# Solving the Robust and Integrated Aircraft Routing and Crew Pairing Problem in Practice – A Discussion of Heuristic and Optimisation Methods

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**Abstract**

We formulate an integrated aircraft routing and crew pairing model that yields solutions for both problems that incur small costs and are robust to typical stochastic variability in airline operations, i.e. effects of delays occurring in operations are minimised.

We propose two new solution methods to solve the integrated model. The first approach is an optimisation based heuristic that is capable of generating good quality solutions quickly, the second approach utilises Dantzig-Wolfe decomposition to solve the integrated model to optimality.

Using data from domestic Air New Zealand schedules, we evaluate the benefits of solving the integrated model on real world problem instances. Our solutions satisfy all rules imposed on aircraft routings and crew pairings and are ready to be implemented in practice. We obtain solutions that dramatically improve costs and robustness of solutions obtained by traditional methods. We also compare our approaches with an existing Benders decomposition approach.

## 1 Introduction

Airline scheduling consists of sequentially solving the following five planning problems: First, marketing decisions in the *schedule design problem* determine the schedule of flights the airline operates. Given the set of flights in a schedule, the solution of the *fleet assignment model* determines which flight is operated by which aircraft type. Next, the *aircraft routing problem* seeks a minimal cost assignment of available aircraft to the flights. Similarly, the *crew pairing problem* (or *tour of duty problem*) allocates generic crew to flights in a minimal cost way. Generic crew pairings are constructed subject to many rules so that each flight is covered by the required number of crew members. The last of the planning problems is *crew rostering*. Based on the constructed crew pairings, a line of work (a sequence of pairings and days off) is assigned to each individual crew member.

Traditionally, all five scheduling problems are solved in sequence although the problems are interdependent. Each successive problem is solved to assign aircraft types, aircraft, and crew to the flights that are determined in the schedule design problem. Once the fleet assignment problem is solved, one aircraft routing problem is solved for each aircraft type. The crew pairing problem depends on the aircraft routing problem, since the connection time between two flights a crew can operate differ depending on whether the crew stays on the same aircraft or not. Since airlines operate in a highly competitive market, the main goal of most these problems is the minimisation of a cost objective. The decline in airline passenger numbers following the events of September 2001, rising fuel prices, and competitive pressure from low-cost airlines increase the need for traditional airlines to operate as efficiently as possible. Solving the five problems sequentially can lead to a suboptimal solution because once one problem is solved, the solution of this problem may restrict the feasible solutions of subsequently solved problems. A large percentage of variable costs of airline operations occurs in the crew pairing problem which is solved rather late in the sequence.

The minimisation of planned costs alone to create a highly efficient schedule neglects the characteristics of the environment in which such a schedule is operated. A very efficient schedule usually features very short ground times between flights for aircraft and crew to keep aircraft utilisation high and crew costs low. During airline operations, however, disruptions are likely to occur because of delayed passengers, aircraft failure, or weather conditions, etc.. When disruptions occur and ground times between inbound and outbound flights operated by the same aircraft are minimal, the flights operated subsequently by the same aircraft will also depart late. If, additionally, crew are changing aircraft on a connection with short ground time after a delayed flight, the flight operated subsequently by the crew will most likely depart late as well. Such a propagation of delay can quickly cause widespread disruptions. We refer to a schedule where effects of an initial disruption on other flights in the schedule are minimal as *operationally robust*. A schedule that is not robust can cause large additional costs for an airline for example arising from requiring reserve crews, re-accommodating passengers, and damaged reputation.

Ehrgott and Ryan [2002] and Yen and Birge [2006] have shown that the robustness of crew pairing solutions can be significantly improved if aircraft changes are only made when ground time between the incoming and outgoing flights is greater than the minimum ground time. This can be achieved in the crew pairing problem by penalising aircraft changes when ground time is short. Robust crew pairing solutions then have "crew following the same aircraft" as much as possible and changing aircraft only when ground time between flights is longer than the minimum. In this sense, the robust crew pairing solution depends on the given aircraft routing solution but such a sequential solution method may result in a suboptimal solution compared to a solution method that considers both problems simultaneously.

Clearly, there exists a trade-off between minimal planned cost and operational robustness. Ideally, we would like to solve a bi-criterion problem with the two objectives cost and robustness considering all airline scheduling problems simultaneously in one integrated model. Such a formulation is currently intractable. Each of the individual problems is already hard to solve and integration increases the complexity of the formulation.

As a step towards integration of all airline scheduling problems, two of the problems are considered in this work: aircraft routing and crew pairing. We expect the largest improvement in robustness by combining these two scheduling problems and do not need to take the increased complexity into account of considering all 5 scheduling problems simultaneously. While the fleet assignment model is important for large airlines with multiple aircraft types, for this work, the fleet can be regarded as homogeneous and fleet assignment can be omitted. The objective function of the crew rostering problem is to maximise crew satisfaction rather than minimising cost. The crew rostering problem has therefore no influence on the cost of the overall solution and is also not considered.

In this work, we investigate whether it is possible to reduce the cost and at the same time increase the robustness by considering the two problems simultaneously rather than sequentially. We formulate an integrated model for the *robust and integrated aircraft routing and crew pairing problem*. This model yields feasible solutions for both problems that are optimal for the integrated model, where the objective function is a weighted sum of cost and a robustness measure which penalises aircraft changes. Because the integrated problem is hard to solve, decomposition methods are proposed in the literature (see for example Mercier et al. [2005]), but excessive computation times are necessary to solve the model to optimality. We propose two novel solution methods for the integrated model: an *iterative approach* and a *Dantzig-Wolfe decomposition approach*.

The iterative approach is an optimisation-based heuristic. In each iteration we solve the aircraft routing problem first, taking into account the current crew pairing solution, i.e. encouraging aircraft to follow the crew. Then, given the aircraft routing solution, we re-solve the crew pairing problem and this time encourage the crew to follow the aircraft. This procedure generates a series of feasible solutions for the integrated model with varying cost and robustness measure. The airline is not required to associate a monetary value with robustness a priori but can observe the trade-off between cost and robustness and then choose a solution they prefer to operate.

While the iterative procedure generates feasible solutions very quickly, it can not guarantee to find a solution of a certain quality specified beforehand. Nevertheless, a (possibly infeasible) lower bound on the crew pairing cost is provided by the algorithm so that the worst case solution quality can be observed. To obtain feasible lower bounds on the solution quality, we propose a Dantzig-Wolfe decomposition approach capable of solving the integrated model to optimality. Both solution approaches do not add constraints to the original problems preserving their structure and not increasing their complexity.

In order to verify the quality of our solution approaches, we apply all solution methods to various domestic airline schedules of Air New Zealand to test the performance of the approaches. The iterative approach yields low cost solutions which are highly robust compared with the traditional sequential approach. Since our main focus is to solve a practical problem, all rules and requirements imposed by Air New Zealand are considered in the solution approach and the solutions we generate are ready to be operated in practice. We compare the quality of the solutions of the iterative approach with optimal solutions obtained by the Dantzig-Wolfe decomposition approach. We also compare the performance of the Dantzig-Wolfe decomposition approach with that of Benders decomposition which is currently known as the most successful approach in the literature.

The remainder of this paper is organised as follows. First, we describe relevant literature in the subsequent Section. In Section 3 we describe the concept of operational robustness. In Section 4 we formulate the integrated model and discuss our solution methods. Computational experiments are presented in Section 5 before we summarise our findings in the Conclusion.

## 2 Literature

Airline scheduling problems have been addressed in an extensive number of publications, see Klabjan [2005] for a detailed overview. We list selected literature addressing airline scheduling problems and describe attempts to integrate the aircraft routing and crew pairing problems. We conclude with recent formulations that incorporate robustness measures.

We are not aware of contributions that discuss the schedule design problem as an independent problem, in the literature it is addressed in combination with fleet assignment. The fleet assignment model is formulated by Hane et al. [1995] as a multi-commodity flow problem. It is called *leg-based* because revenue effects between flight-legs are not modelled. To take such network effects into account, Barnhart et al. [2002] describe an enhanced model using demand forecasts for origin-destination pairs.

An integrated model for schedule design and fleet assignment is presented by Lohatepanont and Barnhart [2004]. They use the origin-destination fleet assignment model and flights are chosen from an optional set of flights to maximise profit.

The aircraft routing problem has been addressed in a number of publications, for example in Clarke et al. [1997], Feo and Bard [1989], Daskin and Panayotopoulos [1989], Gopalan and Talluri [1998], and Grönkvist [2006]. Sarac et al. [2006] consider the aircraft routing problem on an operational level rather than a planning level.

After fuel costs, crew salary is the second largest operational cost an airline has to account for. Therefore finding a minimal cost solution to the crew pairing problem is important. It is a very difficult problem due to the large number of possible pairings, the complicated rule structure and the necessity to find integer solutions. For these reasons the crew pairing problem has received a lot of attention in the literature, see Barnhart et al. [2003] for a detailed description of the crew pairing problem and a review of the literature addressing the problem. Also recently, Gopalakrishnan and Johnson [2005] give a comprehensive overview on state-of-the-art methods to solve the crew pairing problem.

The crew rostering problem can be viewed as a separate optimisation problem with no effect on the cost of the integrated solution. We refer to Ernst et al. [2004] for an annotated bibliography of rostering problems.

The literature reports on the integration of various combinations of airline scheduling problems. Klabjan et al. [2002] partially integrate aircraft routing, crew pairing and schedule design. They reverse the order of the crew pairing and aircraft routing problems. Plane count constraints are added to the crew pairing problem to guarantee the existence of a feasible solution for the aircraft routing problem by ensuring that at most the number of available aircraft is used at any time. Their results are based on a *hub-and-spoke network*. In this network only large airports (hubs) are linked by direct flights and all smaller airports (spokes) are only connected to one hub. Many aircraft meet at the same hub at the same time ensuring the existence of many feasible connections. This property leads to a much larger number of feasible routings than in an *interconnected network*. In interconnected networks many airports are linked with multiple other airports by direct flights. To include schedule design, the departure time of each flight is allowed to vary in some time window. This is done by relaxing feasibility parameters in the crew pairing problem and hence generating a larger set of pairings. Klabjan et al. [2002] solve the crew pairing problem via a linear programming (LP) based branch-and-bound algorithm.

Another model to integrate aircraft routing and crew pairing is proposed by Cordeau et al. [2001] and also by Mercier et al. [2005]. They use Benders decomposition and branch-and-price to solve the model. Employing the crew pairing problem as the subproblem as well as the master problem has been tested, the latter with better success. Both approaches add inequalities to the set partitioning polytopes of the problems. Cordeau et al. [2001] also reverse the sequential approach and try to solve the crew pairing problem first followed by the aircraft routing problem as in Klabjan et al. [2002]. They apply this approach to an interconnected network but are not successful in obtaining feasible solutions for the aircraft routing problem.

Cohn and Barnhart [2003] also integrate aircraft routing and crew pairing. They extend the crew pairing problem by using the aircraft routing problem as a second column generator next to the crew pairing generator. For each solution of the aircraft routing problem one variable is added to the crew pairing problem and a convexity constraint ensures the selection of one of the aircraft routing solutions in the final solution of the problem. LP based branch-and-price is used in this computationally expensive solution method. Mercier et al. [2005] find that their Benders decomposition approach yields better solutions in less computation time than the extended crew pairing model of Cohn and Barnhart [2003].

Sandhu and Klabjan [2007] partially integrate fleet assignment, aircraft routing, and crew pairing with a similar approach as Klabjan et al. [2002] and solve the model with both Lagrangian relaxation and Benders decomposition. Papadakos [2007] integrates the fleet assignment, aircraft routing, and crew pairing problems as an extension of the model of Mercier et al. [2005].

Very recently, Mercier and Soumis [2007] extend their model (Mercier et al. [2005]) and integrate aircraft routing and crew pairing with time windows for the departure times. Flights are allowed to depart five minutes earlier or later than originally scheduled. Binary variables are used to indicate which departure time is assigned to a flight. Equality constraints sum up the binary departure time variables for the crew and aircraft solutions and ensure that the same departure times are used in the solutions of both problems. Again, the authors use Benders decomposition to solve the problem.

Models that focus purely on minimising cost tend to generate solutions that appear brittle in operations. Such solutions incur large recovery costs once disruptions occur. In order to improve the behaviour in operations a number of robustness measures have been introduced.

Schaefer et al. [2005] use expected operational cost for the crew pairings instead of planned cost. Interactive effects between pairings are ignored and a push-back strategy for recovery is used. In this strategy the flights are delayed until crew and aircraft are available. The authors estimate the costs and evaluate the quality of their solutions with SimAir, a Monte Carlo simulation of airline operations, see Rosenberger et al. [2002].

Yen and Birge [2006] formulate the crew pairing problem as a stochastic programming problem that they solve in a computationally expensive approach. Crew switching aircraft are penalised in the objective function. A similar measure of robustness is introduced by Ehrgott and Ryan [2002] in a deterministic approach. Crew pairings are penalised if crew are changing aircraft and the sit-time of the crew is less than the minimal sit-time plus some measure of delay of the incoming flight. Crew who stay on the same aircraft are not penalised. Thus, crew connections where disruptions are likely to propagate onto multiple flights are penalised. Robustness is treated as a second objective function in a bi-criteria approach. Mercier et al. [2005] also penalise crew changing aircraft on restricted connections (see Section 3).

Recently, Shebalov and Klabjan [2006] solve the crew pairing problem first and then maximise the number of move-up crews, i.e. crew that potentially can be swapped, without increasing the planned cost too much. They compare their method with the method of solving the standard crew pairing problem by simulating disruptions and find solutions with lower operational costs if the additional cost allowed for move-up crews is not too high.

A direct comparison between the various approaches is difficult due to different levels of integration, robustness measures and characteristics of schedules and rule-sets used. Cordeau et al. [2001] and Mercier et al. [2005] find their Benders decomposition approach superior to two recent models (Klabjan et al. [2002] and Cohn and Barnhart [2003]). It is also evident that a direct solution approach to the integrated model (see (3) below) for large scale practical problems is much more time consuming than decomposition techniques if not intractable (Cordeau et al. [2001]).

# 3 Operational Robustness

An airline schedule is unlikely to be operated as planned because of disruptions. Delays occur frequently in airline operations and can be caused by late passengers, unscheduled maintenance requirements, bad weather, and so forth. Such disruptions can not be controlled by the airline. Since the aircraft and crew that operate a delayed flight are usually planned to operate further flights that depart later during the day, these flights may also be delayed due to the late running of earlier flights. In this section we describe how we minimise such consequential delays.

A solution, where effects of potential delays are minimal, is called *operationally robust.* The concept of robust solutions is important since an airline is interested in achieving high *on-time performance (OTP)*, i.e. a high percentage of all flights that depart on-time. However, the *planned cost* of a more robust solution is usually higher since slack is built into the schedule to compensate for delays. Bad OTP can incur large additional costs (referred to as *recovery costs*), caused by additionally required crews, compensation for passengers affected by delayed or cancelled flights, and damaged reputation of the airline. These costs may by far exceed the savings of using a solution with less planned cost than using a solution that is more robust. Costs listed here are generally planned costs. We try to identify solutions with low planned costs which are operationally robust, i.e. where disruptions will result in minimal recovery costs.

Before we describe how to obtain operationally robust solutions, we explain the concepts of short and restricted connections as introduced in Mercier et al. [2005]. If two flights can be operated in sequence by the same aircraft or crew, the time between arrival of the inbound and departure of the outbound flight is called *turn-time* for aircraft and *sit-time* for crew.

The minimal time required for an aircraft or a crew to operate a connection is called *minimal turn-time* or *minimal sit-time*, respectively. The required minimal sit-time can exceed the minimal turn-time. For example, crew need enough time to travel from the arrival gate, through the terminal(s), to the departure gate of the next flight. If crew stay on the same aircraft, the minimal turn-time for this connection also applies to crew, instead of the minimal sit-time. A connection between flights $i$ and $j$ is called *short* if

$$(\text{minimal turn-time})_{ij} \leq (\text{sit-time})_{ij} < (\text{minimal sit-time})_{ij}.$$

Thus, in a feasible solution, short connections are only allowed if crew stay on the same aircraft. Additionally, we prefer solutions where crew are not changing aircraft when the turn time is less than some *restricted time.* A connection between two flights $i$ and $j$ is called *restricted* if

$$(\text{minimal sit-time})_{ij} \leq (\text{sit-time})_{ij} < (\text{restricted time})_{ij}.$$

In contrast to short connections, crews are allowed to change aircraft if the connection is restricted, but we try to find solutions in which this occurs as rarely as possible. If crew change aircraft on restricted connections we refer to these connections as *restricted aircraft changes.*

Minimal turn-times are usually operated in aircraft routings to maximise aircraft utilisation and to keep connection times attractive for passengers. However, if many connections with minimal turn-time are operated in sequence by the same aircraft, it is more likely that the flights at the end of this sequence become delayed during operations because no buffer time is available to compensate for delays that occurred early in this sequence.

In our solution approaches we attempt to generate operationally robust aircraft routing solutions by avoiding the occurrence of too many consecutive connections with minimal turn-time (denoted by MinTurnSeq) within an aircraft routing. Such a sequence ends if a connection is operated whose ground time exceeds the minimal turn-time by at least 5 minutes. We refer to connections with minimal turn-time as *minimal turns.*

To obtain operationally robust crew pairing solutions, we minimise the number of restricted aircraft changes that are operated. If the crew change aircraft on a restricted connection after a delayed flight, additional flights might be affected by the initial delay. Due to the insufficient buffer to compensate for the delay, the crew are likely to be late for the next flight they operate. This behaviour can propagate to a large number of delayed flights within a short amount of time.

We use two mechanisms to limit the number of restricted aircraft changes: Firstly, AircraftChange-Cost can be imposed for crew changing aircraft when the sit-time is below some threshold (restricted time). By minimising these costs as part of the objective function we encourage the crew to stay on the same aircraft whenever the sit-time is small. We impose costs that increase linearly with decreasing sit-time. The cost for changing aircraft on a restricted connection $ij$ is denoted by $c_{ij}^{AC}$:

$$c_{ij}^{AC} = (k_1 - ((\text{sit-time})_{ij} - (\text{minimal sit-time})_{ij})) * k_2 \tag{1}$$

Weights $k_1$ and $k_2$ are chosen such that $c_{ij}^{AC}$ equals 7 for a restricted aircraft change with sit-time equal to the minimal sit-time, 6 for a restricted aircraft change with sit-time exceeding the minimal sit-time by 5 minutes, and so on until a weight of 1 is assigned to a restricted aircraft change with sit-time exceeding the minimal sit-time by 30 minutes.

The aircraft change cost $\bar{c}^{AC}$ of a crew pairing solution is the sum over all restricted aircraft changes that are contained in the solution:

$$\bar{c}^{AC} = \sum_{ij \in RAC} c_{ij}^{AC}, \tag{2}$$

where $RAC$ is the set of all restricted aircraft changes.

The second mechanism to limit the number of aircraft changes is the DutyPeriodAircraftChange-Limit (DPACLim) which specifies how often a crew member can change aircraft during one duty period. Air New Zealand introduces this rule instead of penalising aircraft changes in order to limit the total number of aircraft changes of a solution and, hence, increase its robustness, independently of the sit-time of these aircraft changes. It is only possible to count the number of aircraft changes during a duty period if an aircraft routing solution is given as input. Hence, this rule needs special attention in Section 4 when aircraft routing and crew pairing problems are integrated and the aircraft routing solution is no longer fixed.

# 4 Robust and Integrated Aircraft Routing and Crew Pairing Problem

We describe a model that integrates aircraft routing and crew pairing problems where short connections are only permitted if crew stay on the same aircraft. This condition might result in suboptimal or infeasible solutions if the two problems are solved separately: If the crew pairing problem is solved for a fixed aircraft routing solution, the feasible set of connections to be used by crew is limited. But if the aircraft routing problem is solved for a fixed crew pairing solution, it may be infeasible to operate all required (short) connections with the given number of aircraft. Solving the two problems in an integrated model resolves these issues.

The *aircraft routing problem* is the problem of assigning aircraft to a given set of flights in a schedule. We assign one *aircraft routing* to each aircraft such that each flight of the schedule is contained in exactly one routing. Each routing is subject to maintenance requirements and other flying restrictions, and the number of available aircraft is fixed. We consider the MinTurnSeq robustness measure as the only cost of the aircraft routings.

Aircraft routings can be represented as columns of a binary $(m + a) \times n^R$ matrix $A^R$ where $m$ is the number of flights, $a$ the number of available aircraft and $n^R$ the number of possible routings. The elements $(a_{ij})^R$ of the first $m$ rows of matrix $A^R$ are defined as follows:

$$(a_{ij})^R = \begin{cases} 1 & \text{if flight } i \text{ is contained in routing } j \\ 0 & \text{otherwise,} \end{cases}$$

with $1 \leq i \leq m, 1 \leq j \leq n^R$. Additionally, the element $(a_{m+i,j})^R$ is defined as:

$$(a_{m+i,j})^R = \begin{cases} 1 & \text{if routing } j \text{ is operated by aircraft } i \\ 0 & \text{otherwise,} \end{cases}$$

with $1 \leq i \leq a, 1 \leq j \leq n^R$.

Given a flight schedule, the *crew pairing problem* is defined as the problem of assigning generic crews to flights in the schedule such that each flight is operated by exactly one crew. Here it is assumed that all crew members operate as a single unit. A sequence of flights which can be operated by a crew on one working day is called a *duty period*, after which a *rest period* must be assigned to each crew member. An alternating sequence of duty periods and rest periods is called a *crew pairing* or *tour of duty*. Any crew pairing must start and end at the same crew base and is restricted by a number of rules such as rest time regulations or flying time restrictions. A cost is associated with each crew pairing which is a combination of salaries, meal, rest, and travel allowances. In the crew pairing problem we seek a minimal cost set of crew pairings that partition the flights in the schedule, i.e. each flight is contained in exactly one pairing.

The crew pairings can be represented as columns of a binary $m \times n^P$ matrix $A^P$ where $m$ is the number of flights in the schedule and $n^P$ is the number of possible crew pairings. Entries $(a_{ij})^P$ of matrix $A^P$ are defined as follows:

$$(a_{ij})^P = \begin{cases} 1 & \text{if flight } i \text{ is contained in pairing } j \\ 0 & \text{otherwise,} \end{cases}$$

with $1 \leq i \leq m, 1 \leq j \leq n^P$.

*Base-constraints* are needed to consider *base strengths* at the crew bases, i.e. to restrict the number of crew pairings that can start at a crew base in a particular week or on a particular day. To include these restrictions, the columns of matrix $A^P$ are appended by the following entries:

$$(a_{i+m,j})^P = \begin{cases} k_j & \text{if pairing } j \text{ starts at the crew base and in the time interval} \\ & \text{specified by base-constraint } i \\ 0 & \text{otherwise,} \end{cases}$$

with $1 \leq i \leq m^{BC}, 1 \leq j \leq n^P$. The integer value $k_j \in \mathbb{N}$ specifies the number of working days that are necessary to operate crew pairing $j$ and $m^{BC}$ is the total number of base constraints. The base-constraints usually have inequality signs and integer right hand sides ensuring that at least ($\geq$) or at most ($\leq$) a given number of crew pairings start on a particular day or during a particular week from a given crew base.

In order to obtain an optimal solution for the aircraft routing and crew pairing problem we need to consider short and restricted connections in an integrated model. We enumerate all short connections that can be operated by crew and define a binary $m^B \times n^P$ matrix $B^P$ where $m^B$ is the number of short connections. Each pairing is associated with one column of $B^P$, where

$$(b_{ij})^P = \begin{cases} 1 & \text{if short connection } i \text{ is contained in pairing } j \\ 0 & \text{otherwise,} \end{cases}$$

with $1 \le i \le m^B, 1 \le j \le n^P$. For aircraft, a binary $m^B \times n^R$ matrix $B^R$ is defined in an analogous way.

Similarly to short connections, we define a binary $m^D \times n^P$ matrix $D^P$ where $m^D$ is the number of restricted connections:

$$(d_{ij})^P = \begin{cases} 1 & \text{if restricted connection } i \text{ is contained in pairing } j \\ 0 & \text{otherwise,} \end{cases}$$

with $1 \le i \le m^D, 1 \le j \le n^P$. For aircraft, a binary $m^D \times n^R$ matrix $D^R$ is defined in an analogous way.

With this matrix representation the *robust and integrated aircraft routing and crew pairing problem* [see also Mercier et al., 2005] can be formulated as follows:

$$\begin{array}{llllllll} \text{Minimise} & (\boldsymbol{c}^P)^T \boldsymbol{x}^P & + & (\boldsymbol{c}^R)^T \boldsymbol{x}^R & + & p(\boldsymbol{c}^{AC})^T \boldsymbol{d} & & \\ \text{subject to} & A^P \boldsymbol{x}^P & & & & & = & \boldsymbol{b}^P \\ & & & A^R \boldsymbol{x}^R & & & = & \mathbb{1} \\ & B^P \boldsymbol{x}^P & - & B^R \boldsymbol{x}^R & & & \le & 0 \\ & D^P \boldsymbol{x}^P & - & D^R \boldsymbol{x}^R & - & \boldsymbol{d} & \le & 0, \end{array} \tag{3}$$

where $\boldsymbol{x}^P \in \{0,1\}^{n^P}$, $\boldsymbol{x}^R \in \{0,1\}^{n^R}$, and $\boldsymbol{d} \in \{0,1\}^{m^D}$ are binary variables. The decision variable $x_j^P \in \{0,1\}$ has value 1 if pairing $j$ is contained in the solution and 0 otherwise. The entries of $\boldsymbol{b}^P$ have value 1 for all flight partitioning constraints and are integer for base-constraints. The element $c_j^P$ of $\boldsymbol{c}^P \in \mathbb{R}^{n^P}$ is the cost associated with pairing $j$. Besides crew pairing columns and variables, matrix $A^P$ and variables $\boldsymbol{x}^P$ also contain slack and surplus columns and variables to satisfy the equality base-constraints. The element $c_j^R$ of $\boldsymbol{c}^R \in \mathbb{R}^{n^R}$ is the cost associated with routing $j$. The decision variable $x_j^R \in \{0,1\}$ takes value 1 if routing $j$ is in the solution and 0 otherwise. Costs $\boldsymbol{c}^{AC} \in \mathbb{R}_+^{m^D}$ are positive penalties for changing aircraft and value $p \in \mathbb{R}_+$ is a weight to adjust the scale of the aircraft change cost compared to crew pairing and aircraft routing costs. Variable $d_i$ equals 1 if restricted connection $i$ is operated by a crew but no aircraft and 0 otherwise.

The first set of constraints formulates the original crew pairing problem and ensures that all flights are operated by exactly one crew and that base constraints are satisfied. The second set of constraints models the original aircraft routing problem and ensures that each flight is operated by exactly one aircraft and that each aircraft is assigned to exactly one routing. The third set of constraints enforces that short connections which are operated by some crew are also operated by some aircraft. The last set of constraints provokes additional aircraft change cost in the objective function if a restricted connection is operated by a crew but not by an aircraft.

The model yields an optimal solution for given aircraft change cost weight $p$. The model assumes that the DPACLIM rule is relaxed. We describe below how the DPACLIM rule can be considered in each solution approach.

## 4.1 Iterative Solution Approach

In this section we describe an optimisation based heuristic solution method for the robust and integrated aircraft routing and crew pairing problem (3). Since for the schedules we consider, the minimal sit-time is equal to the minimal turn-time for all connections, the short connection constraints are omitted. The two individual problems are alternately solved to optimality. Each problem receives input from the previously solved problem. This process continues until a stopping criterion is reached. A predefined solution quality cannot be guaranteed but a lower bound for the optimal solution value is provided so that the quality is known once the algorithm terminates.

We assume that MinTurnSeq costs are the only aircraft routing costs and the majority of the costs of the integrated solution are crew pairing costs. We search for an integrated solution with small crew pairing costs, small MinTurnSeq costs, and small aircraft change cost $\bar{c}^{AC}$ (2). The lower the aircraft change cost $\bar{c}^{AC}$ of a solution the more robust we expect the solution to be. Initially, we solve the crew pairing problem to cost optimality without considering any aircraft routings. This results in a larger set of feasible crew pairings since feasibility parameters are relaxed. The solution is likely to be infeasible for any aircraft routing solution. However, this initial solution yields a lower bound on the crew pairing cost of a feasible integrated solution. Then, in each iteration the aircraft routing problem is solved first. We consider all restricted connections operated in the current crew pairing solution and force the aircraft routing solution to contain as many of those connections as possible. This will enforce the "aircraft to follow the crew" as much as possible if the connection is restricted. In other words, we solve the aircraft routing problem using the following objective function:

$$\text{Minimise } (\boldsymbol{c}^R)^T \boldsymbol{x}^R - \sum_{ij \in RC} c_{ij}^{AC}, \tag{4}$$

where RC is the set of restricted connections operated in the current crew pairing problem and in the aircraft routing solution. Vectors $\boldsymbol{c}^R$ and $\boldsymbol{x}^R$ are defined as in (3). The first part of objective function (4) minimises the number of consecutive minimal turns and the second part maximises the number of restricted connections in the aircraft routing solution that are operated in the previously solved crew pairing problem. Note that the set $RC$ is determined by the crew pairings $\boldsymbol{x}^P$ and aircraft routings $\boldsymbol{x}^R$ in the solution. Next we solve the crew pairing problem to optimality for the current aircraft routing solution with a weighted sum objective function of crew pairing costs and aircraft change costs:

$$\text{Minimise } (\boldsymbol{c}^P)^T \boldsymbol{x}^P + p\bar{c}^{AC}, \tag{5}$$

where $\boldsymbol{c}^P, \boldsymbol{x}^P, p$, are defined as in (3) and $\bar{c}^{AC}$ is defined as in (2). The solutions of the two problems solved in each iteration yield a feasible solution to the integrated problem. We start with penalty $p$ equal to 0 and increase the penalty in each iteration in order to increase the robustness of the solutions we generate. Note that we do not change the ratio of weights between costs $\boldsymbol{c}^R$ and $\sum_{ij \in RC} c_{ij}^{AC}$ in the aircraft routing problem. Here the ratio is set to reflect the importance of the two robustness measures aircraft change cost and consecutive minimal turns and there is no trade-off with a monetary cost objective as in the crew pairing problem.

Algorithm 1 shows the steps of the iterative approach. The aircraft routing and crew pairing problems are both solved with the simplex algorithm applied to the LP relaxation of each problem combined with column generation methods. We use constraint branching to obtain integer solutions.

Since we always solve the crew pairing problem for a given solution of the aircraft routing problem,

**Algorithm 1** Iterative Algorithm

1: **set** $p = 0$
2: **solve** crew pairing problem with objective function (5){Since no aircraft routings are taken into account a larger set of feasible pairings is generated.}
3: **while** $p \leq p_{max}$ **do**
4: **solve** aircraft routing problem with objective function (4){Minimise cost and maximise the number of restricted connections contained in the aircraft routing solution that are operated in the current crew pairing solution.}
5: **solve** crew pairing problem with objective function (5){Minimise cost and the number of restricted aircraft changes.}
6: **break** if robustness cannot be improved
7: **increase** $p$
8: **end while**

short connections could be considered by removing connections in the underlying network of the crew pairing problem. If short connections are present in the problem, e.g. in problem instances of American or European airlines, Step 2 of Algorithm 1 generally yields an infeasible solution that violates the short connection rule, however, all subsequent iterations yield feasible solutions.

For our problem instances the interdependence between aircraft routings and crew pairings stated above is extended by the DPACLIM rule. Since we solve the crew pairing problem for a given aircraft routing solution, the rule can easily be embedded in the column generation method of the crew pairing problem, a resource constrained shortest path algorithm.

The cost of the crew pairing solution in Step 2 yields a lower bound on the crew pairing costs of a feasible integrated solution since no aircraft routings are taken into account. In our experiments, the crew pairing solution of Step 2 is either infeasible with respect to the DPACLIM rule or contains a large number of restricted aircraft changes.

After the initial steps of the algorithm, we obtain a feasible solution to the integrated problem in each iteration by solving the crew pairing problem (Step 5) for a given aircraft routing solution (Step 4). Once the integrated solution converges to a stable solution, the algorithm stops. For a stable solution, all successive iterations yield identical aircraft routing and crew pairing solutions, despite increasing penalty $p$. Hence, the aircraft change cost can not be improved. The value of $p_{max}$ is chosen such that the aircraft change costs dominate the crew pairing costs in function (5) in the sense that no restricted aircraft changes are contained in the optimal solution if such a solution exists. The sequence of values $p$ we use is $p \in \{0, 2, 5, 10, 20, 50, 100, 500, 1000\}$. In practice, we stop the algorithm once the aircraft change costs are below some threshold.

Once the algorithm terminates, a number of different solutions to the robust and integrated aircraft routing and crew pairing problem are obtained. The trade-off between cost and robustness varies between solutions and the airline can choose which solution to operate (see Figure 1 below).

## 4.2 Dantzig-Wolfe Decomposition Approach

The goal of the Dantzig-Wolfe decomposition approach [Dantzig and Wolfe, 1960] is to solve model (3) to optimality. As in the previous section, we omit the short connection constraints and in the following only describe the method for restricted connections. The short-connection constraints can be considered in a similar way to the restricted connection constraints.

We use Dantzig-Wolfe decomposition to re-formulate (3) as one master problem and two sub-problems. The only constraints of the original formulation that are present in the master problem are the restricted connection constraints:

$$\begin{array}{llcccccccl}
\text{Minimise} & (\boldsymbol{c}^P)^T V^P \boldsymbol{\lambda} & + & (\boldsymbol{c}^R)^T V^R \boldsymbol{\mu} & + & p(\boldsymbol{c}^{AC})^T \boldsymbol{d} & & & & \\
\text{subject to} & \mathbb{1}^T \boldsymbol{\lambda} & & & & & = & 1 & \rightarrow & \pi^P \\
 & & & \mathbb{1}^T \boldsymbol{\mu} & & & = & 1 & \rightarrow & \pi^R \\
 & D^P V^P \boldsymbol{\lambda} & - & D^R V^R \boldsymbol{\mu} & - & \boldsymbol{d} & \leq & 0 & \rightarrow & \boldsymbol{\pi},
\end{array} \tag{6}$$

where $\boldsymbol{\lambda} \in \{0,1\}^{|V^P|}, \boldsymbol{\mu} \in \{0,1\}^{|V^R|}$ and $\boldsymbol{d} \in \{0,1\}^{m^D}$. The columns $\boldsymbol{v}_i^P$ and $\boldsymbol{v}_i^R$ of matrices $V^P = [\boldsymbol{v}_1^P, \boldsymbol{v}_2^P, \cdots, \boldsymbol{v}_k^P]$ and $V^R = [\boldsymbol{v}_1^R, \boldsymbol{v}_2^R, \cdots, \boldsymbol{v}_k^R]$ span the respective polyhedra $P^P = \{\boldsymbol{x}^P \in \mathbb{R}_+^n | A^P \boldsymbol{x}^P = \boldsymbol{b}^P\}$, $P^P = \text{conv}(\{\boldsymbol{v}_1^P, \ldots, \boldsymbol{v}_k^P\})$ and $P^R = \{\boldsymbol{x}^R \in \mathbb{R}_+^n | A^R \boldsymbol{x}^R = \mathbb{1}\}$, $P^R = \text{conv}(\{\boldsymbol{v}_1^R, \ldots, \boldsymbol{v}_k^R\})$. Dual values $\pi^P$ and $\pi^R$ are associated with crew and aircraft convexity constraints, respectively. The entries of vector $\boldsymbol{\pi}$ are the dual values corresponding to the restricted connection constraints. The convexity constraints ensure that exactly one aircraft solution and exactly one crew solution is chosen in an optimal integer solution.

The two subproblems contain all other constraints of the original formulation. The crew pairing subproblem is identical to the original crew pairing problem except for the objective function:

$$\begin{array}{llcl}
\text{Minimise} & ((\boldsymbol{c}^P)^T - \boldsymbol{\pi}^T D^P)\boldsymbol{x}^P & & \\
\text{subject to} & A^P \boldsymbol{x}^P & = & \boldsymbol{b}^P \\
 & \boldsymbol{x}^P & \in & \{0,1\}^{n^P}.
\end{array} \tag{7}$$

It is important to note that this crew pairing subproblem assumes no aircraft routing solution. All connections are assumed to be follow-on (on the same aircraft) connections and minimal sit-time rules are relaxed. Similarly, the aircraft routing subproblem is identical to the original aircraft routing problem except for the objective function:

$$\begin{array}{llcl}
\text{Minimise} & ((\boldsymbol{c}^R)^T + \boldsymbol{\pi}^T D^R)\boldsymbol{x}^R & & \\
\text{subject to} & A^R \boldsymbol{x}^R & = & \mathbb{1} \\
 & \boldsymbol{x}^R & \in & \{0,1\}^{n^R}.
\end{array} \tag{8}$$

The solution process starts by solving the LP relaxation of the restricted master problem and passing the optimal dual values to both subproblems as input. The LP relaxations of both subproblems are solved and one column is generated from each subproblem solution and added to $V^P$ or $V^R$ of the master problem, respectively, if the reduced costs are negative. The process iterates until no columns with negative reduced cost are returned by either subproblem or a specified optimality gap is reached. Both subproblems are solved with column generation and branch-and-price. In this first phase when only the LP relaxations of all problems are solved, an optimality gap can be obtained from the LP optimal solution values of the subproblems.

If fractional solution variables are contained in the optimal solution of the linear relaxation we branch on variables $d$ to obtain an integer solution. Since each variable $d_i$ is associated with a connection $i$, the branching decisions are easily incorporated in the subproblems by forcing or banning connections to be contained in a solution. Both subproblems are solved to integer optimality in this phase.

In our computational experiments we stop the algorithm after the linear relaxation solution is within a specified optimality gap since the integer solutions of the iterative approach are usually within the specified branch-and-bound gap of 2%. Hence, there is no additional benefit of running the IP solution phase. We refer to the computational experiments (Section 5) for more details.

Note that we do not integrate the DPACLim rule in this solution approach. The integration of this rule requires linking particular routings and pairings which could be enforced by additional constraints or a branching strategy. This, however, is computationally difficult and inefficient to enforce. We expect that an operationally robust solution will "almost" satisfy the DPACLim rule. Since the DPACLim is an artificial rule to enforce robustness before the introduction of the aircraft change robustness measure, a slight violation of the rule can be tolerated. We can enforce the rule heuristically by using the aircraft routing solution of the optimal integrated IP solution and generating a DPACLim rule feasible crew pairing solution as in the iterative approach.

## 4.3 Discussion of Approaches

We include a Benders decomposition approach to solve (3) as described in Mercier et al. [2005]. Benders decomposition iterates between a crew pairing master problem and an aircraft routing subproblem. Instead of columns as in the Dantzig-Wolfe decomposition, constraints are added to the master problem in each iteration until an optimal solution is obtained.

In both optimisation approaches, Dantzig-Wolfe and Benders decomposition, a weight must be attached to aircraft change cost a priori. This weight represents the trade-off between monetary costs and operational robustness and is difficult to estimate. In the iterative approach the user can choose a solution after the algorithm terminates depending on the trade-off observed between crew pairing cost and aircraft change cost, no weight is needed a priori.

All solution methods previously discussed in the literature, including Benders decomposition, add constraints to the set partitioning polytopes of the aircraft routing and crew pairing problems. These additional constraints can cause computational difficulties. In the iterative approach and the Dantzig-Wolfe decomposition approach the original set partitioning structures are not disturbed by additional constraints which has two further advantages: Firstly, it is possible to solve aircraft routing and crew pairing problems efficiently with existing methods. In both approaches only the objective function is changed to influence characteristics of the solutions. An airline usually uses an aircraft routing and crew pairing solver as part of the traditional sequential solution approach. Existing solvers can be used in the iterative approach and the Dantzig-Wolfe decomposition with only minor modifications. Secondly, in both approaches the aircraft routing and crew pairing problems must be solved repeatedly. The solution of a previous iteration can be used as a starting basis for the simplex algorithm. Furthermore, if only the objective function changes, the previous solution is still feasible and we expect that only very few iterations are needed until the new optimal solution is found.

The iterative approach and the Dantzig-Wolfe decomposition approach are structurally very similar. In both approaches identical subproblems are solved. The penalties used in the iterative approach to penalise aircraft changes can be thought of as duals $\pi$ corresponding to the restricted connection constraints in the Dantzig-Wolfe master problem. This in fact gives the motivation for the iterative approach: Instead of using optimal duals from an LP solution, heuristically constructed duals are used to guide the solution process of the subproblems in the iterative approach.

We think it is not possible to efficiently integrate the DPACLim rule into the Dantzig-Wolfe or Benders decomposition approaches. This would require comparing particular pairs of routings and pairings and is computationally expensive.

All three approaches provide lower bounds on the optimal solution. For the two decomposition approaches the optimal solution values of the LP relaxations of the subproblems provide lower bounds on the objective value of an optimal solution. In the iterative approach a lower bound for the crew pairing cost is calculated. The minimal aircraft routing costs can be added to obtain a lower bound for the cost of an integrated solution. Benders and Dantzig-Wolfe decomposition provide a guarantee of the solution quality of the LP relaxation of problem (3) while the iterative approach does not.

In practice, it is beneficial to combine the iterative approach and the Dantzig-Wolfe decomposition approach: An initial solution is found by the iterative approach. The solution is added to matrices $V^P$ and $V^R$ of the Dantzig-Wolfe decomposition approach. This approach can then be used to obtain a lower bound for the solution and to improve the solution quality. Using a starting solution can significantly speed up the solution process of the optimisation approach.

Mercier et al. [2005] show that in the Benders decomposition approach only very few iterations are required to obtain optimal LP solutions. In their computational experiments they do not consider base-constraints except for limiting the total number of duties. They also use an approximation of the crew cost function. In our experience, such relaxations greatly simplify the crew pairing problem.

# 5 Computational Experiments

In this section we compare computational results of our implementations of the iterative approach, the Dantzig-Wolfe decomposition approach, and the Benders decomposition approach. All program code is written in C, C++, and FORTRAN. We use basic implementations of Dantzig-Wolfe and Benders decomposition without any speed-up procedures. We also solve both optimisation approaches to LP optimality only and compare the results and run times with the iterative approach.

## 5.1 Iterative Approach

Figure 1 displays a typical set of results of the iterative approach. The horizontal axis shows crew pairing costs and the vertical axis aircraft change costs. The costs of the solution operated by the airline and the solutions generated by the iterative algorithm are compared for the first officer schedule, summer 2005. The diamond shows the objective value of the crew pairing solution that was operated by the airline. This solution is obtained by using the aircraft routing solution that was operated by the airline and generating a cost minimal crew pairing solution with the traditional method. The lower bound for the crew pairing cost is shown which is obtained from the initial step of the iterative approach. The squares show the objective values of the solutions generated by the iterative approach. The labels show the iteration in which the solution is obtained. Starting with very cheap solutions the solutions become more robust during the algorithm and also more expensive. It is remarkable that the first six solutions are all cheaper and more robust than the solution obtained by the traditional approach.

In Table 1 results of the iterative approach are listed in detail for one, three, and seven day scenarios of the first officer schedule, summer 2005. The first column lists the scenario name and the second column lists the iteration in which the results are obtained. For comparison we list the solution obtained by the traditional sequential approach as "airline". Column "$p$" shows the value of $p$ that applies to the iteration. The next four columns show LP and IP values of the crew pairing costs, the gap between LP and IP value ("gap") as a percentage, and the improvement ("impr.") compared to the sequential "airline" solution. The aircraft routing costs are displayed in column "$c^R$". The costs of aircraft changes are listed in column "$c^{AC}$" and the improvements compared to the "airline" solution are listed in the following column ("impr.") as a percentage. All restricted aircraft changes are listed in the following columns. Finally, the total time elapsed since the start of the algorithm is shown for each iteration in seconds. We do not show any aircraft change costs for the lower bound solution of iteration 0 since we

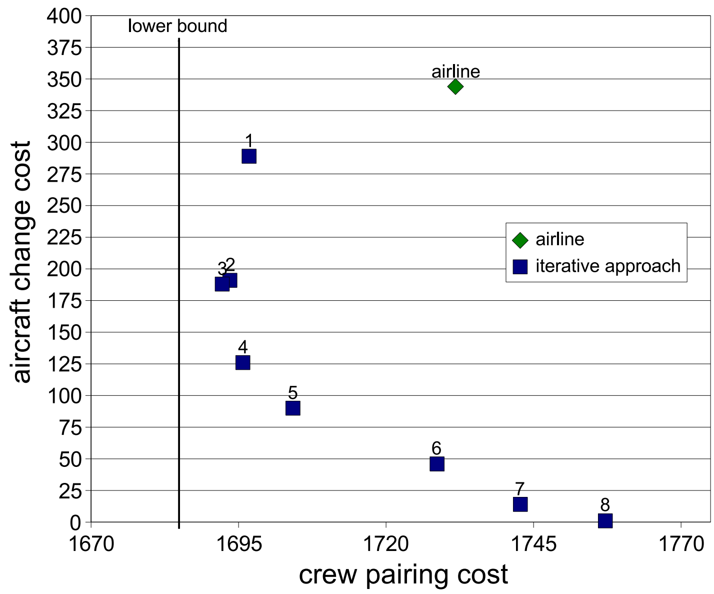


Figure 1: Iterative approach solutions for first officer, schedule summer 2005, 7 days.

can not find an integrated solution satisfying the DPACLim rule and hence this solution is infeasible.

Note that in the crew pairing problem a weighted sum objective of crew pairing costs and aircraft change costs is used. The ratio of crew pairing costs and aircraft change costs in the solution value of LP and corresponding IP solution can differ. Since we only display crew pairing costs we may observe smaller IP solution values than the corresponding LP solution values. Since the ratio also differs from iteration to iteration we observe that crew pairing costs are not strictly increasing during the iterative algorithm.

In Table 1 we can observe for the 7 day solutions, that the lower bound solution obtained in Step 2 of Algorithm 1 (iteration 0) incurs up to 2.34% less crew pairing cost than the airline solution (LP).

The cheapest feasible solution we find (iteration 1) incurs 2.32% less cost than the airline solution (LP) and its cost is almost at the lower bound. Also, the aircraft change cost of this solution is only 289 compared to 344 for the airline solution which is an improvement of 15.99%. The most robust solution that is still cheaper than the airline solution improves the aircraft change cost by 86.63% (iteration 6). Note that the DPACLim is set to 1 for the airline solution while for the iterative algorithm DPACLim is set to 2. Different settings are investigated below (see Figure 2) where we show that for setting DPACLim to 1 the solutions of the iterative approach are also cheaper than the airline solution. These results are remarkable since Air New Zealand is using sophisticated optimisation methods for crew planning, described in detail in Butchers et al. [2001], a finalist entry for the Franz Edelman Award in 2000.

In Tables 2 and 3 we present further results for first officer schedules of winter 2005 and summer 2006, respectively. The overall trend is similar to the solutions for summer 2005 shown in Table 1. We find solutions that incur up to 2.23% (w05, 7 days, iter. 1) less crew pairing cost than the corresponding airline solution. For all 7 day scenarios we obtain solutions with no increase of crew pairing cost but a decrease of aircraft change cost exceeding 90% (w05, 7 days, iter. 7 and s06, 7 days, iter. 7). Aircraft routing costs $c^R$ remain on a similar level during the iterations of the algorithm. This is due to the constant ratio of weights for $c^R$ and $c^{AC}$ in the objective function of the aircraft routing problem. Note that for the summer 2005 scenarios aircraft routing costs are much higher for the airline solutions than for all other scenarios. For these scenarios the airline did not take the objective of minimising minimal turn sequences into account when constructing the aircraft routings.

Although the integer bound gap is set to 2.0% for all scenarios the average bound gap observed over all solutions of the iterative approach is much lower with 0.44%.

Finally, we observe that the total running time of the iterative approach for a scenario of one week ranges in between 1231 and 1667 seconds. These running times are very short for the types of planning problem we try to solve. Note that the increase of weight $p$ during the algorithm is chosen to be conservative. A faster increase of $p$ will decrease the number of iterations and hence the running time of the algorithm considerably. We choose to generate many solutions to allow for better judgement of the trade-off between crew pairing costs and aircraft change costs.

| | | | crew pairing cost ($c^P$) | | | | aircraft | aircraft changes | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | impr. | | gap | routing | | impr. | (minutes exceed. min. sit-time) | | | | | | | run time |
| scenario | iteration | p | lp ($\times 10^2$) | (%) | ip ($\times 10^2$) | (%) | cost ($c^R$) | $c^{AC}$ | (%) | 0 | 5 | 10 | 15 | 20 | 25 | 30 | elapsed (s) |
| s05, | airline | - | 347.13 | - | 347.13 | 0.00 | 2250 | 34 | - | 2 | 1 | 1 | 1 | 0 | 1 | 3 | - |
| 1 day | 0 | - | 345.36 | 0.51 | 345.37 | 0.00 | 260 | - | - | - | - | - | - | - | - | - | 13.97 |
| | 1 | 0 | 345.80 | 0.38 | 346.37 | 0.16 | 300 | 60 | -76.47 | 6 | 0 | 2 | 1 | 0 | 1 | 2 | 21.59 |
| | 2 | 2 | 345.96 | 0.34 | 346.34 | 0.11 | 310 | 27 | 20.59 | 3 | 0 | 0 | 0 | 1 | 1 | 1 | 26.66 |
| | 3 | 5 | 346.44 | 0.20 | 346.44 | 0.00 | 320 | 10 | 70.59 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 31.90 |
| | 4 | 10 | 346.54 | 0.17 | 346.34 | -0.06 | 320 | 13 | 61.76 | 1 | 0 | 0 | 0 | 1 | 0 | 3 | 36.50 |
| | 5 | 20 | 346.68 | 0.13 | 346.68 | 0.00 | 270 | 8 | 76.47 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 42.20 |
| | 6 | 50 | 349.37 | -0.65 | 346.62 | -0.79 | 270 | 8 | 76.47 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 48.96 |
| | 7 | 100 | 349.53 | -0.69 | 352.07 | 0.72 | 270 | 2 | 94.12 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 55.53 |
| | 8 | 500 | 353.76 | -1.91 | 353.76 | 0.00 | 270 | 0 | 100.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 61.58 |
| s05, | airline | - | 892.95 | - | 900.84 | 0.88 | 4020 | 145 | - | 10 | 4 | 2 | 2 | 4 | 5 | 11 | - |
| 3 days | 0 | - | 883.69 | 1.04 | 893.03 | 1.05 | 580 | - | - | - | - | - | - | - | - | - | 62.87 |
| | 1 | 0 | 884.06 | 1.00 | 892.65 | 0.96 | 860 | 159 | -9.66 | 13 | 2 | 1 | 6 | 3 | 4 | 10 | 106.40 |
| | 2 | 2 | 884.44 | 0.95 | 890.66 | 0.70 | 960 | 65 | 55.17 | 3 | 1 | 1 | 2 | 2 | 4 | 11 | 152.94 |
| | 3 | 5 | 885.06 | 0.88 | 897.89 | 1.43 | 920 | 85 | 41.38 | 8 | 0 | 0 | 2 | 2 | 2 | 11 | 196.10 |
| | 4 | 10 | 885.71 | 0.81 | 891.05 | 0.60 | 980 | 60 | 58.62 | 3 | 1 | 2 | 1 | 3 | 2 | 6 | 233.72 |
| | 5 | 20 | 888.06 | 0.55 | 887.63 | -0.05 | 890 | 29 | 80.00 | 2 | 0 | 0 | 0 | 3 | 1 | 4 | 269.05 |
| | 6 | 50 | 888.89 | 0.45 | 891.75 | 0.32 | 930 | 13 | 91.03 | 0 | 0 | 0 | 0 | 3 | 1 | 2 | 304.36 |
| | 7 | 100 | 894.24 | -0.14 | 899.37 | 0.57 | 930 | 8 | 94.48 | 0 | 0 | 0 | 0 | 1 | 1 | 3 | 340.20 |
| | 8 | 500 | 908.73 | -1.77 | 910.67 | 0.21 | 880 | 1 | 99.31 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 368.17 |
| | 9 | 1000 | 914.31 | -2.39 | 921.44 | 0.77 | 880 | 0 | 100.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 416.42 |
| s05, | airline | - | 1725.12 | - | 1731.77 | 0.38 | 8210 | 344 | - | 29 | 5 | 5 | 10 | 4 | 8 | 18 | - |
| 7 days | 0 | - | 1684.77 | 2.34 | 1702.27 | 1.03 | 1470 | - | - | - | - | - | - | - | - | - | 268.26 |
| | 1 | 0 | 1685.10 | 2.32 | 1696.78 | 0.69 | 2250 | 289 | 15.99 | 22 | 3 | 6 | 7 | 6 | 8 | 25 | 455.82 |
| | 2 | 2 | 1686.39 | 2.25 | 1693.53 | 0.42 | 2230 | 191 | 44.48 | 14 | 3 | 1 | 3 | 5 | 8 | 27 | 604.79 |
| | 3 | 5 | 1687.53 | 2.18 | 1692.23 | 0.28 | 2290 | 188 | 45.35 | 14 | 3 | 0 | 4 | 4 | 5 | 34 | 734.08 |
| | 4 | 10 | 1690.88 | 1.98 | 1695.73 | 0.29 | 2250 | 126 | 63.37 | 8 | 2 | 1 | 3 | 4 | 3 | 23 | 902.01 |
| | 5 | 20 | 1696.27 | 1.67 | 1704.22 | 0.47 | 2210 | 90 | 73.84 | 5 | 2 | 0 | 2 | 3 | 1 | 24 | 1052.88 |
| | 6 | 50 | 1712.21 | 0.75 | 1728.64 | 0.95 | 2190 | 46 | 86.63 | 1 | 2 | 1 | 1 | 1 | 1 | 13 | 1242.15 |
| | 7 | 100 | 1729.58 | -0.26 | 1742.76 | 0.76 | 2140 | 14 | 95.93 | 0 | 1 | 0 | 1 | 0 | 0 | 4 | 1392.27 |
| | 8 | 500 | 1748.06 | -1.33 | 1757.16 | 0.52 | 2050 | 1 | 99.71 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1593.49 |

Table 1: Results of iterative approach for first officer, schedule summer 2005.

| | | | crew pairing cost ($c^P$) | | | | aircraft | aircraft changes | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | impr. | | gap | routing | | impr. | (minutes exceed. min. sit-time) | | | | | | | run time |
| scenario | iteration | p | lp ($\times 10^2$) | (%) | ip ($\times 10^2$) | (%) | cost ($c^R$) | $c^{AC}$ | (%) | 0 | 5 | 10 | 15 | 20 | 25 | 30 | elapsed (s) |
| w05, | airline | - | 313.74 | - | 313.93 | 0.06 | 320 | 62 | - | 4 | 1 | 2 | 2 | 1 | 1 | 5 | - |
| 1 day | 0 | - | 310.28 | 1.10 | 310.28 | 0.00 | 220 | - | - | - | - | - | - | - | - | - | 12.96 |
| | 1 | 0 | 310.56 | 1.01 | 310.66 | 0.03 | 240 | 55 | 11.29 | 6 | 0 | 0 | 1 | 0 | 2 | 5 | 19.14 |
| | 2 | 2 | 310.74 | 0.96 | 310.77 | 0.01 | 310 | 23 | 62.90 | 2 | 0 | 0 | 0 | 0 | 2 | 5 | 25.74 |
| | 3 | 5 | 311.03 | 0.87 | 311.03 | 0.00 | 310 | 16 | 74.19 | 1 | 0 | 0 | 0 | 0 | 2 | 5 | 34.49 |
| | 4 | 10 | 311.19 | 0.81 | 311.19 | 0.00 | 310 | 13 | 79.03 | 1 | 0 | 0 | 0 | 0 | 1 | 4 | 43.06 |
| | 5 | 20 | 311.32 | 0.77 | 311.32 | 0.00 | 270 | 12 | 80.65 | 1 | 0 | 0 | 0 | 0 | 1 | 3 | 50.21 |
| | 6 | 50 | 311.91 | 0.58 | 311.91 | 0.00 | 270 | 8 | 87.10 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 55.86 |
| | 7 | 100 | 311.91 | 0.58 | 311.91 | 0.00 | 270 | 8 | 87.10 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 61.33 |
| | 8 | 500 | 322.32 | -2.74 | 322.36 | 0.01 | 270 | 0 | 100.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 67.72 |
| w05, | airline | - | 787.84 | - | 789.61 | 0.22 | 960 | 183 | - | 15 | 3 | 5 | 3 | 4 | 1 | 9 | - |
| 3 days | 0 | - | 779.51 | 1.06 | 785.69 | 0.79 | 600 | - | - | - | - | - | - | - | - | - | 59.56 |
| | 1 | 0 | 779.70 | 1.03 | 780.91 | 0.15 | 1060 | 158 | 13.66 | 13 | 1 | 3 | 5 | 4 | 1 | 12 | 92.96 |
| | 2 | 2 | 780.18 | 0.97 | 781.09 | 0.12 | 1120 | 93 | 49.18 | 7 | 0 | 2 | 2 | 3 | 2 | 13 | 130.44 |
| | 3 | 5 | 780.68 | 0.91 | 781.29 | 0.08 | 1040 | 55 | 69.95 | 4 | 0 | 1 | 0 | 3 | 1 | 11 | 159.81 |
| | 4 | 10 | 782.51 | 0.68 | 782.51 | 0.00 | 920 | 33 | 81.97 | 1 | 0 | 1 | 0 | 3 | 1 | 10 | 191.35 |
| | 5 | 20 | 782.34 | 0.70 | 782.34 | 0.00 | 920 | 33 | 81.97 | 1 | 0 | 1 | 0 | 3 | 1 | 10 | 218.63 |
| | 6 | 50 | 788.03 | -0.02 | 791.54 | 0.44 | 910 | 15 | 91.80 | 0 | 0 | 0 | 0 | 3 | 1 | 4 | 267.80 |
| | 7 | 100 | 789.02 | -0.15 | 793.27 | 0.54 | 870 | 11 | 93.99 | 0 | 0 | 0 | 0 | 3 | 0 | 2 | 313.38 |
| | 8 | 500 | 803.73 | -2.02 | 806.24 | 0.31 | 860 | 0 | 100.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 358.93 |
| w05, | airline | - | 1663.62 | - | 1684.42 | 1.23 | 2220 | 374 | - | 30 | 9 | 8 | 5 | 5 | 8 | 19 | - |
| 7 days | 0 | - | 1625.19 | 2.31 | 1632.37 | 0.44 | 1410 | - | - | - | - | - | - | - | - | - | 234.53 |
| | 1 | 0 | 1626.56 | 2.23 | 1636.52 | 0.61 | 2330 | 289 | 22.73 | 29 | 2 | 3 | 3 | 2 | 6 | 29 | 405.46 |
| | 2 | 2 | 1627.27 | 2.18 | 1630.94 | 0.23 | 2280 | 205 | 45.19 | 19 | 1 | 2 | 4 | 1 | 3 | 31 | 513.00 |
| | 3 | 5 | 1628.49 | 2.11 | 1629.31 | 0.05 | 2370 | 165 | 55.88 | 14 | 1 | 2 | 2 | 2 | 4 | 29 | 614.41 |
| | 4 | 10 | 1631.85 | 1.91 | 1634.70 | 0.17 | 2200 | 112 | 70.05 | 8 | 0 | 2 | 2 | 2 | 3 | 26 | 744.60 |
| | 5 | 20 | 1635.72 | 1.68 | 1645.23 | 0.58 | 2190 | 58 | 84.49 | 4 | 0 | 1 | 2 | 0 | 1 | 15 | 972.41 |
| | 6 | 50 | 1642.06 | 1.30 | 1654.36 | 0.74 | 2190 | 47 | 87.43 | 3 | 0 | 1 | 1 | 1 | 0 | 14 | 1207.04 |
| | 7 | 100 | 1653.67 | 0.60 | 1670.09 | 0.98 | 2190 | 17 | 95.45 | 1 | 0 | 0 | 1 | 0 | 0 | 6 | 1422.26 |
| | 8 | 500 | 1678.67 | -0.90 | 1700.83 | 1.30 | 2190 | 1 | 99.73 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1666.60 |

Table 2: Results of iterative approach for first officer, schedule winter 2005.

| | | | crew pairing cost ($c^P$) | | | | aircraft | aircraft changes | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | impr. | | gap | routing | | impr. | (minutes exceed. min. sit-time) | | | | | | | run time |
| scenario | iteration | p | lp ($\times 10^2$) | (%) | ip ($\times 10^2$) | (%) | cost ($c^R$) | $c^{AC}$ | (%) | 0 | 5 | 10 | 15 | 20 | 25 | 30 | elapsed (s) |
| s06, | airline | - | 314.36 | - | 315.65 | 0.41 | 270 | 104 | - | 10 | 2 | 1 | 2 | 1 | 1 | 4 | - |
| 1 day | 0 | - | 312.70 | 0.53 | 313.09 | 0.12 | 130 | - | - | - | - | - | - | - | - | - | 12.54 |
| | 1 | 0 | 312.89 | 0.47 | 314.06 | 0.37 | 200 | 61 | 41.35 | 5 | 1 | 0 | 1 | 1 | 1 | 11 | 17.26 |
| | 2 | 2 | 313.09 | 0.40 | 313.75 | 0.21 | 230 | 22 | 78.85 | 1 | 0 | 0 | 1 | 1 | 0 | 8 | 23.08 |
| | 3 | 5 | 313.75 | 0.20 | 313.76 | 0.01 | 230 | 21 | 79.81 | 1 | 0 | 0 | 1 | 1 | 0 | 7 | 29.08 |
| | 4 | 10 | 313.83 | 0.17 | 313.83 | 0.00 | 230 | 13 | 87.50 | 0 | 0 | 0 | 1 | 1 | 0 | 6 | 37.66 |
| | 5 | 20 | 314.26 | 0.03 | 314.26 | 0.00 | 230 | 10 | 90.38 | 0 | 0 | 0 | 1 | 0 | 0 | 6 | 43.73 |
| | 6 | 50 | 314.75 | -0.12 | 314.75 | 0.00 | 230 | 8 | 92.31 | 0 | 0 | 0 | 1 | 0 | 0 | 4 | 52.11 |
| | 7 | 100 | 315.01 | -0.21 | 315.01 | 0.00 | 180 | 8 | 92.31 | 0 | 0 | 0 | 1 | 0 | 0 | 4 | 57.05 |
| | 8 | 500 | 322.13 | -2.47 | 322.13 | 0.00 | 180 | 4 | 96.15 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 61.74 |
| | 9 | 1000 | 343.83 | -9.37 | 356.41 | 3.53 | 180 | 0 | 100.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 68.92 |
| s06, | airline | - | 719.62 | - | 721.79 | 0.30 | 860 | 252 | - | 24 | 2 | 7 | 3 | 3 | 1 | 14 | - |
| 3 days | 0 | - | 710.81 | 1.22 | 710.94 | 0.02 | 410 | - | - | - | - | - | - | - | - | - | 54.62 |
| | 1 | 0 | 710.81 | 1.23 | 711.83 | 0.14 | 760 | 98 | 61.11 | 10 | 0 | 0 | 3 | 1 | 1 | 11 | 79.24 |
| | 2 | 2 | 710.85 | 1.22 | 710.85 | 0.00 | 760 | 48 | 80.95 | 3 | 0 | 0 | 3 | 1 | 1 | 10 | 102.66 |
| | 3 | 5 | 711.43 | 1.14 | 711.43 | 0.00 | 750 | 31 | 87.70 | 1 | 0 | 0 | 3 | 1 | 1 | 7 | 128.07 |
| | 4 | 10 | 712.08 | 1.05 | 712.08 | 0.00 | 740 | 30 | 88.10 | 1 | 0 | 0 | 3 | 1 | 1 | 6 | 159.78 |
| | 5 | 20 | 711.16 | 1.18 | 711.16 | 0.00 | 740 | 23 | 90.87 | 0 | 0 | 0 | 3 | 1 | 1 | 6 | 185.73 |
| | 6 | 50 | 714.04 | 0.78 | 711.28 | -0.39 | 690 | 15 | 94.05 | 0 | 0 | 0 | 1 | 1 | 2 | 4 | 215.91 |
| | 7 | 100 | 715.36 | 0.59 | 714.27 | -0.15 | 650 | 12 | 95.24 | 0 | 0 | 0 | 1 | 1 | 1 | 3 | 246.88 |
| | 8 | 500 | 730.64 | -1.53 | 731.12 | 0.07 | 650 | 0 | 100.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 273.81 |
| s06, | airline | - | 1644.17 | - | 1673.21 | 0.55 | 1890 | 385 | - | 35 | 2 | 10 | 10 | 4 | 4 | 18 | - |
| 7 days | 0 | - | 1615.21 | 1.76 | 1617.23 | 0.12 | 850 | - | - | - | - | - | - | - | - | - | 229.73 |
| | 1 | 0 | 1614.43 | 1.81 | 1625.28 | 0.67 | 1450 | 364 | 5.45 | 36 | 3 | 7 | 6 | 1 | 4 | 24 | 370.05 |
| | 2 | 2 | 1618.73 | 1.55 | 1631.20 | 0.76 | 1540 | 212 | 44.94 | 18 | 2 | 2 | 7 | 3 | 5 | 17 | 538.57 |
| | 3 | 5 | 1618.96 | 1.53 | 1620.44 | 0.09 | 1480 | 107 | 72.21 | 7 | 0 | 3 | 4 | 3 | 1 | 16 | 624.55 |
| | 4 | 10 | 1619.83 | 1.48 | 1619.53 | -0.02 | 1450 | 93 | 75.84 | 7 | 0 | 1 | 3 | 3 | 2 | 14 | 705.58 |
| | 5 | 20 | 1621.36 | 1.39 | 1621.36 | 0.00 | 1420 | 68 | 82.34 | 4 | 0 | 1 | 3 | 2 | 2 | 13 | 796.41 |
| | 6 | 50 | 1632.11 | 0.73 | 1634.23 | 0.13 | 1380 | 28 | 92.73 | 0 | 0 | 0 | 3 | 2 | 0 | 10 | 919.37 |
| | 7 | 100 | 1638.91 | 0.32 | 1638.29 | -0.04 | 1370 | 21 | 94.55 | 0 | 0 | 0 | 2 | 1 | 0 | 10 | 1000.55 |
| | 8 | 500 | 1673.11 | -1.76 | 1681.33 | 0.49 | 1370 | 1 | 99.74 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1151.35 |
| | 9 | 1000 | 1674.63 | -1.85 | 1674.73 | 0.01 | 1360 | 0 | 100.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1230.94 |

Table 3: Results of iterative approach for first officer, schedule summer 2006.

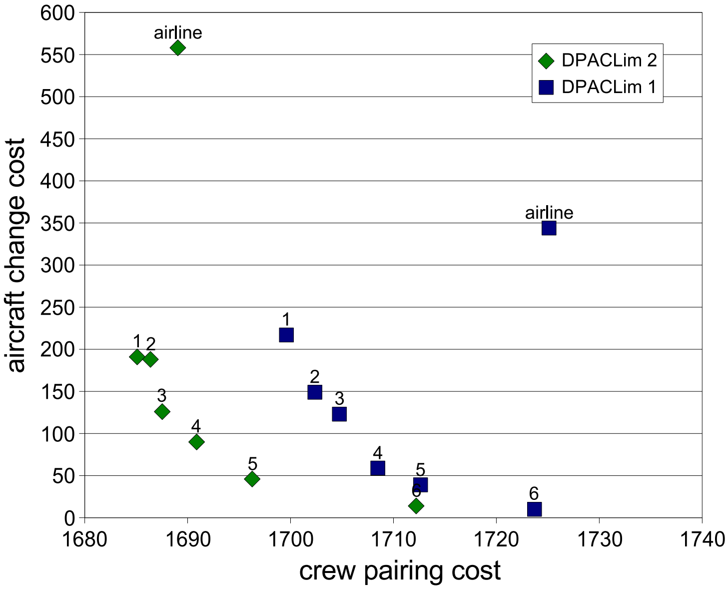


Figure 2: Variation of DPACLim for first officer, schedule summer 2005, 7 days.

Figure 2 shows the impact of the DPACLim rule for the first officer scenario, schedule summer 2005, 7 days. Relaxing the rule from 1 to 2 for the traditional sequential ("airline") approach results in a solution with less crew pairing costs but many more restricted aircraft changes. Hence, it is less robust and could not be operated reliably in practice. In the iterative approach cheaper solutions can be generated if the DPACLim rule is relaxed from 1 to 2. Here the solutions are equally robust for both settings since restricted aircraft changes are always penalised in the iterative approach and thus the cheaper solutions can be operated in practice. Since the DPACLim rule is only used by the airline to increase the robustness of the solutions the rule is relaxed to 2 for the iterative approach since robustness is achieved by means of aircraft change cost. From a practical point of view, multiple aircraft changes can be tolerated if this does not affect robustness, i.e. the aircraft changes occur on connections with long ground times. Note that because of the LP/IP gaps, LP values are displayed in Figure 2 to obtain a more consistent representation.

Table 4 lists some more details about the characteristics of the solutions. Statistics are shown for setting DPACLim to 1 and 2, respectively. We list the airline solutions as well as the solutions generated by the iterative approach. For DPACLim equal to 1 we list the total number of duty periods in the

| | DPACLim 1 | | | | DPACLim 2 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | no. duty periods | | | | no. duty periods | | | | | |
| iteration | all | 1 ac | (rac) | $c^{AC}$ | all | 1 ac | (rac) | 2 ac | (rac) | $c^{AC}$ |
| airline | 210 | 115 | 72 | 344 | 206 | 74 | 52 | 56 | 76 | 558 |
| 0 | 204 | 101 | 54 | 255 | 202 | 77 | 46 | 25 | 24 | 289 |
| 1 | 203 | 98 | 52 | 217 | 202 | 81 | 43 | 17 | 14 | 191 |
| 2 | 205 | 83 | 35 | 149 | 202 | 83 | 40 | 21 | 19 | 188 |
| 3 | 204 | 86 | 34 | 123 | 202 | 76 | 26 | 16 | 11 | 126 |
| 4 | 204 | 81 | 17 | 59 | 203 | 73 | 24 | 16 | 10 | 90 |
| 5 | 205 | 77 | 14 | 39 | 205 | 60 | 12 | 12 | 3 | 46 |
| 6 | 205 | 81 | 5 | 10 | 206 | 69 | 5 | 10 | 1 | 14 |
| 7 | 207 | 83 | 1 | 2 | 207 | 66 | 1 | 17 | 0 | 1 |

Table 4: Variation of DPACLim for first officer, schedule summer 2005, 7 days.

solution ("all"). We show the number of duty periods with 1 aircraft change ("1 ac") and the number of restricted aircraft changes ("(rac)") within these duty periods. Column "$c^{AC}$" shows the aircraft change cost for each solution. For a DPACLim setting of 2 we again show the total number of duty periods. Additionally, we list the number of duty periods with 1 ("ac 1") and 2 ("ac 2") aircraft changes and the number of restricted aircraft changes contained in both types of duty periods, respectively. Finally, column "$c^{AC}$" again displays the aircraft change costs of the solutions. We observe that the relaxation of the rule has no negative impact on the aircraft change cost of the solutions of the iterative algorithm. The reduction in crew pairing cost is achieved by increasing the number of duty periods with two aircraft changes. However, the aircraft change costs of these solutions do not increase significantly. In iteration 3 for example, aircraft change costs are almost identical but we observe (see Figure 2) a decrease in crew pairing cost of 0.77 % for setting DPACLim to 2. This cheaper solution does contain 16 duty periods with 2 aircraft changes but only 11 restricted aircraft changes. This demonstrates that we can achieve a better crew pairing cost and a more robust solution if we allow a small number of duty periods to contain two aircraft changes.

## 5.2 Comparison of Iterative Approach and Optimisation Approaches

In this section we use Dantzig-Wolfe and Benders decomposition methods to solve the LP relaxation of the robust and integrated aircraft routing and crew pairing problem. The purpose of the computational experiments is twofold: Firstly, we want to establish lower bounds for the solution values of the integrated problem. We compare the integer solutions of the iterative approach with the LP lower bound from the decomposition methods. Secondly, we compare the running times of Dantzig-Wolfe and Benders decomposition approaches for solving the LP relaxation to establish which algorithm performs better.

For each of the scenarios investigated in the previous section we apply each decomposition approach twice. In one run we set weight $p$ equal to 2 and in the other run we set weight $p$ equal to 20. This corresponds to the value of weight $p$ in iterations 2 and 5 of the iterative approach. In both decomposition approaches all master problems and subproblems are only solved to LP optimality. Both decomposition approaches provide lower bounds for the optimal solution and once the gap between the lower bound and the best LP solution value found is below 0.5% we stop the algorithm.

Table 5 summarises the results of the experiments. We list scenario name, solution method and value of $p$ for all experiments. In the next two columns the number of iterations and run time needed to reach the stopping criterion are listed. If the optimality gap of 0.5% is not reached in 50 iterations we stop the algorithm. For each optimisation run we list lower and upper bound of the optimal LP solution and the gap between the two values. Note that because of some heuristic elements of the crew pairing solver that is used (typical for practical crew pairing solvers), for the same optimal solution we can sometimes observe two different intervals that do not overlap, for example s05, 7 days, $p = 20$ or w05, 7 days, $p = 20$. Of course, this can not occur if the solution approach for the crew pairing problem is truly optimal. The observed gap between two intervals for the same optimal solution is usually very small ($< 0.1\%$). In one instance, however, the observed gap is 0.33% (s05, 7 days, p 20). For the iterative approach, we show the best solution that is found with respect to the objective function that is used in the optimisation approaches. We apply this objective function to the integer solutions of the iterative approach and display the objective value and the gap with respect to the lower bound obtained by the optimisation approach. We also list the time needed for the iterative approach to obtain the solution.

Note that the lower bound LP relaxation solutions do not necessarily satisfy the DPACLim rule restrictions since the rule is not integrated into the model. The LP/IP gaps observed are partially caused by the violation of the DPACLim rule.

The average gap between the IP iterative approach solution and LP lower bound is very small (0.9%). In most cases this integer solution is found in less computation time than the stopping criterion of either optimisation approach is reached. Using the iterative approach, we always find an integer solution within the 2% branch-and-bound gap and hence, we do not solve the optimisation approaches to IP optimality, as the additionally required run time can not result in significantly improved solution quality. In two cases the Dantzig-Wolfe decomposition indicates a gap exceeding 2% but the gap is smaller than 2% for the lower bound of Benders decomposition in both cases.

In terms of run time, Benders decomposition seems to be slightly superior to Dantzig-Wolfe decomposition. It is noteworthy that the contribution by Mercier et al. [2005] indicates fewer iterations and much faster run times for Benders decomposition than observed in our computational experiments. We believe that this is caused by the authors solving a slightly relaxed problem, while in our approach a real world application is addressed together with all applicable rules. It also seems to be much harder to obtain an optimal solution for both decomposition approaches when the weight $p$ is large. For Benders decomposition, this is caused by larger costs that are associated with the subproblem and hence the subproblem must not only transfer feasibility but also optimality information back to the master problem. The Dantzig-Wolfe decomposition is more difficult when $p$ is large since the sum of crew pairing cost and aircraft routing cost as lower bound for the objective value of the relaxed master problem is further away from an optimal solution than for small values of $p$.

Note that both optimisation approaches are implemented in a basic fashion. No dual stabilisation method is used to speed up Dantzig-Wolfe decomposition, and Benders decomposition is not enhanced with stronger cut generation. We omit these improvements since we do not believe that the run times of the optimisation approaches can be improved sufficiently to compete with those of the iterative approach. In practice we suggest only running the decomposition approach to verify the solution quality of the iterative approach. If this is only done rarely and run time is not very important then Dantzig-Wolfe decomposition can be chosen as the solution method since it is much easier to implement into existing aircraft routing and crew pairing optimisation algorithms than Benders decomposition.

We investigated hot-starting the Dantzig-Wolfe decomposition by generating columns for the restricted master problem from all solutions of the iterative approach but this procedure did not improve run times significantly.

| | | | | | | | | iterative approach | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | number of | | lp cost ($\times 10^2$) | | | ip solution | gap | run time |
| scenario | method | p | iterations | run time (s) | lower bound | upper bound | gap (%) | value ($\times 10^2$) | (%) | elapsed (s) |
| s05, | DW | 2 | 2 | 19.02 | 348.62 | 349.49 | 0.25 | 349.48 | 0.24 | 48.96 |
| 1 day | DW | 20 | 10 | 65.90 | 348.98 | 349.98 | 0.29 | 350.92 | 0.55 | 48.96 |
| | Benders | 2 | 2 | 24.28 | 348.35 | 349.17 | 0.23 | 349.48 | 0.32 | 48.96 |
| | Benders | 20 | 11 | 77.64 | 349.22 | 350.89 | 0.48 | 350.92 | 0.48 | 48.96 |
| s05, | DW | 2 | 2 | 72.26 | 890.20 | 892.96 | 0.31 | 897.11 | 0.77 | 269.05 |
| 3 days | DW | 20 | 19 | 476.95 | 892.97 | 896.49 | 0.39 | 902.33 | 1.04 | 269.05 |
| | Benders | 2 | 2 | 89.48 | 890.88 | 892.79 | 0.21 | 897.11 | 0.69 | 269.05 |
| | Benders | 20 | 16 | 407.30 | 896.60 | 899.95 | 0.37 | 902.33 | 0.64 | 269.05 |
| s05, | DW | 2 | 2 | 303.92 | 1702.51 | 1709.76 | 0.42 | 1718.89 | 0.95 | 734.08 |
| 7 days | DW | 20 | 36 | 3069.00 | 1709.81 | 1716.92 | 0.41 | 1743.43 | 1.93 | 902.01 |
| | Benders | 2 | 2 | 352.04 | 1706.92 | 1708.51 | 0.09 | 1718.89 | 0.70 | 734.08 |
| | Benders | 20 | 15 | 1420.18 | 1722.59 | 1729.63 | 0.41 | 1743.43 | 1.20 | 902.01 |
| w05, | DW | 2 | 2 | 18.28 | 312.89 | 313.80 | 0.29 | 314.16 | 0.40 | 19.14 |
| 1 day | DW | 20 | 17 | 119.11 | 313.70 | 314.93 | 0.39 | 316.21 | 0.80 | 55.86 |
| | Benders | 2 | 3 | 28.40 | 312.50 | 313.67 | 0.37 | 314.16 | 0.53 | 19.14 |
| | Benders | 20 | 14 | 102.74 | 314.22 | 315.49 | 0.40 | 316.21 | 0.63 | 55.86 |
| w05, | DW | 2 | 3 | 102.98 | 786.81 | 789.06 | 0.29 | 792.20 | 0.68 | 218.63 |
| 3 days | DW | 20 | 34 | 886.42 | 789.02 | 792.41 | 0.43 | 798.14 | 1.14 | 218.63 |
| | Benders | 2 | 2 | 80.67 | 788.05 | 789.11 | 0.13 | 792.20 | 0.52 | 218.63 |
| | Benders | 20 | 21 | 521.62 | 791.77 | 795.60 | 0.48 | 798.14 | 0.80 | 218.63 |
| w05, | DW | 2 | 3 | 417.79 | 1641.59 | 1647.30 | 0.35 | 1656.31 | 0.89 | 614.41 |
| 7 days | DW | 20 | 51 | 4526.00 | 1643.77 | 1652.91 | 0.55 | 1678.73 | 2.08 | 972.41 |
| | Benders | 2 | 2 | 324.71 | 1643.62 | 1648.26 | 0.28 | 1656.31 | 0.77 | 614.41 |
| | Benders | 20 | 19 | 2017.75 | 1653.82 | 1661.20 | 0.44 | 1678.73 | 1.48 | 972.41 |
| s06, | DW | 2 | 2 | 17.51 | 314.29 | 315.70 | 0.45 | 316.39 | 0.66 | 37.66 |
| 1 day | DW | 20 | 14 | 89.74 | 314.95 | 316.50 | 0.49 | 318.41 | 1.09 | 57.05 |
| | Benders | 2 | 2 | 19.53 | 314.56 | 316.03 | 0.46 | 316.39 | 0.58 | 37.66 |
| | Benders | 20 | 16 | 109.84 | 315.93 | 317.28 | 0.43 | 318.41 | 0.78 | 57.05 |
| s06, | DW | 2 | 3 | 89.86 | 715.41 | 718.78 | 0.47 | 718.48 | 0.43 | 215.91 |
| 3 days | DW | 20 | 49 | 1055.78 | 715.88 | 719.45 | 0.50 | 721.18 | 0.74 | 215.91 |
| | Benders | 2 | 4 | 124.72 | 715.55 | 717.48 | 0.27 | 718.48 | 0.41 | 215.91 |
| | Benders | 20 | 26 | 584.09 | 718.37 | 721.87 | 0.49 | 721.18 | 0.39 | 215.91 |
| s06, | DW | 2 | 3 | 354.85 | 1625.66 | 1632.10 | 0.39 | 1635.89 | 0.63 | 705.58 |
| 7 days | DW | 20 | 51 | 4219.00 | 1623.71 | 1636.55 | 0.78 | 1649.16 | 1.54 | 796.41 |
| | Benders | 2 | 2 | 328.79 | 1623.52 | 1631.08 | 0.46 | 1635.89 | 0.76 | 705.58 |
| | Benders | 20 | 34 | 2821.00 | 1634.72 | 1642.83 | 0.49 | 1649.16 | 0.88 | 796.41 |

Table 5: Comparison of iterative approach, Dantzig-Wolfe decomposition approach and Benders decomposition approach.

# 6 Conclusion

We propose an iterative approach that couples the two problems aircraft routing and crew pairing heuristically and quickly generates a series of solutions with low crew pairing costs and high robustness. No monetary value needs to be attached to robustness a priori. Instead, the trade-off between costs and robustness can be observed and a preferred solution can be selected. Although optimality of the solutions can not be guaranteed, a lower bound on the optimal crew pairing cost is provided by the algorithm. We obtain solutions that incur less crew pairing costs and are significantly more robust than solutions currently used in practice.

We propose a Dantzig-Wolfe decomposition approach to solve the robust and integrated aircraft routing and crew pairing problem to optimality, which is computationally expensive. Also, to identify a robust solution using the approach, we need to associate a monetary value with non-robustness. The run times of an optimisation approach are much longer than the run time of the iterative approach. The optimisation approaches show that the iterative approach solutions are of very good quality with an average optimality gap over all problem instances of less than 1%. It is complicated to incorporate a rule like limiting the number of aircraft changes per duty period (DPACLim rule) into any optimisation approach because the rule requires to compare individual routings and pairings. We observe that there is no significant disadvantage in using Dantzig-Wolfe decomposition compared to Benders decomposition in terms of running time. We also show that the problem becomes much harder to solve if the weight for robustness is increased in the objective function.

We demonstrate in the computational experiments that it is indeed possible to solve the integrated formulations without disturbing the set partitioning structures of the individual problems. We therefore can employ existing and efficient solution methods to solve the individual problems in an integrated model.

As the main focus of this paper is to solve a real world application, data provided by Air New Zealand was used to measure the performance of the solution approaches. All rules imposed by Air New Zealand are satisfied in the solutions we generate. At the time this paper is written, Air New Zealand is using the iterative approach in their production environment. This required to extend the approach to consider multiple crew pairing problems (for captains, first officers, and cabin crew) replacing the single crew pairing problem as presented in this work. The implementation would not have been possible without considering all rules of a real world application in this research project. Until optimisation methods are improved, we suggest the use of sensible heuristic decision making combined with optimisation methods in order to incorporate all requirements of a practical problem. Despite using data from an Air New Zealand domestic schedule, the proposed methods are sufficiently general so that we expect similar results for similar airlines that operate a domestic schedule with routings and pairings containing many flights per day and many short turn times.

Future research includes the integration of other airline scheduling problems, i.e. time windows for the departure times, fleet assignment, and crew rostering into an integrated problem. Most importantly, passenger flow should also be considered in an integrated model since an aircraft is frequently delayed because of late connecting passengers. Also, the use of additional robustness measures could be investigated, possibly in combination with the ones considered in this work. Finally, the operational counterpart of the problem should be investigated where in case of disruptions decisions on how to recover must be made quickly. Since the iterative approach is very fast, it can be used to simultaneously re-route aircraft, crew, and passengers in an automated fashion.

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