THE IMPACT OF AVIATION CHECKPOINT QUEUES
ON SECURITY SCREENING EFFECTIVENESS

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Abstract

Passenger screening at aviation security checkpoints is a critical component in protecting airports and aircraft from terrorist threats. Recent developments in technology have increased the ability to detect these threats. However, the average amount of time it takes to screen a passenger still remains a concern. This paper models the queueing process for a multi-level airport checkpoint security system, where multiple security classes are formed through subsets of specialized screening devices. An optimal static assignment policy is obtained which minimizes the steady-state expected amount of time a passenger spends in the security system. Then, an optimal dynamic assignment policy is obtained that maximizes the number of true alarms while maximizing passenger throughput. Performance of a two-class system is compared to that of a selective security system. The key contribution is that the resulting optimal assignment policies increase security and passenger throughput by efficiently and effectively utilizing available screening resources.
1 INTRODUCTION

The role of aviation security checkpoint screening is paramount in protecting our nation’s airports, airlines and passengers from terrorist threats. The events on September 11, 2001, when terrorists hijacked four commercial aircraft to attack the World Trade Center twin towers and the Pentagon, demonstrate the catastrophic consequences when terrorist plots succeed. The resulting increase in awareness of terrorist activity helped uncover and prevent a plot to detonate explosives onboard ten United States bound transatlantic flights in August 2006. In response to these terrorist activities, the Transportation Security Administration (TSA) continuously reforms security screening strategies in an effort to adapt to the expanding list of potential threats. The reactive nature of many of these changes, such as the policy of removing jackets and shoes, and the 3-1-1 liquid and gel policy for carry-on baggage, for example, has caused increases in the cost associated with resolving alarms for non-threatening passengers and inconvenience to travelers due to longer screening times.

Aviation security operations began in the 1970’s, when the United States installed basic surveillance equipment and required the use of metal detectors to screen all passengers. Passenger and baggage screening operations remained mostly unchanged until 1996 [1], when the Commission on Aviation Safety and Security (established July 25, 1996) recommended that the aviation industry use existing explosive detection technologies, automated passenger prescreening, and positive passenger-baggage matching to improve the level of airport security. The Federal Aviation Administration (FAA) responded by working with commercial airlines to purchase and deploy explosive detection systems (EDSs) at airports throughout the United States. EDSs and other security screening detection devices, such as magnetometers, X-ray machines, and chemical trace detectors, are used to identify and confiscate threat objects such as guns, knives, and explosives before they enter the airport terminal.

The Computer-Aided Passenger Prescreening System (CAPPS), an automated risk assessment system developed by the FAA, Northwest Airlines, and the United States Department of Justice, was introduced in 1998 to assess passengers’ risk and assist in allocating limited security screening resources [2]. Initially, only the checked baggage of selectee passengers (i.e., those not cleared by CAPPS) was screened by the EDSs, while the checked baggage of nonelectee passengers (i.e., those cleared by CAPPS) received no additional security attention. One consideration for limiting the use of EDSs to selectee baggage was the prohibitive purchase, maintenance, and operational costs of using this expensive technology to screen all baggage.

Immediately following the terrorist attacks on September 11, 2001, the United States Congress enacted the Aviation and Transportation Security Act (ATSA), which in turn created the TSA
within the Department of Transportation (now a part of the Department of Homeland Security). The ATSA required the TSA to acquire and deploy TSA-certified EDSs to screen all checked baggage no later than December 31, 2002 [3]. The purchase and deployment cost for over 2500 EDS devices necessary to ensure 100% checked baggage screening exceeded $2.5 billion and increased labor costs for federally employed screeners [4, 5].

To better assess passenger risk, the TSA developed CAPPS II, an enhanced version of CAPPS for systematically prescreening passengers. CAPPS II partitioned passengers into three risk-associated security classes (as opposed to two classes with CAPPS): selectees, nonselectees and a third class of passengers not permitted to fly based on federal terrorist watch lists. After investing US $100M in its development, the TSA announced (on July 14, 2004) the dismantling of CAPPS II due to privacy concerns [6]. Shortly thereafter, the TSA announced plans to develop Secure Flight, a new system for passenger prescreening. This system maintains terrorist watch lists and matches passengers to these lists to help reduce the number of false alarms (i.e., passengers who are not a threat but are earmarked for additional screening) resulting from CAPPS and CAPPS II [7]. The policies that used CAPPS and CAPPS II incorporated EDSs and explosive trace devices (ETDs) to implement the necessary level of security screening for each passenger [1, 8].

A prescreening system, such as CAPPS, assigns each passenger a perceived risk level based on information involving personal background, prior travel history, etc. This perceived risk may also be influenced by other factors, such as those obtained from the Screening Passengers by Observation Technique (SPOT) program, for example, as part of the Checkpoint Evolution concept aimed at improving security by focusing on people, process, and technology [10]. While a large percentage of passengers could be screened using specialized, costly, and time-consuming devices, the resulting increase in security may carry the additional expense of longer processing times, increased screening device operational costs, and a larger taskforce of security personnel. Moreover, these effects are magnified as the number of travelers per year increases, along with their impatience and dissatisfaction with ever-changing airport security procedures. The entire passenger screening system paradigm can be revamped to provide a solution that balances the tradeoff between maximizing security and minimizing the expected amount of time it takes to screen passengers and baggage through security checkpoints.

This paper develops a new paradigm of passenger and baggage screening operations by formulating a stochastic model for a multi-level security system, capturing the queueing dynamics of the passengers as they proceed through the security checkpoint. An optimal strategy for sequentially assigning passengers to a multi-level security system is developed which balances
the probability of detecting a threatening passenger with the expected amount of time the passenger spends in the security system.

Recent literature related to performing aviation security system decisions use discrete optimization models to develop policies that optimally screen passengers given a set of security screening devices. McLay et al. [11] show that subsets of detection devices can be identified by solving an integer programming model, which defines multiple levels of security based on the device capacities within each security class. Passengers are then assigned to a particular security class based on their level of perceived risk. This model performs the security assignment decisions statically, assuming that all passenger perceived risk levels are known a priori.

Discrete optimization models have also been developed to sequentially assign passengers to a set of security classes, when passenger perceived risk levels are known only upon check-in [12, 13, 14]. The Sequential Stochastic Multilevel Passenger Screening Problem (SSMPSP) [12] is modeled as a Markov Decision Process (MDP), where the optimal policy for SSMPSP is obtained through dynamic programming. Obtaining the optimal solution to SSMPSP is shown to be computationally intractable for large instances of passenger assignments. Lee et al. [13] model the passenger assignment process as a discrete-time difference equation, and then develop an optimal closed-loop passenger assignment algorithm through both a probabilistic analysis and nonlinear control. Nikolaev et al. [14] propose a two-stage model for the Sequential Stochastic Security Design Problem (SSSDP), where the first stage analyzes the purchase of security devices, while the second stage determines the screening assignments of sequentially arriving passengers. Here, the SSSDP is transformed into a deterministic integer program. Babu et al. [15] use linear programming models to investigate the benefit of dividing passengers into multiple levels of screening. Their objective of minimizing the number of false alarms, subject to a maximum false clear probability constraint, demonstrates that using multiple levels of screening is beneficial even when the risk posed by all passengers is the same. Nie et al. [16] extend this model to allow a numerical quantification of passenger perceived risk, and formulate the resulting model as a mixed integer program. They conclude that using multiple levels of screening results in a significantly more efficient security system.

The allocation of customers to a queueing system with multiple servers has been studied extensively in literature. Static flow models analyze the queueing process by means of a steady-state analysis, whereas dynamic flow models incorporate time-dependent behavior. One approach to controlling the average number of customers in the system is by changing the number of servers as a function of the queue length [17]. Another viable control technique involves the allocation of customer arrivals to a queueing system with a finite number
of servers. At each sequential arrival, the customer is assigned to one of the server queues, so as to minimize the queue length or to minimize the expected waiting time. Filipiak [18] solves a Poisson arrival, dynamic traffic assignment problem for multiple exponential servers to obtain the optimal traffic flow pattern which minimizes the waiting time spent in the system. The problem of sequentially assigning customers to multiple servers can be addressed by either modeling the queueing process prior to the server allocation [19, 20], or modeling the queueing process following the routing decision [21, 22]. However, since these models typically focus on problems in computer systems or communication networks, the common objective lies solely on either minimizing the number of customers in the system or minimizing the amount of time the customer spends in the system.

Several queueing networks draw a parallelism to passenger screening systems. Harrison and Wein [23] consider a queueing model where a type A customer is serviced at station 1, while a type B customer is serviced at both station 1 and then at station 2. This resemblance of selectee versus nonselectee screening, containing primary and secondary levels of screening, assumes that each type of customer arrives according to a different general distribution. They obtain a dynamic policy that minimizes the expected number of customers in the system. Schwartz [24] presents a class of queueing models where customers are allowed some freedom to select the lane which to be served. Steady-state analyses are performed to obtain the expected number of customers and the expected waiting times in the system. Nie et al. [25] use a steady-state queueing analysis to model the selectee lane in a selective security system. They base the passenger assignment problem on the number of passengers in the selectee lane and formulate as a nonlinear binary integer program, with the objective of maximizing the probability of detecting a threat.

This paper addresses the dual objective of maximizing security and passenger throughput by formulating the optimal control problem as a nonlinear program, and is organized as follows: Section 2 introduces the sequential passenger assignment problem. Section 3 presents an optimal static assignment policy based on a steady-state queueing analysis of the multi-level security system. Section 4 addresses the transient queueing behavior of the security screening process, and presents an optimal dynamic assignment policy that balances the cost tradeoff between maximizing security and minimizing the expected amount of time a passenger spends in the security system. Section 5 applies these queueing models to analyze the selective security system, and its application within the Black Diamond Self-Select program. Section 6 presents simulation results that compare the optimal static and dynamic assignment policies. Section 7 provides concluding comments and direction for future research.
2 Problem Background

A multi-level security system for an airport terminal checkpoint assigns each passenger to one of several available security classes according to the passenger’s perceived risk level. The security classes are composed of sets of screening devices, such as a magnetometer, an x-ray, and a millimeter-wave imaging machine, for example. These devices individually specialize in detecting specific types of threats, such as guns, knives, and explosives, and operate collectively to screen a passenger or bag at a certain level of intensity. Assume that passengers arrive independently and sequentially at the security checkpoint according to a Poisson process with rate $\lambda > 0$. The problem of assigning passengers to the appropriate security class is explored dynamically, where the assignment decision must be made as each passenger arrives at the security checkpoint for screening, and without knowledge of future passenger risk.

Let $M$ be the number of security classes available for screening passengers, and let $L_m$ denote the security level of class $m = 1, 2, \ldots, M$, with $0 \leq L_1 \leq L_2 \leq \ldots \leq L_M \leq 1$. The value $L_m$ is defined as the conditional probability that an alarm occurs for a passenger who is a threat, given that the passenger is assigned to security class $m$ (i.e., the conditional true alarm rate). Due to the procedures and technologies associated with each screening device, the rates at which passengers are screened vary across the security classes. It is assumed that passengers are screened faster (slower) through security classes that screen passengers at a lower (higher) level of intensity using a set of routine (specialized) devices and procedures. Otherwise, a security class of higher screening intensity with a fast service rate would supersede all security classes of lower screening intensity with slower rates (operational costs aside). Furthermore, assume that due to these sets of screening devices, the security class service rates are exponential, with rates $\mu_1 > \mu_2 > \ldots > \mu_M > 0$. For stability, assume $\lambda < \sum_{m=1}^{M} \mu_m$ (i.e., collectively, the multi-level security system is capable of screening passengers at a rate faster than the arrival rate).

Passengers sequentially arrive at the security checkpoint for screening, indexed as $i = 1, 2, \ldots$ upon arrival at times $t_1, t_2, \ldots$. It is assumed that the interarrival times between successive passengers are strictly positive, such that $t_i \to +\infty$ as $i \to +\infty$. A perceived risk for each passenger is assumed to be obtained through a combination of an automated prescreening system, such as CAPPS, and other risk assessment programs, such as Behavior Detection Officers (BHOs), for example. Through these risk assessment programs, the perceived risk is quantified as an assessed threat value, scaled between zero and one. A passenger’s assessed threat value is considered unknown for determining the security class assignment until this passenger enters the screening process. Let the assessed threat value for passenger $i$ be the (continuous) random variable, $\alpha_i$, where $\alpha_i$, $i = 1, 2, \ldots$ are independent and identically distributed (iid) random
variables with \(0 < \alpha_i \leq 1\), and with \(\alpha_i, \ i = 1, 2, \ldots\) denoting the respective observed values. The associated cumulative distribution function (cdf), \(F(\alpha_i) \equiv F_{\alpha_i}(\alpha_i)\), is defined over \([0, 1]\). The value \(0 < \alpha_i \leq 1\) is defined as the probability that passenger \(i\) is a threat, where a value of \(\alpha_i\) close to one (zero) corresponds to a high-risk (low-risk) passenger.

Figure 1 illustrates the process of assigning passengers to the multi-level security system. Define \(p_m(t_i) = P(X_m(t_i) = 1), m = 1, 2, \ldots, M, i = 1, 2, \ldots\), as the probability that passenger \(i\) entering screening at time \(t = t_i\) is assigned to security class \(m\). The decision to assign passenger \(i\) to security class \(m\) is a function of the passenger’s assessed threat value in relation to a set of security class threshold values, \(b_m(t), m = 1, 2, \ldots, M, t \geq 0\). The value \(b_m(t)\) determines the upper threshold value for security class \(m\), where \(0 < b_1(t) \leq b_2(t) \leq \ldots \leq b_M(t) \equiv 1\) partition the interval \((0, 1]\) into \(M\) subintervals. The security class assignment for passenger \(i\) is represented by the Bernoulli random variable, \(X_m(t_i) = 1\) (0) if passenger \(i\) is (not) assigned to class \(m\), with observed value, \(x_m(t_i)\). The objective is to determine the values \(b_m(t_i), m = 1, 2, \ldots, M\) that assigns passenger \(i\) to the appropriate intensity of screening, subject to security class capacity constraints. Section 3 develops an optimal static assignment policy through a set of constant security class threshold values, while Section 4 develops an optimal dynamic assignment policy through a set of time-varying threshold values.

### 3 Static Passenger Assignment Policy

A static passenger assignment policy is one in which each passenger is assigned to a security class independent of all prior passenger assignments. This implies that the assignment decision operates independently of the number of passengers currently in the security system, and sequentially assigns the passengers according to some predetermined set of constant security class threshold values. It is assumed that each security class has infinite capacity, so as to be capable
of screening all passengers if assigned to that security class. This type of policy can be applied, for example, to the random selection of nonselectee passengers (i.e., those cleared by CAPPS) for higher intensity screening in a two-class system, such that the devices used specifically for screening selectee passengers (i.e., those not cleared by CAPPS) are not under-utilized.

Let $N_a(t)$ be the set of discrete random variables corresponding to the number of passengers who have arrived at screening by time $t$, with observed values $n_a(t)$, let $N_{am}(t)$ be the set of discrete random variables corresponding to the number of passengers who have been assigned to security class $m$ by time $t$, with observed values $n_{am}(t)$, where $N_a(t) = \sum_{m=1}^{M} N_{am}(t)$, and let $N_{dm}(t)$ be the set of discrete random variables corresponding to the number of passengers who have been screened by security class $m$ by time $t$, with observed values $n_{dm}(t)$. Also, let $S_m(t)$ be the set of discrete random variables corresponding to the number of passengers in security class $m$ at time $t$ (i.e., $S_m(t) = N_{am}(t) - N_{dm}(t)$), with observed values $s_m(t)$. Given that passengers arrive at the security checkpoint in accordance with the Poisson process $\{N_a(t), t \geq 0\}$ with rate $\lambda$, then since $p_m = P(X_m(t_i) = 1)$ for the security class threshold values $b_m \equiv b_m(t)$, $m = 1, 2, ..., M$, $t \geq 0$, then $\{N_{am}(t), t \geq 0\}$, $m = 1, 2, ..., M$, are independent Poisson processes with rates $\lambda p_m$, $m = 1, 2, ..., M$ [27].

The multi-level security system is modeled as a stochastic process, where the probability that $s_m$ passengers are in security class $m$ at time $t$ (i.e., $P(S_m(t) = s_m)$) evolves dynamically as passengers enter and exit the security system. For a system with infinite capacity, the steady-state expectation and variance of the amount of time a passenger spends in the security system, $W$, are given by [26]

$$E[W] = \sum_{m=1}^{M} \frac{p_m}{\mu_m - \lambda p_m} \text{ and } Var(W) = \sum_{m=1}^{M} \frac{\mu_m p_m}{\lambda (\mu_m - \lambda p_m)^2}. \tag{1}$$

For the static passenger assignment policy, a set of constant values $p_m = p_m(t_i), i = 1, 2, ...$, can be obtained so as to minimize the steady-state expected amount of time a passenger spends in the security system. This minimum can be obtained by solving the following nonlinear program for $p_1, p_2, ..., p_M$,

**Static Passenger Queueing Problem (SPQP):**

$$\text{minimize } \sum_{m=1}^{M} \frac{p_m}{(\mu_m - \lambda p_m)} \tag{2}$$

subject to

$$0 \leq p_m \leq 1, \quad m = 1, 2, ..., M \tag{3}$$

$$p_m < \frac{\mu_m}{\lambda}, \quad m = 1, 2, ..., M \tag{4}$$

$$\sum_{m=1}^{M} p_m = 1. \tag{5}$$
In practice, the inequality constraint in (4) can be replaced with $p_m + \epsilon_m \leq \mu_m/\lambda$, for some $\epsilon_m > 0$. The minimum values of $p_m$, $m = 1, 2, ..., M$ are obtained by taking the limit as $\epsilon_m \to 0$. The quantity $p_m/(\mu_m - \lambda p_m)$ decreases monotonically as $\mu_m$ increases, and increases monotonically as $p_m$ increases. Since $\mu_1 > \mu_2 > ... > \mu_M > 0$, then a solution to SPQP is obtained by pairing larger values of $p_m$ with larger values of $\mu_m$, and hence, the optimal assignment probabilities obtained from SPQP satisfy $p_1^* > p_2^* > ... > p_M^* > 0$. Theorem 1 provides the minimum solution to SPQP for the special case of a two-class system.

**Theorem 1.** Let the number of passenger arrivals, $\{N^m(t), t \geq 0\}$, be a Poisson process with rate $\lambda > 0$, and the service times of the $M = 2$ security classes be exponential with rates $\mu_m > 0$, $m = 1, 2$, where $\mu_1 + \mu_2 > \lambda$. Let $p_1 = P(X_1(t_i) = 1)$ and $p_2 = P(X_2(t_i) = 1) = 1 - p_1$ for the constant threshold values $b_1 \equiv b_1(t_i)$, $b_2(t_i) \equiv 1$, $i = 1, 2, ...$. Then the solution to SPQP is achieved through the optimal static assignment probability

$$p_i^* = \frac{\mu_1(\lambda - 2\mu_2)}{\lambda(\mu_1 - \mu_2)} + \sqrt{\left(\frac{\mu_1(\lambda - 2\mu_2)}{\lambda(\mu_1 - \mu_2)}\right)^2 + \frac{\mu_1\mu_2}{\lambda^2} - \frac{\mu_1(\lambda - 2\mu_2)}{\lambda(\mu_1 - \mu_2)}}$$

(6)

for $\mu_1 > \mu_2$, such that $0 \leq p_m^* \leq 1$ and $p_m^* < \mu_m/\lambda$, $m = 1, 2$. Moreover, if $\mu_1 = \mu_2$, then $p_1^* = p_2^* = 1/2$.

The proof of Theorem 1 follows by substituting $p_2 = 1 - p_1$ and solving SPQP as a function of the decision variable $p_1$. From (6), the special case $\mu_1 > \mu_2 = \lambda/2$ gives $p_1 = 1 - p_2 = (\sqrt{2\mu_1/\lambda})/2$, and hence, $1/2 < p_1 < 1$ whenever $\lambda/2 < \mu_1 < 2\lambda$. Due to the variability of the queue length produced from fixed assignment probabilities, the throughput of passengers will inevitably decrease whenever the overall level of risk in the passenger population is either lower or higher than expected. Furthermore, the queue length can be used as feedback into a closed-loop, dynamic passenger assignment process to balance the expected amount of time a passenger spends in the security system with the expected number of true alarms. Section 4 presents a discrete-time model of the queueing process for the multi-level security system to obtain such a closed-loop decision policy.

**4 Dynamic Passenger Assignment Policy**

Passengers sequentially arrive at screening, with assessed threat value $\alpha_i$, $i = 1, 2, ...$, according to a Poisson process with rate $\lambda$. Passenger $i$ is assigned to security class $m$, with security level $L_m$, $m = 1, 2, ..., M$, if $\alpha_i \in (b_{m-1}(t_i), b_m(t_i)]$, $m = 1, 2, ..., M$. The conditional probability that
an alarm occurs for passenger $i$, given that passenger $i$ is a threat, is given by

$$P(\text{Alarm} | \text{Passenger } i \text{ is a threat}) = \sum_{m=1}^{M} L_m p_m(t_i), \quad (7)$$

$i = 1, 2, \ldots$. Given that the assessed threat value $\alpha_i$ accurately quantifies the probability that passenger $i$ is a threat via a prescreening system such as CAPPS, then $\sum_{\tau=1}^{i} L_m x_m(t_\tau) \alpha_\tau$ represents the expected number of true alarms from security class $m$ over the first $i$ passengers assigned to class $m$. Moreover, $\sum_{\tau=1}^{i} \sum_{m=1}^{M} L_m x_m(t_\tau) \alpha_\tau$ represents the expected number of true alarms in the security system over the first $i$ passengers.

The optimal dynamic assignment policy that determines the sequence of security class threshold values $\{b_m(t_i), i = 1, 2, \ldots\}, m = 1, 2, \ldots, M$ is the policy which minimizes the expected amount of time passenger $i$ spends in the security system, $E[W(t_i)]$ and maximizes the security measure (7) for each passenger assignment. Clearly, if minimizing $E[W(t_i)]$ is not an issue, then the policy which maximizes (7) is simply to have all passengers undergo maximum screening (i.e., $p_M(t_i) = 1, i = 1, 2, \ldots$). However, by defining a dual objective, a balance must be achieved between security and passenger throughput. Section 4.1 develops a discrete-time queueing model, sampled at the time of each passenger arrival. Section 4.2 obtains the optimal dynamic assignment policy by weighting the costs associated with the dual objectives.

### 4.1 Transient Queueing Analysis

The passenger screening process at an airport security checkpoint is modeled as a multi-level, single-server exponential queueing system, where each security class has finite capacity, $c_m$, $m = 1, 2, \ldots, M$. A set of discrete-time state equations are developed to capture the probability that there are $s_m$ passengers in security class $m$ at the time of each passenger arrival, $t = t_i$, $i = 1, 2, \ldots$. The sequential observations at the moment of each passenger arrival corresponds to a discrete-time Markov process. Define $\Delta_i$, $i = 1, 2, \ldots$ as the passenger interarrival times, where $\Delta_i$ is exponentially distributed with rate $\lambda > 0$, and with observed values $\delta_i = t_i - t_{i-1}$.

For successive passenger arrivals at times $t_i = \sum_{j=1}^{i} \delta_j$ and $t_{i+1} = \sum_{j=1}^{i+1} \delta_j$, the number of passengers arriving at screening during the time interval $(t_i, t_{i+1})$ is exactly one (i.e., $N^a(t_{i+1}) - N^a(t_i) = 1$). If the number of passengers screened by security class $m$ can be measured (i.e., by counting the number of passengers passing through a magnetometer, for example), then $N_m^d(t_i)$, and hence, $S_m(t_i)$, are known, and the realization $s_m$ can be used in obtaining the passenger assignment decision. Otherwise, $N_m^a(t_i)$ and $S_m(t_i)$ are unknown, and the assignment decision for passenger $i$ becomes a function of the expected number of passengers in the security system at time $t_i$. 

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Define $N_m^s(t_i, t_{i+1}) = N_m^d(t_{i+1}) - N_m^d(t_i)$, $m = 1, 2, ..., M$, $i = 1, 2, ...$, as the number of passengers screened by security class $m$ during time interval $(t_i, t_{i+1}]$. The conditional probability, $P(N_m^s(t_i, t_{i+1}) = n_m^s|S_m(t_i) = s_m)$, corresponds to the number of passengers screened by security class $m$ during the time interval $(t_i, t_{i+1}]$, given that $s_m$ passengers are waiting to be screened in security class $m$. Note that the number of passengers screened during $(t_i, t_{i+1}]$ is independent of the time passenger $i$ enters the screening process, but is dependent on the number of passengers in the security system at time $t = t_i$. Given exponential service times, the number of passengers screened during $(t_i, t_{i+1}]$ is Poisson if $s_m > n_m^s(t_i, t_{i+1})$. For the case of screening all passengers in the queue for security class $m$ during $(t_i, t_{i+1}]$, the conditional probability $P(N_m^s(t_i, t_{i+1}) = n_m^s|S_m(t_i) = s_m)$ must also account for the server idle time. The number of passengers in security class $m$ at time $t_i$, $S_m(t_i)$, is formulated as a discrete-time, nonhomogeneous Markov chain, with transition probabilities

\[
P_m^{k,j}(t_i) = \begin{cases} 
(1 - p_{m}(t_i))P(N_m^s(t_i, t_{i+1}) = k|S_m(t_i) = k) \\
+ p_{m}(t_i)P(N_m^s(t_i, t_{i+1}) = k + 1|S_m(t_i) = k) & \text{for } k = 0, 1, ..., c_m, \ j = 0 \\
p_{m}(t_i)P(N_m^s(t_i, t_{i+1}) = 0|S_m(t_i) = k) & \text{for } k = 0, 1, ..., c_m - 1, \ j = k + 1 \\
(1 - p_{m}(t_i))P(N_m^s(t_i, t_{i+1}) = k - j|S_m(t_i) = k) \\
+ p_{m}(t_i)P(N_m^s(t_i, t_{i+1}) = k - j + 1|S_m(t_i) = k) & \text{for } k = 1, 2, ..., c_m, \ 1 \leq j \leq k \\
P(N_m^s(t_i, t_{i+1}) = 0|S_m(t_i) = k) & \text{for } k = j = c_m \\
+ p_{m}(t_i)P(N_m^s(t_i, t_{i+1}) = 1|S_m(t_i) = k) & \text{for } k = j = c_m \\
0 & \text{otherwise,}
\end{cases}
\]

$m = 1, 2, ..., M$, $i = 1, 2, ...$, such that

\[
P(S_m(t_{i+1}) = j) = \sum_{k=0}^{c_m} P_m^{k,j}(t_i)P(S_m(t_i) = k), \ j = 0, 1, ..., c_m,
\]

$m = 1, 2, ..., M$, $i = 1, 2, ...$, with boundary condition $P(S_m(t_1) = s_m) = 1$ ($0$ if $s_m = 0$ (otherwise), $m = 1, 2, ..., M$. The states $s_m = 0, 1, ..., c_m$ are positive recurrent and aperiodic for each time $t_i$. From (9), Lemma 1 provides a recursion for the expected amount of time passenger $i$ spends in security class $m$, $E[W_m(t_i)]$.

**Lemma 1.** Given the finite-dimension, nonhomogeneous Markov chain (9), the expected amount of time passenger $i$ spends in the security system if assigned to security class $m$ is

\[
E[W_m(t_{i+1})] = E[W_m(t_i)] + \frac{p_m(t_i)}{\mu_m} \left[ 1 - P(N_m^s(t_i, t_{i+1}) = 0, S_m(t_i) = c_m) \right] - \frac{1}{\mu_m} E[N_m^s(t_i, t_{i+1})]
\]

(10)
\[ m = 1, 2, \ldots, M, \ i = 1, 2, \ldots, \] with \( E[W_m(t_1)] = 1/\mu_m. \)

From Lemma 1, the expected amount of time passenger \( i \) spends in the security system, \( E[W(t_i)] \), can be obtained from

\[
E[W(t_i)] = \sum_{m=1}^{M} p_m(t_i)E[W_m(t_i)],
\]

\( m = 1, 2, \ldots, M, \ i = 1, 2, \ldots. \)

Let the control law, \( u_m(t_i), m = 1, 2, \ldots, M, \) correspond to the assignment probability for passenger \( i \) to security class \( m \), \( p_m(t_i) = u_m(t_i), m = 1, 2, \ldots, M. \) Since \( p_m(t_i) = F_\alpha(b_m(t_i)) - F_\alpha(b_{m-1}(t_i)) \), then \( F_\alpha(b_m(t_i)) = \sum_{j=1}^{m} p_j(t_i), \) and hence, the set of security class threshold values, \( b_m(t_i), m = 1, 2, \ldots, M, \) are obtained through the inverse cdf, \( b_m(t_i) = F_\alpha^{-1}(\sum_{j=1}^{m} u_j(t_i)), \)

\( m = 1, 2, \ldots, M, \) with boundary conditions, \( b_m(t_1) \in (0, 1], m = 1, 2, \ldots, M. \) Note that the passenger risk distribution is only used to determine the set of security class threshold values, \( b_m(t_i), m = 1, 2, \ldots, M, \) from the closed-loop assignment probabilities, \( p_m(t_i), m = 1, 2, \ldots, M. \)

Define \( p_m^* \) as the optimal, steady-state assignment probability for security class \( m \), and \( w_m^* = 1/(\mu_m - \lambda p_m^*) \) as the optimal, steady-state expected amount of time a passenger spends in security class \( m \), obtained from SPQP in Section 3. By defining the system output for security class \( m \) as \( y_m(t_i) = E[W_m(t_i)] - w_m^*, \) the objective of the dynamic passenger assignment policy is to obtain the optimal control \( u_m^*(t_i) \) which regulates \( y_m(t_i) \) to zero through the set of (deterministic) discrete-time difference equations (9) and (10), while maximizing (7).

### 4.2 Optimal Control: Weighted Least Squares

In this section, the optimal control \( u_m(t_i), m = 1, 2, \ldots, M, \) is obtained by minimizing the mean square cost resulting from the security measure in (7) and the expected amount of time a passenger spends in the security system in (11). The mean square cost associated with the true alarm probability results from the difference between assigning passenger \( i \) to class \( m \) and assigning passenger \( i \) to class \( M \) is defined by

\[
C^Z(t_i) \equiv \left( 1 - \sum_{m=1}^{M} \frac{L_m - L_1}{L_M - L_1} p_m(t_i) \right)^2, \quad i = 1, 2, \ldots, \text{where } 0 \leq L_1 < L_2 < \ldots < L_M \leq 1. \] The cost \( C^Z(t_i) \) is normalized such that \( 0 \leq C^Z(t_i) \leq 1, \ i = 1, 2, \ldots, \) where \( C^Z(t_i) = 0 \) when all passengers are assigned to security class \( M \) (i.e., the highest level of screening intensity) with \( p_M(t_i) = 1, \ i = 1, 2, \ldots. \) Conversely, \( C^Z(t_i) = 1 \) when all passengers are assigned to security class \( 1 \) (i.e., the lowest level of screening intensity) with \( p_1(t_i) = 1, \ i = 1, 2, \ldots. \)
The mean square cost associated with the expected amount of time passenger \( i \) spends in the security system is defined by

\[
C^W(t_i) = \left( \frac{1}{M-1} \sum_{m=1}^{M-1} \left( 1 - \frac{p_m(t_i)}{p^*_m} \right)^2 \right),
\]

\( i = 1, 2, ... \), where \( w^* = \sum_{m=1}^{M} p^*_m w^*_m \) is the optimal, steady-state expected amount of time a passenger spends in the security system, as determined by SPQP in Section 3. The cost \( C^W(t_i) \) is normalized such that \( 0 \leq C^W(t_i) \leq 1, \ i = 1, 2, ... \), where \( C^W(t_i) = 0 \) with \( p_m(t_i) = p^*_m \) and \( E[W_m(t_i)] = w^*_m \). Since \( \sum_{m=1}^{M} p_m(t_i)E[W_m(t_i)] \) is convex, then its maximum is achieved at the boundary conditions when \( p_m(t_i) = 1(0) \) for the security class with \( \max_{m=1,2,...,M} \{E[W_m(t_i)]\} \) (otherwise). Therefore, \( C^W(t_i) = 1 \) results using the normalizing factor \( \max_{m=1,2,...,M} \{E[W_m(t_i)]\} - w^* \). Note that if \( \max_{m=1,2,...,M} \{E[W_m(t_i)]\} = w^* \), then \( C^W(t_i) = 1(0) \) if \( M \geq 2 \) (otherwise). In addition, the mean square cost associated with the optimal assignment probability error, \( p_m(t_i) - p^*_m \), is defined by

\[
C^P(t_i) = \frac{1}{M-1} \sum_{m=1}^{M-1} \left( 1 - \frac{p_m(t_i)}{p^*_m} \right)^2,
\]

\( i = 1, 2, ... \), where \( p^*_m = \max_{m=1,2,...,M} \{p_m\} \). The cost \( C^P(t_i) \) is normalized such that \( 0 \leq C^P(t_i) \leq 1, \ i = 1, 2, ... \), where \( C^P(t_i) = 0 \) with \( p_m(t_i) = p^*_m \), and where \( C^P(t_i) = 1 \) with \( p_m(t_i) = 1(0) \) for security class \( m = 1 \) (otherwise). Note that the assignment probabilities are only summed over the first \( M-1 \) security classes, since by definition, \( p_M(t_i) = 1-\sum_{m=1}^{M} p_m(t_i) \).

The scalar weight \( \eta_1 \) is used create a cost tradeoff between security versus passenger throughput, while the scalar weight \( \eta_2 \) creates a balance between the costs associated with regulating to the optimal expected amount of time and the optimal assignment probability,

\[
C(t_i) = (1 - \eta_1)C^Z(t_i) + \eta_1 \left( (1 - \eta_2)C^W(t_i) + \eta_2 C^P(t_i) \right), \quad 0 \leq \eta_1 \leq 1, \ 0 \leq \eta_2 \leq 1.
\]

\( i = 1, 2, ... \). Since \( p_m(t_i) = u_m(t_i) \), then \( u_m(t_i) \) must also satisfy the constraint \( \sum_{m=1}^{M} u_m(t_i) = 1, \ i = 1, 2, ... \). The control law which minimizes the cost function (12) can be obtained by solving the nonlinear program for \( u_m(t_i), \ m = 1, 2, ..., M \).

**Dynamic Passenger Queueing Problem (DPQP):**

\[
\begin{align*}
\text{minimize} & \quad C(t_i) \\
\text{subject to} & \quad 0 \leq u_m(t_i) \leq 1, \quad m = 1, 2, ..., M \\
& \quad \sum_{m=1}^{M} u_m(t_i) = 1,
\end{align*}
\]

for each successive passenger \( i = 1, 2, ... \) arriving at the security checkpoint.
If \( S_m(t_i) = c_m \) for passenger \( i \), then the assignment probability \( p_m(t_i) = u_m(t_i) = 0 \) ensures that the queue length for class \( m \) does not exceed capacity. Also, due to the security class capacity constraints, \( P(S_m(t_i) \geq c_m + 1) = 0 \), and \( 0 \leq u_m(t_i) \leq 1 \), the system output for security class \( m \) is bounded,

\[
y_m(t_i) = E[W_m(t_i)] - w_m^* \leq (c_m + 1)/\mu_m - w_m^* < +\infty.
\]

Therefore, the closed-loop control, \( u_m(t_i) \), does not allow the quantity \( E[W_m(t_i)] - w_m^* \) to grow infinitely large. In general, a solution to DPQP is difficult to obtain for multi-level systems with a large number of security classes. However, the analysis of a two-class system is beneficial (as shown in Section 5) for comparison with a selective security system, containing primary and secondary levels of screening. Theorem 2 shows the optimal dynamic assignment policy that minimizes cost function (13) for a two-class system.

**Theorem 2.** Let the number of passenger arrivals, \( \{N^a(t), t \geq 0\} \), be a Poisson process with rate \( \lambda \), and the \( M = 2 \) security classes, with security levels \( L_m, m = 1, 2 \), follow exponential service times with rates \( \mu_m, m = 1, 2 \), where \( \mu_1 + \mu_2 > \lambda \). Let \( p_1(t_i) = 1 - p_2(t_i) = u_1(t_i), \)

\( i = 1, 2, ..., \) where \( p_1(t_i) = P(X_1(t_i) = 1) \) and \( p_2(t_i) = P(X_2(t_i) = 1) \). Given the optimal, steady-state assignment probabilities, \( p_m^*, m = 1, 2 \), from (14), and \( w^* = \sum_{m=1}^{2} p_m^*/(\mu_m - \lambda p_m^*) \), the optimal control becomes

\[
u_1^*(t_i) = \frac{\eta_1((1 - \eta_2)(E[W_2(t_i)] - w^*)(E[W_2(t_i)] - E[W_1(t_i)]) + \eta_2 \gamma^2(t_i)/p_1^*)}{(1 - \eta_1)\gamma^2(t_i) + \eta_1((1 - \eta_2)(E[W_2(t_i)] - E[W_1(t_i)])^2 + \eta_2(\gamma(t_i)/p_1^*)^2)} \tag{14}
\]

where \( \gamma(t_i) = \max\{E[W_1(t_i)], E[W_2(t_i)]\} - w^* \), such that \( 0 \leq u_i^*(t_i) \leq 1, i = 1, 2, ..., \) for some \( 0 \leq \eta \leq 1 \). The set of optimal security class threshold values are \( b_1^*(t_i) = F_{\alpha_1}^{-1}(u_1^*(t_i)), \)

\( i = 1, 2, ..., \) and \( b_2^*(t_i) = 1, i = 1, 2, .... \)

The proof of Theorem 2 follows by substituting \( u_2(t_i) = 1 - u_1(t_i) \) and solving DPQP as a function of the decision variable \( u_1(t_i) \) at time \( t_i \). When \( \eta_1 = 0 \), the optimal control in (14) reduces to \( u_1^*(t_i) = 0, i = 1, 2, ... \). Therefore, \( p_2^*(t_i) = 1 - u_1^*(t_i) = 1 \) implies that all passengers are assigned to the security class with the highest level of screening intensity, and the total security (i.e., expected number of true alarms) is maximized without regards to the expected amount of time each passenger spends in the security system. However, if \( \mu_2 \leq \lambda \), then the steady-state expected number of passengers in the security system becomes infinite. When \( \eta_1 = 1 \) and \( \eta_2 = 1 \), the optimal control in (14) reduces to \( u_1^*(t_i) = p_1^* \), which corresponds to the optimal static assignment policy obtained in Section 3.

In practice, it is not desirable to place passenger \( i \) with assessed threat value \( \alpha_i = 1 \) into any security class other than class \( M \). Thus, it is assumed that the TSA holds the right to override
the decision obtained by either the static or dynamic assignment policies. This implies that the security class capacity constraint may, in fact, be exceeded due to an imminent threat. With fixed capacities, there exists the potential for terrorists to game the system by overloading the higher levels of security with a group of high-risk operatives, thereby increasing the probability that a later-arriving, lower-risk terrorist will be insufficiently screened. However, this type of scenario can be addressed through a separate pattern detection algorithm, for example, to signal a cautionary alarm for particular sequences of passenger risk observations. Section 5 next outlines the selective security system, containing primary and secondary levels of screening.

5 Security Systems with Primary and Secondary Screening

This section outlines the selective security system, containing primary and secondary levels of screening, as well as its application within the Black Diamond Self-Select program [10]. Figure 2 illustrates the structure of the selective security system. Passengers arrive at screening and join the queue for primary screening, where all passengers and carry-on baggage are screened through a set of routine devices, such as a magnetometer and an X-ray machine, for example. Then, selectee passengers are directed to join the queue for secondary screening, consisting of specialized devices and procedures such as pat-down search, hand wand, or explosive trace detection devices, among others. Nonselectees who clear primary screening are permitted to exit the screening process. Additionally, nonselectees who clear primary screening may be directed toward secondary screening, with probability \( p(t_i) \), based on the passenger’s assessed threat value and the secondary screening queue length at the time the secondary screening decision is made. Assume that an alarm occurring in primary screening either uncovers a threat item (i.e., a true alarm) and the passenger is prevented from entering the airport terminal, or the false alarm is resolved by an appropriate procedure. Moreover, if a nonselectee passenger produces an alarm during primary screening where the threat cannot be immediately identified, this passenger’s risk status can be changed to selectee prior to the decision for allocating secondary screening.
Let the number of nonselectee and selectee passengers arriving at the security checkpoint, \{N_{NS}^a(t), t \geq 0\} and \{N_S^a(t), t \geq 0\}, follow a Poisson process with rates \(\lambda(1 - p_s) > 0\) and \(\lambda p_s > 0\), respectively, where \(0 \leq p_s \leq 1\) is the fraction of passengers who are designated as selectees. The total number of passengers arriving at the security checkpoint, \{N^a(t) = N_{NS}^a(t) + N_S^a(t), t \geq 0\}, is Poisson with rate \(\lambda > 0\). Assume that if passenger \(i\) is designated as a selectee (nonselectee), then the observed passenger’s assessed threat value is \(\alpha_i = 1\) (0 < \(\alpha_i < 1\)). Furthermore, assume that the primary and secondary screening service times are exponential with rates \(\mu_1\) and \(\mu_2\), and have security levels \(L_1\) and \(L_2\), respectively. Define the Bernoulli random variable \(X(t_i) = 1(0)\) if passenger \(i\) is (not) directed to secondary screening, (i.e., \(X(t_i) = 1\) if \(b(t_i) < \alpha_i \leq 1\), \(i = 1, 2, \ldots\)).

Define \(N_1^d(t)\) and \(N_2^d(t)\) to be the number of passengers departing primary and secondary screening by time \(t\), respectively. Also, define \(N_2^a(t)\) to be the number of passengers assigned to undergo secondary screening by time \(t\). Furthermore, let \(S_1(t) = N^a(t) - N_1^d(t)\) represent the number of passengers in primary screening, while \(S_2(t) = N_2^d(t) - N_2^a(t)\) represent the number of passengers in secondary screening, and hence, \(S(t) = S_1(t) + S_2(t)\) represents the total number of passengers in the security system.

In the selective security system in Figure 2, the primary screening queue length is a result of both the passenger arrival rate and the primary screening service rate. Therefore, assume \(\mu_1 > \lambda\) to ensure stability. Also assume that secondary screening has finite capacity \(c_2\) exceeding the number of passengers designated as selectees. Rather than sampling the queueing system at each passenger arrival as in Section 4, suppose that the system is sampled when each passenger enters the decision point for assigning secondary screening. This point in time also corresponds to when a passenger completes primary screening, and since a decision for secondary screening cannot be made if there are no passengers remaining in primary screening, the number of passengers entering the secondary screening decision point, \{\(N_1^d(t), t \geq 0\}\), is not Poisson. However, general arrival distributions can be addressed by the asynchronous sampling of the queueing process at time \(t_i\) when passenger \(i\) exits primary screening. Define \(\Delta_i, i = 1, 2, \ldots\) as the interarrival time for passenger \(i\) at secondary screening, with observed value, \(\delta_i = t_i - t_{i-1}\). Therefore, the number of passengers who undergo primary screening during the time interval \((t_i, t_{i+1}]\) is exactly one (i.e., \(N_1^d(t_i, t_{i+1}) \equiv N_1^d(t_{i+1}) - N_1^d(t_i) = 1\)). The number of passengers in primary screening is formulated as an infinite dimension, nonhomogeneous, discrete-time Markov chain,

\[
P(S_1(t_{i+1}) = j) = \sum_{k=0}^{\infty} P(S_1(t_i) = k) P_1^{k,j}(t_i), \quad j = 1, 2, \ldots,
\]

(15)
\( i = 1, 2, \ldots \), with boundary condition \( P(S_1(t_1) = s_1) = 1 \) (0) if \( s_1 = 0 \) (otherwise), and with transition probabilities,

\[
P_{1}^{k,j}(t_i) = \begin{cases} 
    e^{-\lambda t_i} \frac{(\lambda t_i)^{j+1}}{(j+1)!} & \text{for } k = 0, \ j \geq 0 \\
    e^{-\lambda t_i} \frac{(\lambda t_i)^{j+1-k}}{(j+1-k)!} & \text{for } k = 1, 2, \ldots, \ j \geq k - 1 \\
    0 & \text{otherwise,}
\end{cases}
\]

\( i = 1, 2, \ldots \), where states \( s_1 = 0, 1, \ldots \) are positive recurrent and aperiodic at each time \( t_i \).

Define \( N_2^d(t_i, t_{i+1}) \equiv N_2^d(t_{i+1}) - N_2^d(t_i) \), \( i = 1, 2, \ldots \) as the number of passengers who undergo secondary screening during the time interval \((t_i, t_{i+1}]\). The probability that \( s_2 = 0, 1, \ldots, c_2 \) passengers are in secondary screening, \( P(S_2(t_{i+1}) = s_2) \), when passenger \( i + 1 \) arrives at the decision point is formulated as a finite dimension, nonhomogeneous, discrete-time Markov chain,

\[
P(S_2(t_{i+1}) = j) = \sum_{k=0}^{c_2} P(S_2(t_i) = k) P_{2}^{k,j}(t_i), \quad j = 1, 2, \ldots, \tag{16}
\]

\( i = 1, 2, \ldots \), with boundary condition \( P(S_2(t_1) = s_2) = 1 \) (0) if \( s_2 = 0 \) (otherwise), and with transitional probability components, \( P_{2}^{k,j}(t_i) \), as defined in (8).

By sampling the queueing process only at the time when each passenger exits primary screening, and not at the time that this passenger arrives at the screening checkpoint, the amount of time each passenger spends in the queue for primary screening is unknown. However, since the queueing model for primary screening is a \( M/M/1 \) process, the steady-state expected amount of time a passenger spends in primary screening is

\[
E[W^p] = \sum_{s_1=0}^{\infty} s_1 P_{s_1}^{ss} = \frac{1}{\mu_1 - \lambda},
\]

where \( P_{s_1}^{ss} \equiv \lim_{t_i \to +\infty} P(S_1(t_i) = s_1) \).

Optimization of the secondary screening assignment decision for the selective security system in Figure 2 is equivalent to that of a two-class security system in (14) with \( L_1 = 0 \), \( E[W_1(t_i)] = 0 \), \( p_*^{1} = 1 \), and \( w^* = 0 \). Since \( \gamma(t_i) = \max\{E[W_1(t_i)], E[W_2(t_i)]\} - w^* = E[W_2(t_i)] \), then from (14), the optimal probability for assigning (nonselectee) passenger \( i \) exiting primary screening reduces to

\[
p^*(t_i) = 1 - \frac{w^*(t_i)}{1 - p_0} = \begin{cases} 
    1 - \frac{\eta_1}{1 - p_s} & \text{if } s_2(t_i) < c_2 \\
    0 & \text{otherwise,}
\end{cases} \tag{17}
\]

\( i = 1, 2, \ldots \), while \( p^*(t_i) = 1 \) if passenger \( i \) is designated as a selectee. The optimal assignment probability in (17) is time-invariant whenever capacity remains in secondary screening, producing the (constant) optimal threshold value \( b^* = b^*(t_i) = F_{\alpha}^{-1}(p^*(t_i)) \). The security achieved for a passenger who undergoes only primary screening is \( L_1' = L_1 \), while the security achieved for a
passenger who undergoes both primary and secondary screening is given by 
\[ L'_2 = L_1 + L_2 - L_1 L_2, \]
which corresponds to the probability that the threat is detected during either primary or sec-
ondary screening. Here, it is assumed that a threat (i.e., a prohibited item) detected in primary 
screening does not need to be confirmed in secondary screening to be confiscated.

Primary and secondary levels of screening are also a basic component of the Black Diamond 
Self-Select program, a concept currently being tested for domestic terminals at a limited 
number of United States commercial airports [28]. The Black Diamond Self-Select program is 
part of the TSA’s Checkpoint Evolution concept, which seeks to increase passenger through-
put, decrease alarm rates, integrate flexibility, and create a positive environment during the 
checkpoint screening process. Passengers arrive at the security checkpoint and choose to enter 
one of following three lanes for screening based on their travel experience: the Black Lane, 
for expert travelers who know the security procedures well, the Blue Lane, for casual travelers 
who are not very familiar with the security procedures, and the Green Lane, for families and 
passengers with special needs. Assume that passengers enter the Black, Blue, and Green Lanes 
according to Poisson processes with rates \( \lambda^e \), \( \lambda^c \), and \( \lambda^f > 0 \), corresponding to expert, casual, 
and family, respectively. Since the choice of lane is made by the passenger, an expert traveler 
may choose to enter either the casual or family designated lanes if either of these lanes are 
much shorter than the one for expert travelers. Similarly, casual travelers or families may enter 
alternate lanes; however, these cases may be more infrequent due to their inexperience or need 
for special assistance.

Presently, the TSA does not distinguish risk among these three groups of passengers, and 
structures each Self-Select lane as a selective security system with primary and secondary 
levels of screening. However, since a large portion of passengers are business travelers who 
are familiar with the security process, one area of secondary screening is reserved exclusively 
for the Black Lane, while a second area of secondary screening is shared between the Blue and 
Green Lanes. Assume that the service rates for primary screening are exponential with rates \( \mu^e_1 \), 
\( \mu^c_1 \), and \( \mu^f_1 \), while the service rates for secondary screening, are exponential with rates \( \mu^e_2 \) and 
\( \mu^c,f_2 \), corresponding to the expert, casual, and family oriented lanes, respectively. Furthermore, 
due to the experience of passengers in the Black Lane and the special assistance required by 
passengers in the Green Lane, assume that \( \mu^e_1 > \mu^c_1 > \mu^f_1 \) and \( \mu^e_2 > \mu^c,f_2 \). The security levels 
for primary and secondary screening are \( L_1 \) and \( L_2 \), respectively.

Let the queueing process for the Self-Select system be sampled at each time a passenger 
exits primary screening in either the Black, Blue or Green Lanes, denoted by \( t_i, i = 1, 2, \ldots \). 
Assume that at most one secondary screening assignment decision is performed at any given
time. If passengers exit primary screening in two or three lanes simultaneously, then priority in the secondary screening decisions goes from Black to Blue to the Green Lane. Within each of the three lanes, passenger $i$ exiting primary screening is directed toward secondary screening with probability $p^b(t_i)$, $p^c(t_i)$, or $p^f(t_i)$, if passenger $i$ is in the Black, Blue, or Green Lane, respectively. As in the selective security system model, nonselectee passengers may be directed toward secondary screening based on the secondary screening queue length and the passenger’s assessed threat value.

Since the Black Lane is separate from the Blue and Green Lanes, the optimal dynamic policy for assigning passenger $i$ to secondary screening, $X_c(t_i)$, is (17). Furthermore, the assignment decision in the Blue Lane is a function of the number of passengers currently in the queue for secondary screening, and is independent of the next passenger to exit primary screening in the Green Lane. Therefore, the Blue and Green Lanes also operate as independent selective security system, where the queue length for the shared secondary screening may increase faster than each individual exiting rate of primary screening. Thus, if passenger $i$ is a nonselectee, the optimal policy for assigning passenger $i$ to secondary screening in the Blue and Green Lanes, $X_c(t_i)$ and $X_f(t_i)$, respectively, is

$$p^*(t_i) = p^c(t_i) = p^f(t_i) = \begin{cases} \frac{1-m}{1-p_s} & \text{if } s_2(t_i) < c_2 \\ 0 & \text{otherwise} \end{cases}$$

(18)

$i = 1, 2, ..., $ while $p^*(t_i) = 1$ if passenger $i$ is designated as a selectee. If passengers who are part of a family or require special assistance in the Green Lane pose less of a risk than casual passengers in the Blue Lane, then from (18), more passengers from the Blue Lane, on average, are directed to secondary screening. Furthermore, a high risk passenger attempting to beat the system gains no advantage by joining the Green Lane, since (18) ensures that this passenger is directed to receive secondary screening with the same probability as that in the Blue Lane.

6 Computational Results

This section reports computational results that compare the performance of the optimal static and dynamic passenger assignment policies in Sections 3 and 4 for both two-class and selective security systems. Since the Black Diamond Self-Select passenger screening program consists of three parallel lanes of primary and secondary levels of screening, the results can be extended to infer the performance of the Black Diamond Self-Select program against that of a multi-level security system.

The distribution of assessed threat values, $F_\alpha(\alpha_i)$, for the perceived risk of passenger $i$ can
be depicted as a uniform, triangular, exponential, or a set of discrete values. For the purpose of
the analysis in this section, assume that the passengers designated as nonselectees have assessed
threat values distributed exponentially, truncated between zero and one. Therefore,

\[ F_{\alpha_i}(\alpha_i) = \frac{1 - e^{-\alpha_i/\theta}}{1 - e^{-1/\theta}}, \quad 0 < \alpha_i \leq 1, \]

\( i = 1, 2, \ldots, \) where \( E[\alpha_i] \approx \theta \) for small values of \( \theta > 0 \), corresponding to the overall level of
passenger risk. Since the details surrounding passenger prescreening program (e.g., CAPPS)
are confidential, the truncated exponential distribution with parameter \( \theta = 0.05 \) provides a
reasonable approximation for a population containing a majority of low-risk passengers (i.e.,
P(\alpha \leq 0.05) = 0.63) and a small percentage of those posing a higher risk (i.e., P(\alpha > 0.2) =
0.02). Assume that all passengers designated as selectees have assessed threat value \( \alpha_i = 1 \),
which ensures that these passengers undergo the highest level of screening. Since the number of
passengers designated as nonselectees or selectees is also security sensitive information, suppose
that \( p_s \) passengers are designated as selectees, and \( 1 - p_s \) are designated as nonselectees. If all
passengers are designated as selectees, then the security system screens all passengers equally,
and with the highest available set of screening devices. If all passengers are designated as
nonselectees, with assessed threat values \( 0 < \alpha_i < 1, \) \( i = 1, 2, \ldots, \) then the dynamic assignment
policy allocates the available security resources, given the distribution of passenger risk.

For the purpose of simulating the security screening process, consider the first \( N = 1000 \)
passengers to enter a \( M = 2 \) class security system. The passengers arrive at screening as
a Poisson process with rate \( \lambda = 2.5 \) passengers per minute, with selectee and nonselectee
passengers arriving with rates \( \lambda p_s \) and \( \lambda(1 - p_s) \), respectively. Let the service times of the
security classes be exponential with rates \( \mu_1 = 3 \) and \( \mu_2 = 1 \) passengers per minute, where
primary screening in the selective security system satisfies the stability condition \( \mu_1 > \lambda \). The
security level, \( L_m \), for class \( m = 1, 2 \), as well as for primary and secondary screening, captures
the probability that a true alarm occurs for a passenger who is a threat. This value is security
sensitive, since terrorists may use it to compute the probability of a successful attack or to
locate vulnerabilities in the security system. Set these values to be \( L_1 = 0.75 \) and \( L_2 = 0.9 \)
for security class \( m = 1, 2 \) and for both primary and secondary screening. In addition, let
the maximum capacity of the security classes be \( c_1 = 60 \) and \( c_2 = 40 \), which are chosen
sufficiently large such that capacity limitations do not to prevent high-risk passengers from
receiving the necessary level of screening. Likewise, smaller capacity values (i.e., holding areas)
can be analyzed; however, the resulting analysis becomes trivial for higher selectee counts, since
secondary screening resources then become used only for selectee passengers. The computations
were performed using MATLAB v7.1 on a 3GHz Pentium IV with 2GB RAM. The simulation
First, consider the optimal static passenger assignment policy for a two-class security system. The solution to SPQP, which minimizes $E[\mathbf{W}]$ produces the optimal assignment probability $p_1^* = 0.82$ and corresponding security class threshold value $b_1 = b_1(t_i) = 0.086$, yielding a steady-state expected amount of time a passenger spends in the two-class system at $E[\mathbf{W}] = 1.19$ minutes. Next, consider the steady-state response of the optimal dynamic passenger assignment policy obtained by computing the Markov transition probabilities in (8). Figure 3 depicts the sensitivity of the steady-state value $E[\mathbf{W}]$ with respect to the scalar weight $\eta_1$, using the optimal control in (14) with $\eta_2 = 0.5$ and $p_s = 0$. Values of $\eta_1$ near zero place more emphasis on obtaining the highest true alarm rate, while values of $\eta_1$ near one place more emphasis on minimizing the expected amount of time each passenger spends in the security system. For the given values of $L_m$, $m = 1, 2$ and service rates, $\mu_m$, $m = 1, 2$, the steady-state expected number of passengers in the system is minimized for $\eta_1 = 1$.

Performance comparisons between the two-class and selective security systems can be made through variations in the scalar weight $\eta_1$. Table 1 compares the overall measure of security (i.e., the true alarm rate) achieved for both the two-class and selective security systems across several values of $1 - \mu_2/\lambda \leq \eta_1 \leq 1$, obtained over a set of 60 independently seeded replications. Let the fraction of selectee passengers be $p_s = 0.1$ and the scalar weight $\eta_2 = 0.5$. For the two-class system, the security measure is given by $\sum_{i=1}^{N}(L_1 x_1(t_i) + L_2 x_2(t_i)) \alpha_i$, where $x_1(t_i) = 1 - x_2(t_i)$,
Table 1: Performance of Two-Class and Selective Security Systems.

### Two-Class Security System

<table>
<thead>
<tr>
<th>(\eta_1)</th>
<th>True Alarm Rate</th>
<th>Time Spent in Class 1 (minutes)</th>
<th>Time Spent in Class 2 (minutes)</th>
<th>Time Spent in System (minutes)</th>
<th>Number of Passengers</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.60</td>
<td>0.86 (0.002)</td>
<td>0.69 (0.685)</td>
<td>9.80 (7.017)</td>
<td>3.29 (5.681)</td>
<td>608 (392)</td>
</tr>
<tr>
<td>0.65</td>
<td>0.86 (0.002)</td>
<td>0.75 (0.738)</td>
<td>6.15 (4.490)</td>
<td>2.18 (3.478)</td>
<td>621 (379)</td>
</tr>
<tr>
<td>0.70</td>
<td>0.85 (0.002)</td>
<td>0.79 (0.768)</td>
<td>4.83 (3.310)</td>
<td>1.78 (2.583)</td>
<td>636 (364)</td>
</tr>
<tr>
<td>0.75</td>
<td>0.85 (0.003)</td>
<td>0.83 (0.826)</td>
<td>3.74 (2.617)</td>
<td>1.49 (1.991)</td>
<td>646 (354)</td>
</tr>
<tr>
<td>0.80</td>
<td>0.85 (0.003)</td>
<td>0.86 (0.852)</td>
<td>3.18 (2.059)</td>
<td>1.33 (1.624)</td>
<td>660 (340)</td>
</tr>
<tr>
<td>0.85</td>
<td>0.84 (0.003)</td>
<td>0.92 (0.910)</td>
<td>2.77 (1.797)</td>
<td>1.26 (1.433)</td>
<td>670 (330)</td>
</tr>
<tr>
<td>0.90</td>
<td>0.84 (0.003)</td>
<td>0.95 (0.908)</td>
<td>2.52 (1.616)</td>
<td>1.20 (1.304)</td>
<td>680 (320)</td>
</tr>
<tr>
<td>0.95</td>
<td>0.83 (0.003)</td>
<td>1.01 (0.925)</td>
<td>2.34 (1.566)</td>
<td>1.18 (1.242)</td>
<td>693 (307)</td>
</tr>
<tr>
<td>1.00</td>
<td>0.83 (0.003)</td>
<td>0.98 (0.911)</td>
<td>2.21 (1.473)</td>
<td>1.13 (1.187)</td>
<td>693 (307)</td>
</tr>
</tbody>
</table>

### Selective Security System

<table>
<thead>
<tr>
<th>(\eta_1)</th>
<th>True Alarm Rate</th>
<th>Time Spent in Primary (minutes)</th>
<th>Time Spent in Secondary (minutes)</th>
<th>Time Spent in System (minutes)</th>
<th>Number of Passengers</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.60</td>
<td>0.93 (0.003)</td>
<td>2.01 (2.018)</td>
<td>29.30 (22.092)</td>
<td>14.95 (20.813)</td>
<td>1000 (442)</td>
</tr>
<tr>
<td>0.65</td>
<td>0.92 (0.003)</td>
<td>1.90 (1.862)</td>
<td>11.35 (9.805)</td>
<td>6.33 (8.472)</td>
<td>1000 (390)</td>
</tr>
<tr>
<td>0.70</td>
<td>0.91 (0.004)</td>
<td>1.89 (1.765)</td>
<td>5.14 (5.255)</td>
<td>3.61 (4.265)</td>
<td>1000 (336)</td>
</tr>
<tr>
<td>0.75</td>
<td>0.89 (0.005)</td>
<td>1.95 (1.988)</td>
<td>3.45 (3.601)</td>
<td>2.92 (3.145)</td>
<td>1000 (279)</td>
</tr>
<tr>
<td>0.80</td>
<td>0.87 (0.005)</td>
<td>1.98 (2.070)</td>
<td>2.27 (2.250)</td>
<td>2.48 (2.499)</td>
<td>1000 (222)</td>
</tr>
<tr>
<td>0.85</td>
<td>0.85 (0.006)</td>
<td>1.93 (1.950)</td>
<td>1.69 (1.732)</td>
<td>2.21 (2.173)</td>
<td>1000 (167)</td>
</tr>
<tr>
<td>0.90</td>
<td>0.83 (0.006)</td>
<td>1.94 (2.004)</td>
<td>1.41 (1.429)</td>
<td>2.10 (2.104)</td>
<td>1000 (112)</td>
</tr>
<tr>
<td>0.95</td>
<td>0.80 (0.006)</td>
<td>1.97 (1.927)</td>
<td>1.20 (1.255)</td>
<td>2.03 (1.971)</td>
<td>1000 (57)</td>
</tr>
<tr>
<td>1.00</td>
<td>0.75 (0.000)</td>
<td>1.97 (2.135)</td>
<td>0 (0)</td>
<td>1.97 (2.135)</td>
<td>1000 (0)</td>
</tr>
</tbody>
</table>

while for the selective security system, the security measure is \(\sum_{i=1}^{N}(L_1 + L_2\pi(t_i))\alpha_i\). Table 1 also compares the average number of passengers assigned to each security class, as well as the average amount of time passengers spend in each security class. The variance is attributed to the uncertainty in passenger risk, passenger arrival and security class screening times. As \(\eta_1\) increases from zero to one in the two-class security system, less passengers are assigned to receive higher intensity screening, thereby lengthening the queue in security class 1, while shortening the queue in security class 2. For \(\eta_1 = 0.9\), the mean true alarm rate for the two-class and selective security systems are approximately equal. However, the selective security system achieves this level of security by screening one third as many passengers with higher intensity. These high-risk passengers who do undergo secondary screening increase the true alarm rate since they undergo both primary and secondary levels of screening. Moreover, for a given set of
screening devices, a higher level of security can be achieved using a security system with these devices partitioned into primary and secondary screening, while a faster passenger throughput can be achieved by partitioning the devices into the two-class structure.

7 Conclusions

This paper presents both steady-state and transient queueing analyses for a multi-level security system, by capturing the dynamical behavior of passengers sequentially arriving at a security checkpoint and the rates at which the passengers are screened in each of the security classes. The results are then extended to analyze a selective security system, consisting of primary and secondary levels of screening. The passenger interarrival times and the security class screening times are assumed to be exponentially distributed. The queueing process is formulated as a Markov process, where the expected amount of time a passenger spends in the security system is obtained recursively. Optimal policies are obtained for assigning passengers to the multi-level security system by maximizing the probability that a true alarm occurs, while minimizing the expected amount of time a passenger spends in the security system.

Numerical simulations illustrate the effectiveness of both the optimal static and dynamic assignment policies in reducing the expected amount of time a passenger spends in the security system, thereby improving passenger throughput. By incorporating a second objective of maximizing the probability of detecting a threat (i.e., a true alarm), the optimal dynamic assignment policy creates a balance between maximizing security and passenger throughput.

The key contributions include a passenger assignment procedure that efficiently utilizes the set of available screening devices in a multi-level security system, and is implementable in real-time with minimal computational requirements. As a consequence, these passenger assignment policies expedite passenger screening and further strengthen aviation security operation performance through an increase in the probability of threat detection. This dual objective provides significant improvements in both air transportation safety and passenger satisfaction with the security screening process, and produces a system that is reliable, efficient, and cost effective.

In practice, service times do not follow an exponential distribution, since there exists a minimum amount of time to screen a passenger or bag. Future directions include modeling the screening process using more complex service time distributions, which may also include either a time or queue length dependency on the service rate. Also, interactions between the security classes can be modeled, where passengers who signal an alarm in one security class are directed to join the queue for a separate security class. This interaction would facilitate the use of highly specialized, time-consuming screening devices for high-risk passengers as well as
for resolving alarms occurring from lower risk passengers.

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