A Mixed Integer Programming Approach to the Aircraft Weight and Balance Problem

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Abstract

The Aircraft Weight and Balance Problem (AWBP) is a real-world combinatorial optimisation problem in which an aircraft should be loaded with containers in such a way that the total cargo value is maximised. At the same time the centre of gravity should approach an optimal point. Flying optimally reduces fuel consumption and thus results in a financial and environmental gain. This paper introduces a mixed integer programming approach to solve the AWBP. Experimental results show that the model enables an increase of the cargo value compared to the result obtained by an experienced planner. Moreover, the mixed integer programming approach achieves better balanced solutions in only a few seconds.

Keywords: Aircraft Weight and Balance Problem, Mixed Integer Programming

1 Introduction

This paper describes a solution approach to the Aircraft Weight and Balance Problem (AWBP). It involves maximising the total cargo profit and minimising the difference between the Centre of Gravity (CG) of the loaded aircraft and its optimal value in the longitudinal as well as in the lateral direction. Loading cargo is not only an issue of cargo flights but it also strongly affects passenger flights. In fact, it is one of the major revenue sources of airline companies and obviously the core business of cargo carriers. The AWBP can be modelled as a combinatorial optimisation problem. From a set of available cargo units, the most profitable selection has to be loaded into the aircraft while satisfying a large number of safety constraints.

The problem is highly constrained mainly due to safety concerns. When too much weight is loaded into the front, the aircraft is said to be nose-heavy, which results in take off difficulties. On the other hand, the aircraft will tip on its tail when too much weight is loaded into the back. Other safety constraints are imposed to avoid abnormal stresses on the structure of the aircraft. If the aircraft is not correctly loaded, its flying characteristics may change, which, in the worst-case scenario, results in a crash [4]. It is up to the weight and balance expert to manually modify the loading scheme as long as it generates violations of the safety constraints. The aircraft can fly safely only when all the loading constraints are satisfied. However, within the feasible region, large differences exist in fuel consumption. That depends upon the distance between the longitudinal and lateral CGs and their optimum values. Flying optimally results in a financial gain for the airline since less fuel is needed to transport the same cargo. At the same time, an environmental gain is achieved by reducing carbon dioxide emissions.

Currently, weight and balance engineers manually determine the position of the cargo in the aircraft. First, in the case not all cargo can be loaded, they make a selection of the available cargo. They use charts or software tools to calculate the CG, based on the positions of the cargo. Next, they check if the CG is within permitted limits and if the total cargo weight is below the maximum value. Various other safety checks concerning the strength of the aircraft are performed. Only when all checks are cleared, the cargo selection and the position of the containers comply with the safety measures. This manual planning process is time consuming and it leads in most cases to suboptimal solutions.

Amiouny et al. [1] are the first and, to the best of our knowledge, only to report on the aircraft load balancing problem in the academic literature. Their heuristic algorithm only focuses on a one-dimensional balancing problem, and ignores the cargo value optimisation. Other related literature addresses subproblems that precede the AWBP. In aircraft cargo loading, it is important to pack as much cargo onto a container or a pallet as possible. Chan et al. [2] present a three-dimensional bin packing approach for pallet loading. The distribution of the weight in each container also plays an important part in container transportation. That problem is addressed by Davies and Bischoff [3].

The contribution of the paper is a decision support model to automate aircraft loading in such a way that the cargo profit is maximised and that the optimal CGs are approached in both the longitudinal and lateral dimension without violating any safety constraints. The uniqueness of our approach lies in the combination of bin packing and load balancing. The rest of the paper is organised as follows. First, a detailed description of the AWBP is given in Section 2. Section 3 proposes a mixed integer model for the AWBP. Section 4 thoroughly evaluates the proposed model. Finally, Section 5 concludes the paper and points out ideas for further research.

2 Problem Description

In this section, we describe each of the characteristics that determine the AWBP:

- the characteristics of the cargo and the position slots that hold the cargo in the aircraft;
- the aircraft weight and the CG at different loading stages;

- the fuel and cargo weights;
- the different weight and balance restrictions;
- the different objectives.

From a set of available Unit Loading Devices (ULDs), an optimal selection with respect to the total cargo value has to be loaded into an empty aircraft. A ULD can be a container or a pallet. It is characterised by the type, size, weight, height and the profit gained for transporting the ULD. As opposed to pallets, containers all have a standard height.

An aircraft consists of multiple decks, that each have multiple loading configurations of position slots for ULDs. A position slot determines particular aircraft space that can hold exactly one ULD. Each position slot is limited to a set of allowable ULD types. Position slots belonging to a so-called "loading configuration" should not overlap, whereas position slots belonging to different loading configurations on the same deck may overlap. A position slot is characterised by its longitudinal Balance Arm (BA) and its lateral BA. The longitudinal BA is a longitudinal coordinate referenced to a datum (BA zero), which is commonly defined as a fictitious point in the nose of the aircraft. The lateral BA is the lateral coordinate referenced to the centre of the aircraft. In the rest of this paper, our main attention goes to the longitudinal BA, unless lateral is explicitly stated.

We consider the aircraft weight at different loads: the Dry Operating Weight (DOW) is the weight of the empty aircraft, including the staff; the Zero Fuel Weight (ZFW) equals the DOW plus the weight of the cargo; the Taxi Weight (TW) equals the ZFW plus the total fuel weight; the Take Off Weight (TOW) is the TW minus the weight of the taxi fuel; the Landing Weight (LW) is the TOW minus the weight of the trip fuel. The fuel is divided among several tanks. In each tank the fuel weight at the different aircraft weights depends on the fuel-use scheduling strategy, which is known beforehand. The total fuel weight is the sum of the fuel weight of the different tanks at TW. The taxi fuel weight is the total fuel weight minus the sum of the fuel weights of the different tanks at TOW. The trip fuel weight is the total fuel weight minus the sum of the fuel weights of the different tanks at LW.

Basic principles of rotational mechanics allow to determine the CG at the different reference weights. A CG is calculated by dividing the sum of the moments of forces by the sum of the forces. The moment of the gravitational force is obtained by multiplying the force by the corresponding BA. For the problem tackled in this paper, we assume that the gravity acceleration is constant and that height differences between decks are negligible. The CG or BA at DOW is determined by physically weighing the aircraft. The CG of the aircraft at ZFW is determined the division of the sum of the moments of the ULDs and the moment of the aircraft at DOW by the ZFW. The CG of the aircraft at TW, TOW and LW is calculated by dividing the sum of the moments introduced by the fuel tanks and the moment of the aircraft at the appropriate weight by that weight. The BA of the fuel weight of a tank depends on the physical shape of the tank and on the amount of fuel.

The AWBP is subject to a large number of constraints. Position slots belonging to the same configuration are non-overlapping, while those belonging to different configurations may overlap. If a ULD is placed on a particular position slot, the overlapping position slots should remain empty. A position slot has a maximum weight capacity, a maximum height and a set of allowed ULD types.

The ZFW, TW, TOW and LW are constrained by maximum values. These maximum weights depend on several variables such as the amount of lift the wings and engines can provide, the length of the runway, etc.

The CG is limited by minimum and maximum values that depend on the total weight of the aircraft. The CGs for the different reference weights have to be within boundaries that are called the envelope. At each reference weight, the CG boundaries depend on the actual weight.

For each deck individually, a number of load restrictions are defined in terms of BA boundaries: cumulative load constraints limit the total weight to be within the BA boundaries; linear load constraints limit the total weight divided by the length; floor load constraints limit the total weight divided by the surface of the ULDs. The maximum values of these load constraints are constants, while in the case of cumulative and linear load constraints the maximum values are sometimes determined by linear functions of the CG at ZFW. In case the BA limit intersects with a position, the weight of the load on that position is partially taken into account. This set of load constraints are imposed by the structural design of the aircraft. Cumulative and linear load constraints also apply to positions on multiple decks. Counter balance constraints, in addition, define minimum weights per zone.

Finally, the lateral CG has to be checked as well. Lateral imbalance constraints and unsymmetrical load constraints define per zone the maximum relative difference between the weight loaded onto the left and right positions. Lateral imbalance constraints are related to flight stability, whereas unsymmetrical load constraints are induced by the mechanical characteristics of the floor deck.

The objective of the AWBP is to maximise the flight's profit. That is realised by maximising the cargo value and minimising fuel consumption. Cargo is divided into priority classes: e.g. first class luggage has priority over regular cargo. The second part of the objective involves optimising the weight balance. Both the longitudinal and lateral CG at ZFW should get as closely as possible to their optimum values.

3 Mathematical Model for the AWBP

This section introduces a mathematical model for the AWBP. Before explaining the details of the objective function and the constraints, we introduce a number of constants and variables that determine the problem.

3.1 Constants

- N_{ULD} the total number of ULDs available for loading;
- S_i the size of the floor surface of ULD i;
- T_i the type of ULD i;
- W_i the weight of ULD i;
- H_i the height of ULD i;

- G_i the profit gain for transporting ULD i;
- N_{POS} the total number of positions on the aircraft;
- BA_j the longitudinal balance arm of position j;
- $BA_{lat,j}$ the lateral balance arm of position j;
- MW_j the maximum weight capacity of position j;
- MH_j the available height at position j;
- T_j the set of allowed ULD types at position j;
- A_{DOW} the DOW of the plane;
- N_{FUEL} the total number of fuel tanks of the plane;
- $FW_{t,w}$ the weight of the fuel in tank t at aircraft weight w;
- $FBA_{t,w}$ the balance arm of the fuel in tank t at aircraft weight w;
- BA_{DOW} the balance arm or centre of gravity of the aircraft at DOW;
- BA_{OPT} the optimal balance arm of the aircraft at ZFW;
- $MaxWeight_w$ the maximum weight of the aircraft, for $w \in \{ZFW, TW, TOW, LW\}$;
- $MinCG_w$ and $MaxCG_w$ the minimum and maximum CG, for $w \in \{ZFW, TW, TOW, LW\}$;
- N_{LLC}, N_{CLC}, N_{FLC}, N_{CBC} the number of linear load, cumulative load, floor load and counter balance constraints;
- P_{jk} the percentage position j is accounted for in constraint k;
- C_k and D_k define the linear function of the a_{ZFW} for linear and cumulative constraints k;
- MFL_k the maximum floor load for constraint k;
- $MinW_k$ the minimum weight for constraint k;
- N_{LIBC} and N_{ULC} the number of lateral imbalance and unsymmetrical loading constraints;
- $MaxDelta_k$ the maximum absolute weight difference between left and right positions for constraint k;

3.2 Variables

- Binary decision variable x_{ij} is 1 when ULD i is placed onto position j, it is 0 otherwise.
- Aid variable a_{ZFW} represents the ZFW and is equal to

$$a_{ZFW} = A_{DOW} + \sum_{i=1}^{N_{ULD}} \sum_{j=1}^{N_{POS}} W_i x_{ij}$$

- Aid variables a_w represent the TW, TOW, LW and are equal to

$$a_w = a_{ZFW} + \sum_{t=1}^{N_{FUEL}} FW_{t,w} \qquad w \in \{TW, TOW, LW\}$$

- Aid variable m_{ZFW} represents the moment at ZFW and is equal to

$$m_{ZFW} = A_{DOW} BA_{DOW} + \sum_{i=1}^{N_{ULD}} \sum_{j=1}^{N_{POS}} W_i BA_j x_{ij}$$

- Aid variables m_w represent the moments at TW, TOW, LW and are equal to

$$m_w = m_{ZFW} + \sum_{t=1}^{N_{FUEL}} FW_{t,w} FBA_{t,w} \qquad w \in \{TW, TOW, LW\}$$

- Two deviation aid variables $m_{long}d^-$ and $m_{long}d^+$ define the negative and positive deviation from the actual longitudinal moment to the optimal moment at ZFW. If $m_{long}d^- > 0$, then $m_{long}d^+ = 0$ and vice versa;
- Two deviation aid variables $m_{lat}d^-$ and $m_{lat}d^+$ define the negative and positive deviation from the actual latitudinal moment to the centre of the aircraft. If $m_{lat}d^- > 0$, then $m_{lat}d^+ = 0$ and vice versa;

3.3 Objective Function

As stated above, the main objective of the AWBP is to maximise the profit by transporting the load. This formulation has induced the introduction of priority classes of cargo: the gain of a high priority ULD should be higher than the sum of all the gains of the ULDs with lower priority. The second objective is to optimise the weight balance along the longitudinal dimension. It means that the longitudinal CG at ZFW should should be as closely as possible to the optimal BA. The third objective is to optimise the lateral weight balance. It involves that the absolute value of the lateral CG (or the lateral moment) at ZFW should be minimised. A few aid variables are used in order to model the second and third objective. They allow to express that the deviations between the actual moment at ZFW and the optimal one are to be minimised.

$$Max \quad \sum_{i=1}^{N_{ULD}} \sum_{j=1}^{N_{POS}} G_i x_{ij} - \frac{m_{long}d^- + m_{long}d^+}{K_1} - \frac{m_{lat}d^- + m_{lat}d^+}{K_2} \tag{1}$$

The factors K_1 and K_2 have been introduced to handle the relative importance of the different objectives. Optimising the longitudinal weight balance allows to ignore the envelope constraints and avoids non-linear constraints in the model.

3.4 Constraints

$$BA_{OPT}a_{ZFW} = m_{ZFW} + m_{long}d^{-} - m_{long}d^{+}; \tag{2}$$

$$\sum_{i=1}^{N_{ULD}} \sum_{j=1}^{N_{POS}} W_i BA_{lat,j} x_{ij} = m_{lat}d^{+} - m_{lat}d^{-}; \tag{3}$$

$$\sum_{j=1}^{N_{POS}} x_{ij} \leq 1; \qquad \forall i = 1, \ldots, N_{ULD} \tag{4}$$

$$\sum_{i=1}^{N_{ULD}} x_{ij} \leq 1; \qquad \forall j = 1, \ldots, N_{POS} \tag{5}$$

$$\sum_{i=1}^{N_{ULD}} (x_{ij}+x_{ik}) \leq 1; \qquad \forall j = 1, \ldots, N_{POS}, \forall k = 1, \ldots, N_{POS} | k \text{ overlaps } j \tag{6}$$

$$W_i x_{ij} \leq MW_j; \qquad \forall i = 1, \ldots, N_{ULD}, \forall j = 1, \ldots, N_{POS} \tag{7}$$

$$H_i x_{ij} \leq MH_j; \qquad \forall i = 1, \ldots, N_{ULD}, \forall j = 1, \ldots, N_{POS} \tag{8}$$

$$i \in T_j; \forall i = 1, \ldots, N_{ULD}, \forall j = 1, \ldots, N_{POS} | x_{ij} = 1 \tag{9}$$

$$a_w \leq MW_w; \qquad \forall w \in \{ZFW, TW, TOW, LW\} \tag{10}$$

$$\sum_{i=1}^{N_{ULD}} \sum_{j=1}^{N_{POS}} W_i P_{jk} x_{ij} \leq C_k a_{ZFW} + D_k, \qquad \forall k = 1, \ldots, N_{CLC} + N_{LLC} \tag{11}$$

$$\sum_{i=1}^{N_{ULD}} \sum_{j=1}^{N_{POS}} W_i P_{jk} x_{ij} \leq MFL_k \sum_{i=1}^{N_{ULD}} \sum_{j=1}^{N_{POS}} S_i P_{jk} x_{ij} \qquad \forall k = 1, \ldots, N_{FLC} \tag{12}$$

$$\sum_{i=1}^{N_{ULD}} \sum_{j=1}^{N_{POS}} W_i P_{jk} x_{ij} \geq MinW_k, \qquad \forall k = 1, \ldots, N_{CBC} \tag{13}$$

$$\begin{gathered} -MaxDelta_k \leq \sum_{i=1}^{N_{ULD}} \sum_{l} W_i P_{lk} x_{il} - \sum_{i=1}^{N_{ULD}} \sum_{r} W_i P_{rk} x_{ir} \leq MaxDelta_k, \\ \forall l = 1, \ldots, N_{POS} | l \text{ is left position slot}, \\ \forall r = 1, \ldots, N_{POS} | r \text{ is left position slot}, \\ \forall k = 1, \ldots, N_{LIBC} + N_{ULC} \end{gathered} \tag{14}$$

$$x_{ij} \in \{0, 1\}; \qquad \forall i = 1, \ldots, N_{ULD}, \forall j = 1, \ldots, N_{POS} \tag{15}$$

$$m_{long}d^- \geq 0; m_{long}d^+ \geq 0; m_{lat}d^- \geq 0; m_{lat}d^+ \geq 0; \tag{16}$$

Constraint (2) expresses the relation between the actual and the optimal longitudinal moment at zero fuel weight. Constraint (3) expresses the deviation of the actual lateral moment at zero fuel weight. All the deviation variables (16) should be greater than or equal to zero. Constraints (4) state that each ULD can be loaded at most once. Constraints (5) state that at most one ULD can be placed onto a position slot. Constraints (6) state that at most one ULD can be assigned to each pair of overlapping position slots. A position slot has a maximum weight capacity and a maximum height, as stated by Constraints (7) and (8). Constraints (9) state that the type of ULD i should belong to the set of admitted types at position j, if ULD i is placed onto position j. Constraints (7), (8) and (9) are checked beforehand: if the placement of ULD i is not allowed on position j, x_{ij} is omitted from the model. Constraints (11) define the linear and cumulative load limits. Constraints (12) state the floor load restrictions. Constraints (13) state the counter balance limits. Lateral imbalance constraints and unsymmetrical load constraints are implemented by (14).

4 Experimental Results

This section presents the experimental results of solving the proposed model with a Mixed Integer Programming (MIP) approach using the CPLEX 11.0 solver. The model is solved for ten cases involving the following aircraft: Boeing 757, Airbus A300 and Tupolev 204. Table 1 presents the characteristics of the different problem instances.

Table 1: Problem Characteristics

Testset	N_{ULD}	N_{POS}	N_{CLC}	N_{LLC}	N_{FLC}
TUP6404_20070403	20	37	14	0	0
BCS963_20070402	15	20	17	10	0
WO8284_20070402	37	116	31	17	2
BCS4445_20070405	29	90	42	6	0
BCS7466_20070410	31	90	42	6	0
GCO081_20070402	34	87	25	13	0
GCO081_20070403	34	87	25	13	0
NPT893_20070402	7	45	0	0	18
BCS2562_20070401	15	40	16	12	0
BCS4291_20070401	17	40	16	12	0

These cases have been derived from real loading schemes. The ULDs are those that have effectively been loaded onto the aircraft, including the ULDs that have been left behind. Section 4.1 presents the results of the MIP approach and compares them with the solutions obtained by an expert manual planner who applies a graphical software tool that indicates the longitudinal CG and the constraint violations. In Section 4.2 the problem instances are modified by generating extra ULDs for loading, which increases the complexity of the problem as the number of binary decision variables increases. The results of the experiments with excess ULDs give an indication of the time needed to solve more complex instances. The time results presented in Table 4 are averaged over ten runs. All experiments are performed on an Intel Xeon 2.5GHz. K_1 is equal to 10^5 while K_2 is equal to 10^6. These values have been arbitrarily chosen for the experiments. However, if the results would turn out not to respect the relative importance of the three objectives, these values need to be altered in order to meet the domain expert's expectations.

4.1 MIP Result Compared to Expert Solution

Table 2 compares the number and the weight of the loaded ULDs, which results from the MIP algorithm, to the solution obtained by the manual planner. The last two columns present the improvement of the CPLEX approach to the manual planner. In four of the ten cases the algorithm enabled to load an extra ULD onto the aircraft. In fact, the ULD was left behind, as a result of the suboptimal solution of the manual planner. We wish to notice that in the datasets no additional containers (but occasionally one) were available for loading. The outperformance of our approach to the expert might even be larger in real-life.

	MIP		Manual		Improvement	
Problem Instance	#ULDs	weight	#ULDs	weight	#ULDs	weight
TUP6404_20070403	19	27047	19	27047	0	0
BCS963_20070402	15	17382	14	16682	1	700
WO8284_20070402	37	63501	37	63501	0	0
BCS4445_20070405	29	24677	29	24677	0	0
BCS7466_20070410	31	36131	30	35666	1	465
GCO081_20070402	34	56647	34	56647	0	0
GCO081_20070403	34	63081	33	61901	1	1180
NPT893_20070402	7	2428	6	2304	1	124
BCS2562_20070401	15	12928	15	12928	0	0
BCS4291_20070401	17	22622	17	22622	0	0

Table 2: Comparison of the MIP and the manual approach with respect to the number of loaded ULDs and the loaded cargo weight (expressed in kg)

Table 3 presents the balance results of the algorithm, compared to the solution of the manual planner. The CG columns of MIP and Manual display the difference between the longitudinal and optimal CG. The LI columns of MIP and Manual display the lateral imbalance, or the lateral CG. The CG and LI columns of Improvement display the improvement of the MIP approach.

In all of the ten test cases the longitudinal CG is improved. The average improvement is 29.22 cm. In all test cases the optimal CG is approached which lies within the envelope. The envelope constraints were checked afterwards and were never found violated. In five test cases the lateral CG is equal to zero. This is due to the fact that no twin row configurations are available in these aircraft. The lateral BA for all position slots is zero and consequently lateral moments could be induced. In the other five cases, the lateral CG is improved in 2 cases and worse in 3 cases. If the lateral CG has worsened, this is due to the settings of

	MIP		Manual		Improvement	
Problem Instances	CG	LI	CG	LI	CG	LI
TUP6404_20070403	0.06	0.00	11.26	0.00	11.20	0.00
BCS963_20070402	0.77	0.00	66.92	0.00	66.16	0.00
WO8284_20070402	1.34	0.20	6.34	2.25	5.00	2.05
BCS4445_20070405	0.04	1.42	40.61	0.63	40.57	-0.79
BCS7466_20070410	0.49	1.83	21.71	0.84	21.22	-0.99
GCO081_20070402	0.24	0.02	54.81	0.30	54.57	0.28
GCO081_20070403	2.16	0.52	7.67	0.45	5.51	-0.07
NPT893_20070402	0.49	0.00	14.91	0.00	14.42	0.00
BCS2562_20070401	5.13	0.00	37.35	0.00	32.22	0.00
BCS4291_20070401	2.30	0.00	43.65	0.00	41.35	0.00

Table 3: Balance Comparison

K_1 and K_2, which determine the relative importance of the different objectives. In these cases, the algorithm uses a twin row configuration in order to obtain a better longitudinal CG. However, this twin row configuration introduces lateral moments. The manual planner sticked to single row configurations, which led to a suboptimal longitudinal CG, but no lateral moments and thus a better lateral CG.

4.2 Selecting a Subset of Available ULDs

The execution times needed for solving the ten test cases varies from 15.3 ms to 390 ms. The complexity of the problems (and the subsequent execution times) is expected to increase with the number of available ULDs. In order to create more complex (and more realistic) problem instances, the problem instances (from Section 4.1) are modified by adding extra ULDs. A multiplication variable is introduced that determines the number of extra ULDs to make available for loading. The target number of ULDs is the original number times the multiplicator. Until that target number is reached, the following procedure is applied to duplicate an existing ULD and add it to the collection of available ones. Pick a random ULD from the original collection and copy all its characteristics, apart from the weight. The weight is sampled from a Gaussian distribution with μ equal to the weight of the original ULD and σ equal to 100.

multiplicator	1	2	3	4	5
min (ms)	15	27	42	46	56
avg (ms)	135	1 137	1 848	1 739	2 083
max (ms)	390	3 804	7 574	5 292	9 962

Table 4: Execution time in case of excess ULDs for loading

Table 4 presents the results of experiments with a multiplicator ranging from 1, which corresponds to the original test cases, to 5. The average execution time ranges from 135 ms to 2 s. The maximum execution time is less than 10 s, which is acceptable for real-life decision support.

5 Conclusions and Future Research Directions

This paper introduces the Aircraft Weight and Balance Problem as a strongly constrained combinatorial optimisation problem, in which the total profit of the cargo to transport has to be maximised and the centre of gravity has to approach the optimal centre of gravity. We have modelled the problem in such a way that it can be solved to optimality by Mixed Integer Programming. The problem instances have been derived from real loading schemes. In four of the ten instances, the algorithm enabled loading more cargo than an experienced manual planner. On average, the longitudinal centre of gravity was moved considerably closer to the optimal centre of gravity. The lateral centre of gravity shows a marginal improvement of 0.05 on average. Execution times are 135 ms on average for the problems in which all the available cargo can be loaded onto the aircraft. As it takes the manual planner ten minutes to achieve a suboptimal solution. The Mixed Integer Programming approach achieves solutions, in which the value of the load is optimised, within an excellent response time. At the same time, due to the improved centre of gravity position, less fuel will be needed to transport the cargo, which reduces operational costs and carbon dioxide emissions.

In the current model the envelope constraints have been ignored. The rationale was twofold. Firstly, considering the envelope constraints would unavoidably result in a non-linear model, which would have a bad effect on its solvability by Mixed Integer Programming. Secondly, we estimate that optimising both the longitudinal and the lateral centre of gravity, leads to solutions in which the centre of gravity is situated near the centre of the envelope. Future work includes researching the feasibility of a model with explicit envelope constraints. We will consider a piecewise linear function to approximate the envelope constraints, in order to avoid a non-linear model. In addition, passenger aircraft loading requires cargo unloading priorities in the future model: baggage containers are to be unloaded first and therefore should be loaded near the door, in decreasing order of priority: transfer, first class, rest, business class and economy class luggage. For safety reasons, the load should be crushable at some positions in some aircraft. The cargo content may impose extra constraints. Some dangerous cargo should for example not be loaded in the vicinity of vulnerable goods. Finally, the model should enable multiple bulk items to be loaded into the belly position, which is only subject to volume and weight constraints.

Taking all these extra constraints into account will result in a model that enables outperforming the loaded cargo value and the fuel use compared to the current practice.

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