A Dynamic Programming Framework for Aircraft Design and Airline Network Design Incorporating Passenger Demand Models

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Abstract

Conceptual design of aircraft and the network routes they serve on are inextricably linked to passenger driven demand. Many factors influence passenger demand for Origin-Destination (OD) pairs that include socio-economic factors and airline operations in terms of frequency of service and number of stops. As an airline allocates new and existing aircraft to its routes, passenger demand responds reflexively based on observations of price, schedule/frequency, size of the aircraft and number of stops between the origin and destination airports. The proposed research will develop a conceptual dynamic framework that builds upon previous works to concurrently design aircraft and the airline's operational network. The major new addition is the incorporation of passenger demand models to this framework. Additionally, the proposed research will explore the reflexive demand condition as part of the demand feedback mechanism. Formulation and solution of a conceptual scenario exhibits an example of the methodology intended for this research

Nomenclature

AR	=	aspect ratio
$C^{A,B,C}_{ij}$	=	fixed cost of aircraft type '*A*,*B*, or *C*' (respectively) flying route (*i*-*j*) ($/trip
W/S	=	wing loading (lb/ft^2)
T/W	=	thrust to weight ratio
X^{P}_{qij}	=	number of passengers with OD ticket 'q' traveling from origin (i) to destination (*j*)
$X^{A,B,C}_{ij}$	=	number of aircraft type '*A*,*B*,or *C*' (respectively) allocated to route (*i*-*j*)
r	=	range (nmi)
y_A	=	passenger capacity of aircraft type 'A'
t_{ij}	=	ticket price for travel from origin (*i*) to destination (*j*)
q	=	passenger ticket itinerary (commodity)
M	=	set of total passenger ticket itineraries
N	=	set of city nodes
$outflow_{qj}$	=	set of passengers departing from city node (*j*) on itinerary (*q*)
$inflow_{qj}$	=	set of passengers departing from city node (*j*) on itinerary (*q*)
$netflow_{qj}$	=	the net flow of passengers travelling on itinerary (q) at city node (*j*)
S_{ij}	=	number of stops between city (*i*) and city (*j*)
j	=	waypoint index

I. Introduction

With increasing air traffic demand both aircraft manufacturers and operators are challenged to provide better services, while meeting their need to capture an optimal market share. . A strategy that incorporates design methodologies, allocation strategies, and models of passenger travel demand could provide solutions that include a description of both a best new aircraft and a best new route structure; including the passenger demand model will find the best aircraft / best route solution that actually affects demand in a way that could maximize profit over a finite time horizon. . Such a paradigm can reduce the "handoff" between aircraft design and airline allocation functions. Currently, aircraft design takes place relatively independent of an airline's allocation. Commercial passenger aircraft design seeks to maximize airplane-specific metrics, such as cost, weight, or another

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performance metric with the indirect goal of maximizing the profit of airlines who purchase the new aircraft. Coupling the design with the allocation can make the design of the aircraft seek to maximize the airline's profit. The airline's operation is naturally subject to the scrutiny of passengers who dictate the amount of demand on various routes. This demand arises from passengers' observations of metrics that include cost (ticket prices) routes, frequency of service and size of aircraft on a route. Naturally, other important factors such as passenger demographics and geographical considerations are also an integral part of the demand.

The notion of recursive demand feedback has long existed in sociological, economic and finance circles within the context of *reflexivity*[1]. The salient point is that supply and demand are inextricably linked entities as opposed to the traditional view that considers them independent factors that achieve some constant equilibrium state. The airlines and the Federal Aviation Administration (FAA) have used various econometric models[2] to quantitatively categorize and analyze key factors that contribute to airline passenger demand. These factors are often expressed in terms of airline operations, local geography and passenger demographic metrics to predict demand trends for city pairs given past information on key factors. These predictions have examined causalities of interactions at various levels of the airline transportation system that range from passenger choices to terminal-specific and even system-wide levels of interactions. The current research work amalgamates aircraft systems design research previously by Mane, Crossley, Nusawadharna[3] and by de Weck and Taylor[4] with the use of econometric models reflected in works by Bhadra[5], and by Wei and Hansen[6] - to achieve deterministic demand feedback. This serves as a framework to incorporate reflexive demand influences in the design of the aircraft and in the routes that the fleet will operate on.

II. Problem Statement and Methods Used

The scope of this research is to simultaneously determine the design of a potential new commercial passenger aircraft, the network route structure and the resulting aircraft allocation that maximizes the profit obtained while transporting passengers to their destinations, while incorporating reflexive demand trends. The approach uses a subspace decomposition method as previously established[3]; this decomposition has shown to make the overall optimization problem computationally tractable. The decomposition method has described aircraft design features that provide better performance for the airline. The monolithic form of the problem that represents a larger optimization problem includes the aircraft sizing variables along with the allocation variables. This would make the cost of flying an aircraft on a given route a function of the sizing variables, resulting in a nonlinear fleet cost function. This monolithic form of the optimization problem would be a mixed integer (allocation variables) non-linear (objective function and constraint functions) problem which is very difficult to solve.

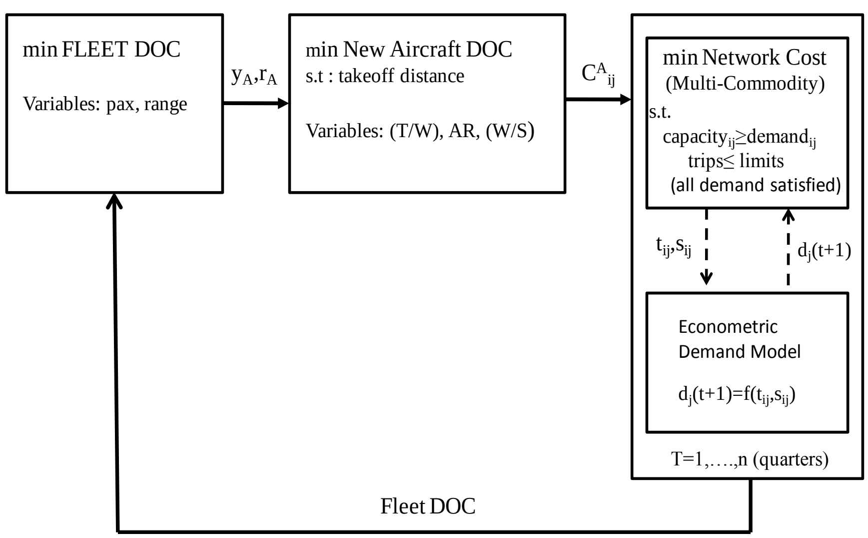


Figure 1: Architecture of overall concurrent design scheme

The monolithic optimization problem is broken into smaller sequences that begins with a systems level optimization problem, proceeds to an aircraft design level optimization problem and then on to the multi-commodity network flow problem. Here, the econometric demand feedback works in conjunction with the minimum cost multi-commodity solution by providing estimated demand for a chosen network route configuration. The overall architecture is as illustrated in Figure 1.

A top-level problem coordinates the entire process. Because both the aircraft design sub-problem and the airline allocation sub-problem need to know the passenger capacity, y_X, and the design range, r_X, of the new aircraft, these are decision variables for this top-level problem. The objective is to maximize the profit of the airline. Because this is a work in progress, the current formulation seeks to minimize direct operating cost of the airline as a surrogate for maximizing profit. . . These are subject to the constraints of what typical interval values of aircraft range and passenger capacity. This is an unconstrainted two variable mixed integer problem (MIP) with the range as a continuous variable and passenger capacity as integer, and can be solved either through enumeration or a branch and bound algorithm. Solutions at this level represent the optimal solution for the aircraft to be designed in terms of passenger range and capacity.

The aircraft sizing sub-problem is a non-linear programming problem; it finds the values of the aircraft sizing variables that minimize the direct operating cost of the aircraft that can perform the design mission defined by the top-level values of payload and range. The resulting aircraft design here provides the cost of operating this aircraft on the routes served by the airline. The NASA developed Flight Optimization System (FLOPS ver. 6.1)**Error! Bookmark not defined.** code is used to size the aircraft and estimate its takeoff gross weight (TOGW) and operating costs for a given passenger capacity, design range, and cruise velocity. In this case, the cost per trip is generated for each aircraft given all the possible ranges it could serve on the network. Trip lengths that are beyond the range of the aircraft are assigned a very high cost per trip coefficient value that serves as a penalty for the proceeding network optimization. The aircraft design variables are aspect ratio (AR), thrust to weight ratio (T/W), and wing loading (W/S). The resulting nonlinear programming problem (NLP) is solved within the FLOPS software package using an array of nonlinear optimization algorithms[7]. Future research will use an surrogate model that can be used in order to reduce computational burden of the cyclic optimizations is through the use of response surfaces that can also be generated with the FLOPS software package.

. The resulting cost coefficients for each route encompassing both the existing ($C^{B,C}_{ij}$) and to be designed aircraft (C^{A}_{ij} - *from the aircraft design sub problem*) are then passed to the network design problem, where a multi-commodity flow problem is modeled using the GAMS software package[8] that utilizes CPLEX solve the integer programming (IP) problem. The airline allocation sub-problem is an integer programming problem that uses a multi-commodity formulation to allocate both the yet-to-be-designed aircraft and the airlines existing fleet to meet demand on the various routes. Via the multi-commodity formulation, the problem will determine if it is more effective to provide direct service between cities in the airline's network or to provide connecting service. The result of this problem is the maximum profit or minimum cost to meet passenger demand, while also satisfying balance and count constraints

The econometric model is a regression-based equation that projects demand on each of the potential itineraries. The model provides passenger demand between two city pairs in terms of number of passengers desiring travel as a function of the ticket price, frequency of service, size of aircraft, and number of stops. (Current results so far present preliminary results that include only ticket price and number of stops. This will be increased to include other pertinent factors.) So, for a given solution to the multi-commodity flow problem, the econometric model represents how potential passengers view the service offered by the airline; the projected demand from the econometric model then becomes the demand for solving the multi-commodity flow problem at the next time step over a given time horizon

A. Concurrent Aircraft Design and Network Design

The network is modeled as a multi-commodity flow problem. In this case, the ‘commodities’ are the origin-destination (OD) pair tickets demanded by passengers. To get to a destination, the ticket holder needs to travel on a particular itinerary. This can simply be thought of as the tickets (commodities) traveling on the routes of the airline network. Modeling of the network design portion as a multi-commodity problem expounds upon assumptions and modeling of network flows in preceding literature[94]. This formulation enables the tracking and minimization of overall cost subject to requirements of each individual ticket itinerary as opposed to the ‘general flow’ of passengers as a whole on the network as otherwise previously done.

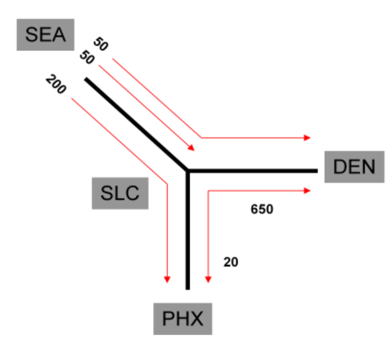


Figure 2: Demand realization for 4 city network

Because this is a work in progress, a very simple illustrative example establishes the approach for concurrent aircraft and network design using the multi-commodity formulation. A hypothetical case of demand between four cities with the corresponding demand itinerary realizations appears in Figure 3. This model is an extension of a similar, sample problem based on an example from Bazaraa and Jarvis[10] to develop and demonstrate the decomposition approach for concurrent aircraft design and airline allocation. Here, this sample problem now includes hypothetical itinerary information rather than simple demands on a hub-and-spoke network. Demand is assumed symmetric with an equal passengers traveling on return trip itineraries (e.g. if 200 people wish to travel between Seattle and Phoenix on the representative day, then 200 people also wish to travel between Phoenix and Seattle). Actual passenger demand is not exactly symmetric, but is reasonably close. This approximation reduces the size of the multi-commodity problem, but does not preclude the approach from addressing non-symmetric demand.

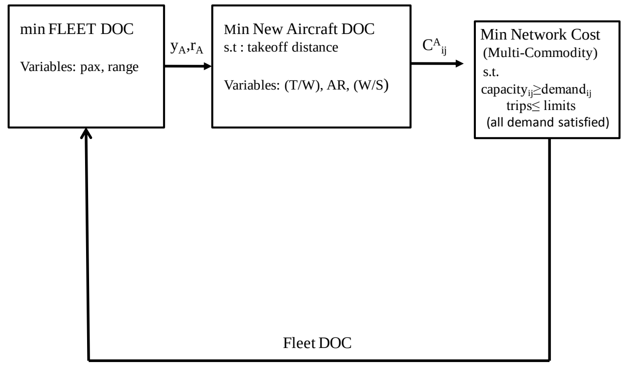


Figure 3: Concurrent Aircraft and Network Design

As in the original example problem, the airline already owns two types of aircraft. In this investigation, these are based upon the Boeing 737-400 and 737-500 with capacities of 140 and 150 passengers, respectively. The airline may also use a third aircraft; this type is the newly designed aircraft. The objective is to minimize the direct operating cost of transporting passengers to their intended destinations.

$$\min \sum_{i}^{n} \sum_{j}^{n} C^A_{i,j} X^A_{i,j} + C^B_{i,j} X^B_{i,j} + C^C_{i,j} X^C_{i,j} \tag{3.1}$$

subject to:

$$\sum_{j}^{n} X^A_{i,j} Y_A + X^B_{i,j} Y_B + X^C_{i,j} Y_C \geq \sum_{q} X^P_{i,j} \tag{3.2}$$

$$\sum_{i}^{n} X^A_{i,j} Y_A + X^B_{i,j} Y_B + X^C_{i,j} Y_C \geq -\sum_{q,i} X^P_{i,j} \tag{3.3}$$

$$\sum_{j}^{n} X^A_{i,j} Y_A + X^B_{i,j} Y_B + X^C_{i,j} Y_C \geq \sum_{q,j} X^P_{i,j} \tag{3.4}$$

$$\sum_{q} \sum_{i} X^P_{q,i,j} - \sum_{q} \sum_{j} X^P_{q,i,j} = netflow_{q,j} \tag{3.5}$$

$$\sum_{i,j}^{n} X^A_{i,j} \leq \text{triplimit}_\text{A} \tag{3.6}$$

$$\sum_{i,j}^{n} X^B_{i,j} \leq \text{triplimit}_\text{B} \tag{3.7}$$

$$\sum_{i,j}^{n} X^C_{i,j} \leq \text{triplimit}_\text{C} \tag{3.8}$$

$$X^P_{q,i,j}, X^A_{i,j}, X^B_{i,j}, X^C_{i,j} \geq 0 (non-negativity) \tag{3.9}$$

The preceding equations (.1-3.9) represent the integer programming (IP) multi-commodity sub problem statement for the network portion of the design. The objective function in equation (3.1) represents the total cost of flights performed on a representative day. Equations (3.2) and (3.3) enforce the condition that the total passenger capacity due to allocation of aircraft on routes sufficiently services the passengers entering and leaving any node point (airport). Hence, nodes with passengers departing for a chosen destination would result in a negative number per ticket itinerary at each node. Equation (3.4) explicitly deals with flow balance of passengers within the network and ensures that it is equal to the net passenger flow at each node. This assumes a perfect balance of passengers within the system, making it a conservative network flow. Nodes with passengers departing on a particular itinerary will result in a positive flow at the departure node and a negative flow number at the destination. A simple example is that if a passenger is traveling on a ticket (commodity, q) from DEN to SEA, then the net flow of that commodity 'q' of 'DENSEA' will be (+50) at the origin Denver and (-50) at the destination Seattle.

The trip number constraints in Equations. 3.6-3.8 are a simplification of the "count" constraints that ensure the airline is not using more aircraft than it owns. This can reflect the maximum number of flights given the speed and/or maintenance constraints for that aircraft. These constraints also reflect a set of balance conditions where a round trip assumption is made which means that if an aircraft is allocated to a route, it will also fly the 'return' trip home. (e.g. an aircraft assigned SLC-DEN, will also fly DEN-SLC). This assumption also simplifies the problem, but does not limit the approach from other aircraft assignment strategies.

Figure 4: Resulting route structure when:

(a) when concurrently optimizing aircraft and network **(b) when optimizing the network only**

A study compared the potential savings that the concurrent optimization of both a yet-to-be-designed aircraft and its network would have over optimizing the network itself without a new aircraft. Figure 4 exhibits the two resulting route configurations. The network optimization without a new aircraft yielded trips solutions that resulted in a directly connected network, where the introduction of a newly designed aircraft facilitated a more serially-connected configuration. This is, of course, a simplified situation where there are no further restrictions on the network configuration beyond the issues of aircraft range and net demand. The cost savings were notably significant with the concurrent aircraft and route yielding a minimum fleet DOC of $63,834 for the representative day compared to the network only optimization that yielded a minimum fleet DOC of $83,997.

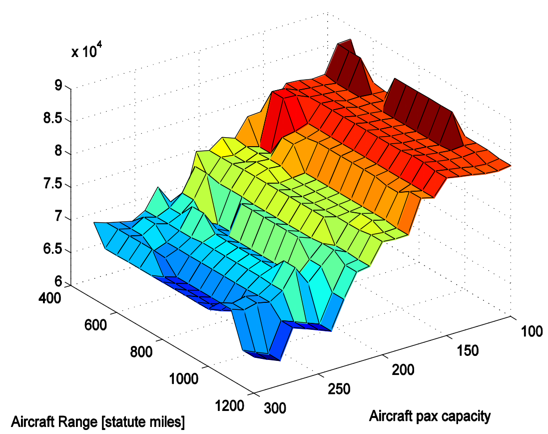


Figure 5: Solution space for concurrent aircraft and network optimization

The top level optimization of the overall fleet direct operating costs can be performed as an unconstrained minimization problem. However, the example problem size is small enough to enable the use of simple enumeration. The solution space is mapped out and lowest cost is then selected as the optimum design point with the corresponding passenger capacity and range. Figure (5) shows the solution space from the enumeration performed for the concurrent aircraft and network optimization problem. The resulting optimum aircraft design for the modified Bazaraa and Jarvis problem corresponds with an aircraft with passenger capacity of 280 passengers and a range of 1200 miles. A comparison of the main aircraft characteristics are exhibited in Table 1 with Aircraft A being the optimal design to be introduced. A worthy note on the observed solution space is that it is clearly multimodal with multiple minima existing throughout it. For a larger class of problem size, the use of a branch and bound or simulated annealing will be more useful in finding the optimal solutions due to the large computational expense.

	New Aircraft (A)	Aircraft B	Aircraft C
Capacity (pax)	280	140	150
Range (nmi)	1200	1000	1300
Takeoff Distance (fixed) (ft)	7780	7780	7780
Takeoff Gross Weight (TOGW) (lbs)	227060	114430	125280
Aspect Ratio (AR)	10.673	11.507	11.101
Thrust to Weight (T/W)	0.291	0.283	0.285
Wing Loading (lb/sqft)	132	125	130
Fuel Weight (lbs)	34969	16561	20887

Table 1: Comparison of aircraft designs

B. Passenger Econometric Demand Modeling

Various methods have been previously used in order to estimate passenger travel between origin-destination pairs[2] . The techniques employed range both in geographical scope and temporal considerations (daily, quarterly, yearly estimations). The FAA utilizes a network flow based Fratar-TAF implementation[11] in order to estimate such traffic flows on a daily basis. Recent research[11] has increased the fidelity of such methods and incorporated more sophisticated methods such as neural networks in order to improve the estimation at the terminal level. Other methods include a more econometric approach that employs regressions in order to statistically determine correlations and causality between passenger travel preferences, demographics and airline operations.

Previous literature has shown success in the use of log-linear demand models[5] to reflect passenger demand preferences for travel in the continental United States given factors such as ticket price, gross domestic product, private income, presence of low cost carriers and aircraft size among examined factors. The literatures cited have shown promising results in modeling demand at both the macro-scale (passenger demand for contiguous regions) and studies even at the airport hub/terminal scale[6]. An added benefit in log-linear models is also the intuitive interpretation of the coefficients that reflect the elasticity/sensitivity of the variable of interest to the regressors. For simplicity and as an illustrative example, a log-linear demand model with two factors of interest was used to serve as an econometric model for the four cities of choice. Demand for the six bidirectional routes served between the four chosen cities was modeled as a series of equations.

$$\begin{Bmatrix} \ln(D_{SEAPHX}) \\ \ln(D_{SEASLC}) \\ \ln(D_{SEADEN}) \\ \ln(D_{SLCDEN}) \\ \ln(D_{SLCPHX}) \\ \ln(D_{PHXDEN}) \end{Bmatrix} = \begin{bmatrix} 1 & x_{SEAPHX} & \ln(S_{SEAPHX}) \\ 1 & x_{SEASLC} & \ln(S_{SEASLC}) \\ 1 & x_{SEADEN} & \ln(S_{SEADEN}) \\ 1 & x_{DENSEA} & \ln(S_{SLCDEN}) \\ 1 & x_{SLCPHX} & \ln(S_{SLCPHX}) \\ 1 & x_{PHXDEN} & \ln(S_{PHXDEN}) \end{bmatrix} \begin{Bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \end{Bmatrix} \tag{5}$$

A separate log-linear equation is used for each route to mitigate some of the effects that heteroskedasticity may have on trying to perform the regression with a single equation to serve all routes – resulting in the above system of equations. The symmetric nature of demand reduces the 12 possible routes to a system of 6 equations as shown in Equation (5). A subscript such as *'DENSEA'* denotes travel both ways between Denver and Seattle.

Additionally, the generated coefficients can be directly interpreted as the route specific demand elasticity with respect to ticket price and number of links. Some key simplifications and assumptions are made for the demand model presented. First, is that only the two most perceived significant factors are chosen – average ticket prices and numbers of links between origin and destination. There are naturally other factors at play such as gross domestic product (GDP), private income (PI), population density and such. The greatly varying distances of travel and demographic variances for each city pair can also give rise to differing preferences towards ticket pricing, travel links and even the possibility of alternatives to air travel. Previous research has also indicated the impact that aircraft size has on both the operations and passenger preference for longer haul flights. Such complexities can be addressed with the use of more sophisticated models and a selection of factors that are apropos to the analysis. The inclusion of regression variables directly related to the attributes of the aircraft can also be used to influence the econometric feedback mechanism. However, for the purposes of simplicity, the above model is used within a limited frame of collected data to highlight the conceptual framework that is introduced.

Data from the Bureau of Transportation Statistics (BTS) DB1B and T100 Market schedules were compiled and used to generate a quarterly data set comprising of average ticket prices and number of links for the years 2002 and 2003. Total passenger travel for itineraries between the four chosen cities were taken from the T100 Market data that publishes such information among other metrics. The average ticket price was determined from the DB1B Market (10%) passenger sampling data that furnishes anonymous individual passenger costs (ticket paid price) on a monthly basis. This was then aggregated quarterly for all airlines serving origin-destination routes between the four cities resulting in 8 observations of data. Explicit information on the number of stops was not furnished. Therefore, the difference between passenger miles flown and actual miles travelled was used to infer the number of stops. It was assumed that there were two flight links/segments for itineraries that reported greater miles flown than actual travel miles. An ordinary least squares regression (OLS) is performed and the resulting statistical properties of the demand model are listed in Table 2. Issues of biasness, consistency and efficiency in the regression are not considered for the OLS regression performed here as only a small sample data set (8 observations) is used for conceptual purposes..

DEN-SEA	*Coeffs.*	*t Stat*	*P-value*	*DEN-SLC*	*Coeffs.*	*t Stat*	P-value
Intercept	4.2475	1.6373	0.1625	Intercept	8.4866	1.0242	0.3527
Avg. fare	-1.7347	-2.3317	0.0671	Avg.fare	-0.3146	-1.9379	0.1104
Avg. links	4.7170	2.2275	0.0764	Avg. links	-2.4471	-0.2954	0.7796
SEA-SLC	*Coeffs.*	t Stat	P-value	*PHX-SLC*	*Coeffs.*	*t Stat*	*P-value*
Intercept	8.2538	3.1054	0.0267	Intercept	22.5719	1.5704	0.1771
Avg. fare	0.4143	0.7301	0.4981	Avg. fare	-2.2302	-1.4489	0.2070
Avg. links	-3.7844	-1.9648	0.1066	Avg. links	-12.6288	-1.0809	0.3291
SEA-PHX	*Coeffs.*	*t Stat*	*P-value*	*DEN-PHX*	*Coeffs.*	*t Stat*	*P-value*
Intercept	4.7079	1.6689	0.1559	Intercept	5.8574	6.8091	0.0010
Avg. fare	0.3594	-0.3834	0.7171	Avg. fare	-0.1827	-0.4598	0.6649
Avg. links	-0.0343	0.03419	0.9740	Avg. links	0.0051	0.4860	0.6475

Table 2: OLS regression coefficients

The *t*-statistic values are used as justifications for inclusion of significant factors and appropriateness of the overall model to be employed. The presented regression results are encouraging in that most of the cities exhibit acceptable t-statistics that have an absolute value over 1 for the generated coefficients. This indicates that the factors are significant in the model. Additionally, the coefficients make intuitive sense in that passengers are generally averse to ticket price increases and number of links. It is also very worthy to note that there are still significant effects that have not been accounted for as the *t*-statistic of the intercept is greatly significant in most of the regressions. However, there are some noted poor regression coefficients that were generated, namely in the case of the *SEA-PHX* and *DEN-PHX* routes that suffer from bad *t*-statistics In the case of the *DEN-PHX* route, it is encouraging that there is an adversity to the price increase but somewhat of an indifference to the average number of links. The strong t-statistic for the error term (intercept) indicates a strong presence of other factors at work as well. In the case of the *SEA-PHX* regression, there overall model is also poor in that although there is an intuitive sense on the adversity to price increases; the strong positive correlation to the average number of links does not seem to bear any intuitive logic to it. For overall consistency of the model to be employed, coefficients of both routes are still incorporated into the econometric model. For actual situations, a better analysis of factors may need to be done to yield more intuitive correlations and causalities for the demand.

Generally, econometric approaches to model demand such as in this four city case within a *supply-demand* framework by means of estimating a single equation has been referred to as the 'identification problem[5]'where variables in demand also exist as endogenous variables in the supply side of the set of resulting simultaneous equations. Here, the objective is however to estimate a single equation on the demand side as given. Previous literatures have used more sophisticated modeling and regression approaches such as Limited Information Maximum Likelihood Estimation (LIML)[5], 3 stage least square (3SLS) and multinomial logit models that deal with an array of possible issues such as heteroskedasticity, multi-collinearity, selectivity biases and choice modeling to name a few.

C. Concurrent Aircraft and Network Design Incorporating Econometric Feedback

The concurrent engineering of the aircraft assets and airline routes represent the 'supply' side of the economic model where airline decisions to optimize routes are modeled through the multi-commodity network flow problem. The regression model on the other hand is the 'demand' side where passenger decision and responses are aggregated over a statistical model - in this case a simple OLS. The feedback loop between the two as exhibited in Figure 6 is the mechanism behind the idea of reflexive demand feedback. Quarterly decisions on the allocation (#links for ticket itineraries and average ticket prices) are looped recursively to the econometric regression to generated new quarter demands.

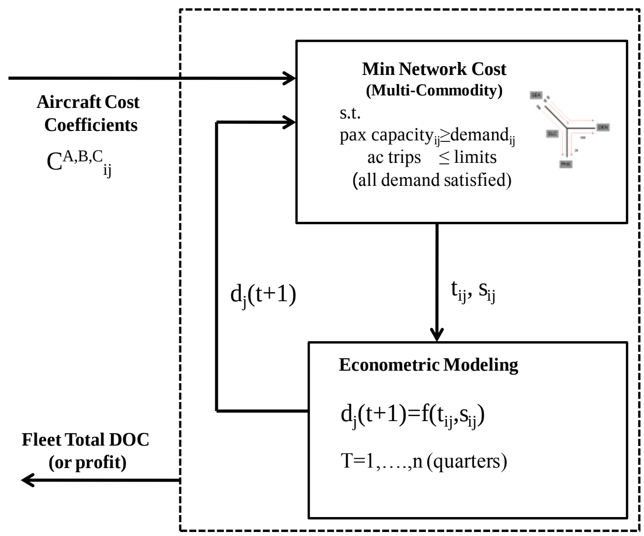


Figure 6: Econometric feedback loop

The objective now is maximum profit given reflexive demand traits within the overall established concurrent engineering framework. The problem statement is incorporate a 'yet to be designed' aircraft within an existing fleet's operations and to predict expected future profit using the econometric regression model. Again, to simplify the problem, it is assumed that a nominal profit of $100.00 per passenger is added on top of the per passenger cost that is calculated using cost coefficients from FLOPS. This is indeed a simplification as additional costs that are not associated with the direct operating costs are not included here. Initial demand for each city pair is set at demand levels (2002 first quarter) as determined from the BTS T100 database[12] for the four cities established. The overall optimization routine was performed with recursive demand being projected over a period of 8 quarters (2 years). Typically, the demand projection should be more reflective of a period consistent with the expected operational life of the aircraft to be designed. However, for the purpose of this simplified example, the projection is truncated to 8 quarters to reduce computation time. From an econometric analysis, the demand elasticity for specified factors changes with time due to aforementioned factors. This would require more sophisticated time series regression techniques and econometric considerations.

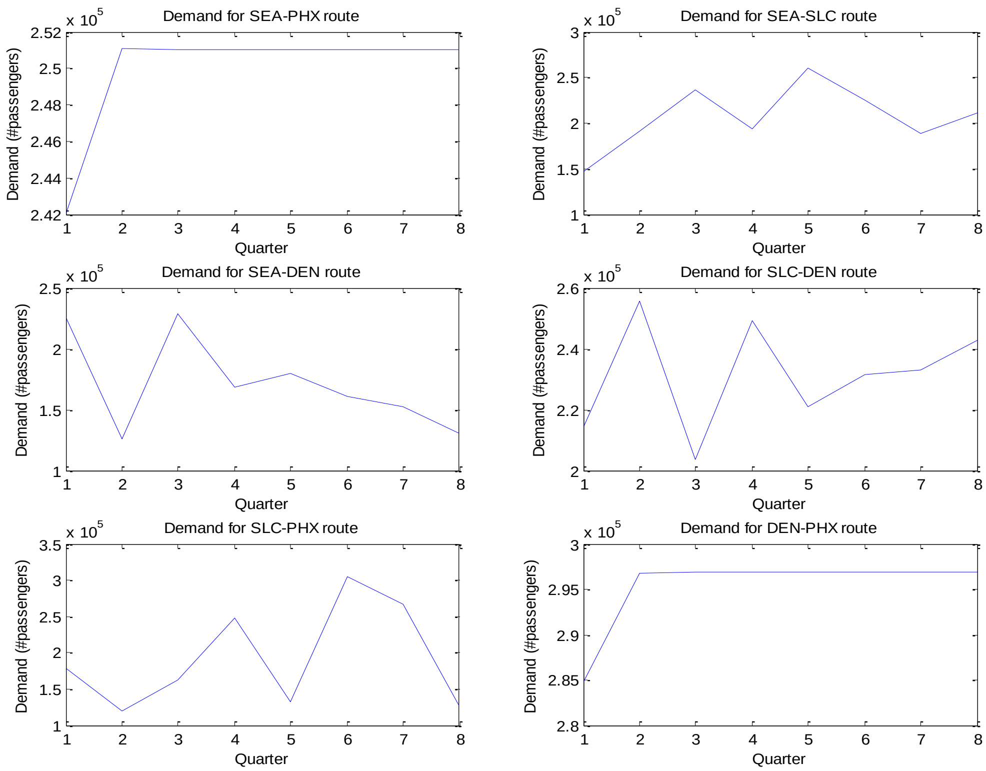


Figure 7: Reflexive demand behavior observed

Error! Reference source not found. shows computational results of the proposed framework where decisions on airline operations for the fleet over the network results in changing demand due to the nature of this feedback mechanism. This is naturally a simplification as there are other factors that directly influence changes in demand, including, economic conditions, seasonality, market competition, jet fuel prices and even geographic considerations such as weather. It can be clearly seen that demand is always in a state of flux as passengers choices change in line with quarterly airline decisions on which routes to service with varying aircraft combinations. The sensitivities of demand for each origin destination pair is reflected with the changing average fares and number of links. Most notably are the "DENPHX" and "SEAPHX" routes where demand does not change very much due to the low coefficients/sensitivities to both ticket price and number of links . The "DENSEA" and "PHXSLC"routes however, experience more significant fluxes as expected. Due to the relative simplicity of the regression model and limited data used, it is not possible to provide an objective comparison to actual demand trends that exist.

A distinct advantage in the design of a new aircraft and associated routes of operation in this manner is that the modeling incorporates explicit sensitivities and passengers preferences to travel service provided. In this case, the two most prominent factors of fare and number of links are examined and this can of course be expanded to include demographic information as well. As external fluxes affect demand either directly (due to change in aircraft allocation on routes served) or indirectly (economic factors etc), there needs to be such information to facilitate the allocation of aircraft more efficiently.

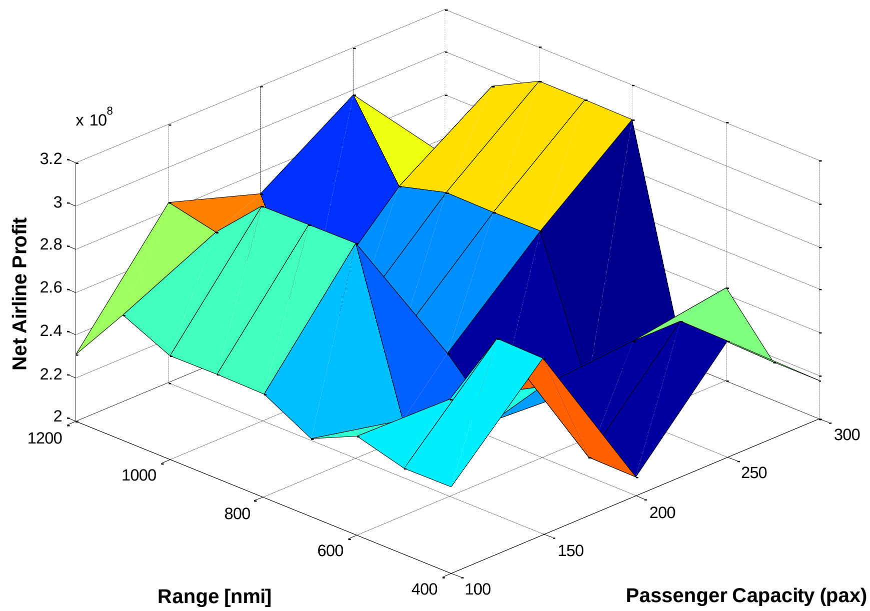


Figure 8: Solution space for profit maximization

Figure 8 above is the result of the total profits earned by the airline with the introduction of a new aircraft design. The solution space is a result of the top level enumeration as carried out previously using the subspace decomposition. The maximum profit is calculated as the summation of all profits for the simulated 8 quarters of the aircraft life with the econometric demand feedback. Since this is now a maximization problem, the peaks are thus the potential optimal points of interest and evidently there are multiple peaks. The maximum profit corresponds to total revenue of $304 million with the optimal aircraft having a range of approximately 900 nmi and passenger capacity of 300 pax. Note that better (more accurate) solutions may be yielded with a finer enumeration mesh or with the use of global optimization methods that can solve (MIP) problems.

III. Dynamic Programming for Sequential Allocation

Dynamic programming is a current and very actively researched area with its roots in control theory (economics and engineering and asset allocation. This class of problems involves solving recursive systems of equations that relate the value of being in a particular state in time to the value in the next point in time. In short, decisions now to maximize (or minimize) value affects subsequent decisions and associated values in the next step. This can be a daunting problem to solve given the exponential increase in the number of states that exist with each subsequent propagated decision for a given horizon. This sequencing is mathematically represented by the equation below, also referred to as the Hamilton-Jacobi-Bellman equation (HJB).

$$V_t(S_t) = \max_{x_t}(C_t(S_t, x_t) + V_{t+1}(S_{t+1}))$$

where C(St, xt) is the cost of effecting a set of decisions (xt) at state (St). Vt + 1 is the 'added benefit' when the state St becomes S(t + 1) due to decisions made in x_t.

The generalized stochastic form of the equation is stated as:

$$V_t(S_t) = \max_{x_t}(C_t(S_t, x_t) + \gamma^t E\{V_{t+1}(S_{t+1})\,|S_t\})$$

where the solution of the above form is computed by recursively computing the optimality equations backward through time. represents the"discount factor" to the expectation of future value added. In the case of the current proposed research, the calculation of the 'expectation' portion of the demand part is effectively done through the use of an econometrics model to estimate relevant factors and the next realization of demand for subsequent quarters. The connotation of using an econometric feedback in an ADP approach would be thus a maximization of profit/minimization of cost in a long term horizon.

The optimal set of policy decisions (allocation of aircraft routes) must thus arise from the maximization of some profit based objective function. The following adapts the above HJB generalized equations to become an objective function for the maximization of profit over a long term profit horizon.

$$profit_t(d_{ij}) = \max\left(c\left(d_{ij}(t), X_{ij}^{A,B,C}(t)\right) + \gamma E\left\{profit(d_{ij}(t), X_{ij}^{A,B,C}(t)\right\}\right)$$

The preceding equation suggests the maximization of some profit that is a function of the current observed state of demand required for city pairs and the allocation of aircraft to meet those needs (Xij's). The second portion of the equation reflects the expected profit as a result of satisfying projected demand at a later time period (say typically every quarter) that is a function of the chosen routes from the preceding step. Estimation of the future demand is done and dependant on the econometric of the immediate rolling horizon. This demand state transition is more formally given by the following state transition equation.

$$d_{ij}(t+1) \rightarrow d_{ij}^{t+1}(d_{ij}(t), X_{ij}^{A,B,C}(t), P_{ij}(t))$$

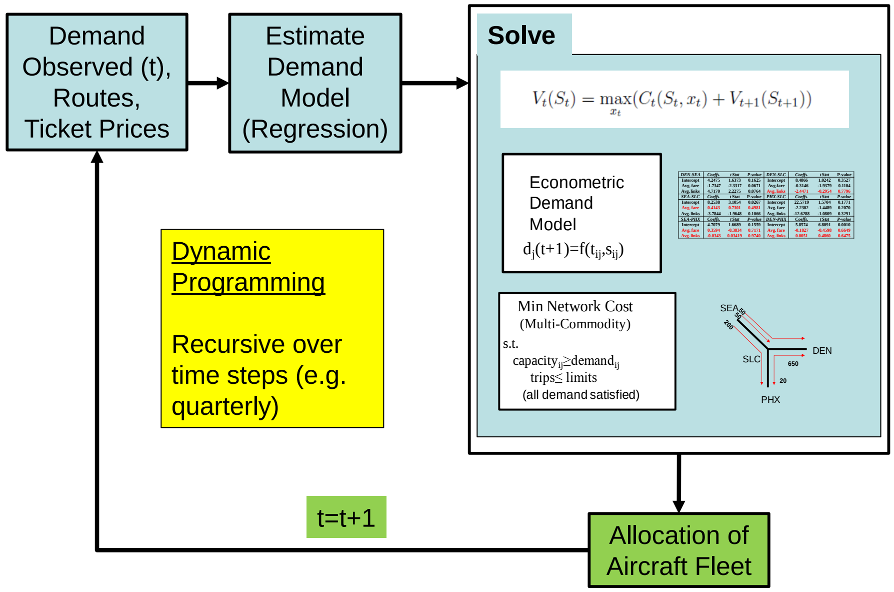
where demand at some future time (say a typical quarter) is dictated by a self updating model that can be of similar form as exhibited earlier but modified as the following:

$$\ln D_{ij}(t) = \alpha_0(t) + \alpha_1(t)\ln(f1) + \alpha_2(\mathrm{t})\ln(f2) + \ \ldots$$

where

D_{ij} – Passenger demand between city i and city j for quarter t
α_i – Factor coefficients at quarter t (updated dynamically)
f_i - Factor variable (e.g. ticket prices, number of stops, aircraft size) at quarter t

The following figure illustrates the overall architecture of the intended dynamic programming sequence to be implemented in order to maximize airline profits by 'encouraging' the next step of demand to maximize the profit horizon of immediately satisfying demand of a current time step and balancing such immediate gains with the projected gains from the effect that such an allocation will have on the next time step (typically an airline quarter) . This rolling horizon optimization represents a forward dynamic approach to maximizing profits for an airline using the notion of this 'reflexive' demand feedback.



IV. Summary and Conclusions

The work in progress presented in this paper exhibits a unifying framework that incorporates concurrent aircraft and network design with the econometric demand feedback. The notion of reflexive behavior is also introduced as part of the feedback mechanism to represent shifts in demand due to observed changes in airline operations by passengers. The architecture encompasses the design of both the aircraft and its operations to meet the expectations of demand trends for routes served. This can be thought of as incorporating the sensitivity (elasticity) of passenger demands to factors such as ticket prices and changes in operations (among other possible factors), within the design space. The inclusion of such information in effect enables newly designed aircraft and operations to be customized for the targeted passenger market. All of this is done within the framework of previously established subspace decomposition method.

The example model provided is limited and makes several prominent simplifications. Among these is the consideration of only two primary factors in a limited data set for a network of four selected cities. Simplifying assumptions are made through use of an OLS regression for the econometric modeling. These considerations are to facilitate the conceptual introduction of econometric modeling within a systems design context. The intended contributions in this paper are more focused on the underlying architecture proposed, the implications of customizing aircraft design to meet demand trends and the notion of reflexive demand feedback.

The potential advantages to such an approach are the added benefits of designing the aircraft and operations tailored to either maximize profit or minimize cost. The reflexive nature of demand can potentially be extended to better examine passenger travel trends and for strategic planning of airline policy and asset allocation to meet such demand characteristics. This naturally makes it amenable to a dynamic programming class of problems for both strategic and tactical asset design and allocation.

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