C0251000600	0	0	0	0	19
C0252000600	0	0	0	0	0
C0502000900	0	0	0	0	44
C0502001800	0	0	0	0	72
C0502003600	0	0	0	0	166
C0504001800	0	0	0	0	1
C0504003600	0	0	0	0	0
C0652000900	0	0	0	0	38
C0652001800	0	0	0	0	10
C0652003600	0	0	0	0	5
C0654001800	0	0	0	0	54
C0654003600	0	0	0	0	4
C0752000900	0	1	1	1	80
C0752001800	0	1	0	0	14
C0752003600	0	0	0	0	37
C0754001800	0	0	0	0	16
C0754003600	0	0	0	0	19
C1004003600	0	0	0	0	6
C1006003600	0	0	1	1	80
C2004001800	2	18	15	18	311
C2004003600	0	4	3	4	58

TABLE VI
Computational results for the mixed 0–1 model

CASO	z_{lp}	z_s	z_{ip}	GAP_{lp}	GAP_s	nn	t_{lp}	t_s	t_{ip}	t_t	nc
C0100500300	-442.02	-25.00	-25.00	-	-	0	0	0	0	0	0
C0100500600	-702.86	-44.00	-44.00	-	-	0	0	0	0	0	0
C0101000300	-197.56	-20.00	-20.00	-	-	0	0	0	0	0	0
C0101000600	-486.63	-50.00	-50.00	-	-	0	0	0	0	0	0
C0102000600	-387.03	-31.00	-31.00	-	-	0	0	0	0	0	0
C0200500300	-1490.52	-58.00	-58.00	-	-	0	0	0	0	0	6
C0200500600	-3892.76	-258.00	-258.00	-2080.65%	0.00%	0	0	0	0	0	10
C0201000300	-1005.62	-33.00	-33.00	-	-	0	0	0	0	0	0
C0201000600	-1929.84	-210.00	-210.00	-1405.56%	0.00%	0	0	0	0	0	14
C0202000600	-962.81	-56.00	-56.00	-	-	0	0	0	0	0	0
C0250500300	-2502.83	-94.00	-94.00	-3090.24%	0.00%	0	0	0	0	0	2
C0250500600	-6635.38	-544.00	-544.00	-1132.49%	0.00%	0	0	1	0	0	56
C0251000300	-1601.03	-89.00	-89.00	-3297.22%	0.00%	0	0	0	0	0	0
C0251000600	-3254.44	-437.00	-437.00	-772.58%	0.00%	0	0	0	0	0	22
C0252000600	-1375.16	-169.00	-169.00	-	-	0	0	0	0	0	53
C0502000900	-8628.41	-1244.65	-1241.00	-779.14%	-0.20%	4	0	0	1	1	133
C0502001800	-12670.84	-4914.69	-4821.00	-204.00%	-1.65%	3	0	1	0	0	85
C0502003600	-16327.43	-8069.55	-7578.00	-126.69%	-4.81%	141	0	1	5	5	169
C0504001800	-6555.91	-1993.00	-1993.00	-447.78%	0.00%	0	0	0	0	0	67
C0504003600	-12816.86	-7624.00	-7259.00	-76.33%	-3.96%	117	0	1	2	3	83
C0752000900	-17517.54	-3214.14	-3213.00	-514.68%	-0.04%	0	0	2	2	3	307
C0752001800	-29704.67	-12036.53	-11348.00	-172.10%	-5.80%	435	0	5	24	24	677
C0752003600	-33618.61	-18632.84	-17065.90	-104.55%	-9.41%	482	0	4	30	31	627
C0754001800	-14087.53	-4059.69	-3985.00	-308.12%	-3.63%	3	0	1	1	1	108
C0754003600	-24435.39	-13424.85	-12601.00	-104.32%	-6.22%	173	0	2	9	9	338