Noise Load Management at Amsterdam Airport Schiphol

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Abstract

Amsterdam Airport Schiphol is one of the five primary hub-airports in Europe. All flight movements are controlled by Air Traffic Control the Netherlands (LVNL), whose main objective is to guarantee safety, efficiency, and protection of the environment, that includes noise load. To this end, a number of enforcement points is located in the vicinity of Schiphol. Each aircraft movement contributes to the noise load at these points. If the cumulative load in an aviation year at an enforcement point exceeds its maximum, the civil aviation authority may impose severe sanctions, such as fines, or a reduction in the number of aircraft movements. The latter is a typical restriction for Schiphol.

Runway selection is carried out via the preference list, an ordered set of runway combinations such that the higher on the list a runway combination, the better this combination is for maintaining the noise load limit. The highest safe runway combination in the list will actually be used. This paper has formulated the preference list selection process in the mathematical framework of Stochastic Dynamic Programming that enables determining an optimal strategy for preference list selection taking into account future and unpredictable weather conditions, as well as safety and efficiency restrictions. The size of the problem is determined by the number of enforcement points.

The work described in this paper is intended as a feasibility study. Numerical results indicate that, although our algorithm in Matlab on a regular PC has a running time that is not practical for direct implementation in the decision process, our algorithm is capable of

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describing optimal decisions for a smaller set of enforcement points, and therefore that the approach followed in this paper is a feasible solution for the optimal preference list problem.

1 Introduction

Amsterdam Airport Schiphol is the main airport in the Netherlands, and is one of the five primary hub-airports in Europe. As a primary hub it serves as a mainport –a junction in the national and international network of transportation of people, goods and services– and is an important driving force for the Dutch economy. All flight movements are controlled by Air Traffic Control the Netherlands (LVNL), that supports the development of the mainport by aiming at an optimal utilization of its aerodrome capacity within limiting conditions, such as weather, runway availability, and environmental restrictions. The main objective of air traffic control is to guarantee *safety*, i.e., that flight trajectories stay well separated.

Air traffic control involves complex interactions between multiple human operators, procedures and technical systems [1]. With the increasing number of flight movements over the years, efficient handling of the traffic volume has become a second major objective. Airports tend to be the bottle-neck, since their infrastructure has not evolved in proportion with the increasing number of flight movements. This has forced airports, airlines and air traffic control to improve the *efficiency* of arrival, departure, stand allocation and turn-around management [6].

Safety and efficiency are two main aspects that determine capacity and characterize the service provided by air traffic control centers worldwide. In the Netherlands a third major task is added. Protection of the *environment* –including pollution, smell, third party risk and especially noise– increasingly influences airport operations. The current Dutch Aviation Act restricts runway and route usage, limits the total noise load produced by aircraft during one year of operation, and enforces a certain distribution of noise load over the direct surroundings of Schiphol [8].

To monitor the noise load distribution, a number of enforcement points is located in the vicinity of Schiphol. Each arriving and departing aircraft contributes to the *noise load* at these enforcement points. To this end, a dB(A) scale is used that allows adding the amount of dB produced by one aircraft to the cumulative noise load of preceding aircraft [7]. Note that noise load is not physically measured but calculated only. Noise load limits are strictly enforced per enforcement point [8]. If the cumulative load in an aviation year (November 1st – October 31st, coinciding with the airline scheduling year) at an enforcement point exceeds its maximum, the civil aviation authority may impose severe sanctions, such as fines, temporary closure of a runway or a reduction in the number of aircraft movements, causing substantial economic damage to the airport and airlines involved. Hence, noise load needs to be managed during the year to prevent an excess in one or more enforcement points. Within this legal and economic framework, air traffic controllers provide the safe and efficient handling of traffic.

Schiphol has five major runways, that allow for different runway combinations. The operation per runway varies depending on the traffic and dependencies among runways. Not all conceivable runway combinations are feasible due to safety and efficiency restrictions. Availability of the individual runways depends on weather criteria. Hence, during operations the choice for the active runway combination is limited in the first place by the weather. When more than one runway combination, satisfying all weather criteria, is available, the one is utilized that is most preferred with respect to noise load management [9]. This preference is laid down in a predetermined ordered set of runway combinations: the preference list.

A preference list is an ordered list of runway combinations such that the highest on the list is used when weather conditions allow its safe use. Otherwise, the next on the list is used, and so on. Each month, based on the realized noise load at the enforcement points, the preference list is implemented that is expected to produce the most balanced accumulation of noise load in the enforcement points with the aim to avoid exceeding noise load limits: preference list changes result in changes of the traffic distribution around the airport, thus leading to a different noise load distribution. *The preference list is the main steering method in air traffic control for Schiphol.* As the weather conditions in the Netherlands are highly unpredictable, optimal selection of preference lists, taking into account the future (and uncertain) weather conditions is of the utmost importance for efficient allocation of Schiphol's capacity.

Preference lists are used at several airports with a more complex lay-out of runways. An airport with a simple layout, e.g. parallel runways, does not have the need for preference lists, since its operation and the noise load development is very predictable. Schiphol has evolved into a complex airport, with runways in different directions, that would unevenly impact communities in its environment if not for the use of preferred runway usage. Airports with similarly complex layouts, such as Logan International Airport in Boston and John F. Kennedy International Airport in New York, also make use of a preference list to control noise load in its surroundings. At these airports, noise load balancing is carried out on a voluntary basis. The Netherlands is unique in the fact that the noise restrictions are enforced by law, making noise load a main steering parameter.

This paper proposes an efficient noise load management scheme for Schiphol that can be used in the monthly preference list selection process. It further allows for a fast evaluation of the trade-off between different runway combinations taking into account future weather conditions, as well as safety and efficiency restrictions. The use of different preference lists directly affects the surroundings of Schiphol. Adequate noise load management will allow Schiphol to coexist with the population in its vicinity. Proper distribution of noise load is imperative to increase the level of acceptance, making this a societally relevant topic.

This paper introduces a mathematical decision model for preference list selection. Each month an optimal preference list is generated based on the realized noise load, flight schedules, and available runway combinations, taking into account uncertain weather conditions. The mathematical framework is that of Stochastic Dynamic Programming [5], which can be directly mapped to the decision process. The size of the program is mainly determined by the number of enforcement points. Schiphol has 35 enforcements points, a number that prohibits an exact evaluation of the dynamic program. Therefore, some approximations are described that allow a numerical evaluation of the preference list selection process.

Schiphol is faced with one of the most extensive set of constraints to noise load in the world. Since noise abatement is becoming increasingly important in the aviation sector, more airports may develop similar noise load management procedures to control noise load in their environments. Due to the versatility of the proposed model, it can easily be implemented for any airport.

The paper is organized as follows. Section 2 presents the noise load management problem and its current implementation at Schiphol. Our mathematical framework is described in Section 3, which also reveals the optimal decision rules. Implemented issues due to e.g. the size of the problem are discussed in Section 4. Section 5 presents a feasibility study that includes a number of examples and a comparison with the current method for preference list selection. Finally, Section 6 provides conclusions, aims for further research, and recommendations.

2 Noise load management

Noise load is the main steering parameter for the selection of preferred runway combinations. To monitor the realized noise load a number of enforcement points is placed in the vicinity of Schiphol. The locations have been chosen to represent an appropriate noise contour surrounding the airport relevant for the population in its vicinity. For each aircraft, the noise load contribution to these enforcement points is calculated based on its flight path, and added to the already realized noise load.

Environmental noise is commonly expressed in A-weighted decibels dB(A), which is an expression for the relative loudness of sounds in air as perceived by the human ear. To reflect the sensitivity of the human ear, decibel values of sounds at low frequencies are reduced, as compared to unweighted decibels, in which no correction is made for audio frequency. Noise load per day is determined by means of values for L_{den} and L_{night}, the noise descriptors for respectively the day-evening-night period and the night period only. L_{den} represents an equivalent sound level over 24 hours at a certain location in which sound levels during the evening (19:00h-23:00h) are penalized by an increase of 5 dB(A) and those during the night (23:00h-07:00h) of 10 dB(A) to reflect people's extra sensitivity to noise during the night and the evening.

Two types of limitations exist with respect to the noise load produced at Schiphol. Firstly, the total volume of noise load produced by all aircraft is limited. This total volume is defined as the arithmetic average of the calculated values of both L_{den} and L_{night} at 33 imaginary points around a single imaginary runway that handles all traffic during the year. Secondly, the distribution of noise load is imposed by a set of enforcement points. An enforcement point is a government-defined location in the vicinity of Schiphol where a limit exists to the yearly cumulates of L_{den} and L_{night}. To L_{den} this is applicable for 35 points, which are shown in Figure 1, to L_{night} for a separate set of 25 points. This paper focuses on the 35 enforcement points for L_{den}.

Noise load needs to be managed during the year to prevent an excess in one or more enforcement points. Since different preference lists distribute noise load differently, they are used as a steering measure to balance the

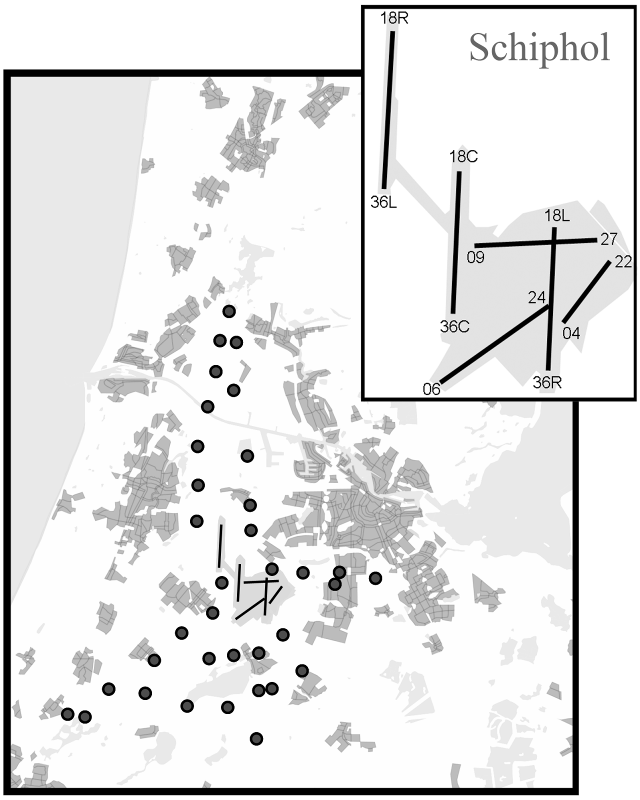

Figure 1: Enforcement points for L_{den} and layout Schiphol

noise load over the enforcement points. In principle, excess in a single enforcement point results in government measures irrespective of the amount of dB(A) by which the noise limit is exceeded. Furthermore, in principle, an excess in multiple points does not intensify the government measures. As a consequence, steering of noise load via preference lists is undertaken to avoid an excess of the noise load limit in all enforcement points, or since weather conditions are uncertain, to minimize the risk of exceeding in one or more enforcement points. To this end, it is essential that measures are taken that result in a balanced development of noise load. On a monthly basis it is decided whether or not a change of the preference list is necessary to achieve a balanced noise load distribution for the remaining period. This decision is made by the Environmental and Economic Management Committee (E^{2}MC), a consultative body consisting of representatives of the airlines, Schiphol and LVNL, and balances interests and demands of parties in the aviation sector.

The preference lists that are available for implementation at Schiphol consist of lists of runway combinations per period of the day. Schiphol is a hub-airport, which is characterized by traffic arriving and departing in waves. The supply of inbound and outbound traffic varies over time, resulting in several inbound peaks, outbound peaks and intermediate-periods and a night-period. For these different periods, different runway combinations are utilized. A complete description of the preference lists is omitted, since these consist of a complex aggregate of lists for runway use for arriving and departing traffic per period. To obtain insight into the affected area surrounding Schiphol using a certain preference list, the primary runways for the highest preference position on the lists are shown in Table 1. These runways, as displayed in Figure 1, are expected to process the largest part of traffic. Less preferred runway combinations vary per preference list and hence affect the surroundings differently. To this end, the enforcement points are shown that are expected to be affected most by the implemented preference list. These enforcement points are depicted in Figure 2 in Section 5.1. Combination of Figures 1 and 2 then reveals the relation between the enforcement points and runway combinations. Although the preference lists seem to coincide, these lists differ in less preferred runways, similar to the difference between preference list 1 and 2 primary runways. In the Appendix, preference lists 1 and 2 are included as illustration. More details can be found in [4].

Pref.	inbound peak			outbound peak			largest exp. contrib.				
list	dep.	arrival		departure		arr.	enforcement points				
1	36L	06	36R	36L	36C	06	18	19	8	9	21
2	36L	06	36R	36L	09	06	21	20	19	22	9
3	36L	06	36R	24	36L	27	22	21	9	8	7
4	36L	06	36R	36L	09	06	21	20	19	22	23
5	24	18R	18C	24	18L	18R	5	4	19	25	31
6	24	18R	18C	24	18L	18R	5	4	19	31	22
7	24	18R	18C	24	18L	18R	21	20	19	22	25
8	24	18R	18C	24	18L	18R	21	20	19	5	4

Table 1: Available preference lists (primary runways)

Mainly due to uncertainty in the weather, differences in the expected and actual realization of noise load develop rapidly. Obviously, noise load realization in the enforcement points is particularly sensitive to the implemented operation. For effective noise load management it is essential that noise load is not allowed to grow at a disproportionally high rate at any enforcement point, as this may yield a situation where an excess cannot be avoided in an operationally acceptable manner. This calls for constant monitoring and steering of the noise load development by the implementation of different preference lists.

The current control strategy uses the average monthly weather conditions, obtained from data collected since 1971. Taking into account the current noise load realization, a preference list is determined. This heuristic and its decision-making process of a stylized problem is discussed in [2], and clearly takes into account the expected noise load, but does not take into account (i) a possible change in future weather conditions, i.e., the probability distribution of the weather development, and (ii) the possible adaptation of the preference list due to these weather developments. In short, the heuristic obtains a single optimal preference list for each subsequent month in the remaining decision period based on constant (average) weather conditions, and the current noise load realization. In this paper, we introduce an approach to obtain the current decision for the preference list to be used in the next month, that takes into account (i) the probabilistic nature of the weather conditions, and (ii) the resulting possibly different preference lists used at subsequent decision epochs.

We introduce a mathematical framework that each month generates an optimal preference list based on the realized noise load, flight schedules, and available runway combinations, taking into account uncertain weather conditions. The mathematical framework is that of Stochastic Dynamic Programming, which can be directly mapped to the decision making process at E^2MC.

3 Noise load optimization

Stochastic Dynamic Programming (SDP) provides a mathematical framework for optimization of decisions in situations where outcomes are uncertain, but partly under the control of the decision-maker. It is a discrete time stochastic control process characterized by a set of states. In each state there are several actions from which the decision-maker may choose. For a state and an action, a state transition function determines the transition probabilities to the next state. The process has the Markov property, which means that when the current state is known, transitions to new states are independent of all previously visited states.

Preference lists are selected each month based on the realized noise load at the enforcement points, the available runway combinations, and the weather conditions in the remaining part of the aviation year. In an SDP setting, the realized noise load at the beginning of month n (the decision epoch) is the *state* of the system at time n. A *decision* at time n is the selection of a preference list from all possible preference lists, i.e., from all available ordered sets of available runway combinations. For each decision, the evolution of the noise load in month n is determined by the weather conditions. As these are uncertain, this evolution is characterized by transition probabilities. The goal is to select each month, in each state, a preference list such that the probability of not exceeding the noise load limit at all enforcement points at the end of month 12 (the end of the aviation year) is maximized.

In a slightly more general setting we have the following mathematical characterization of our decision process, that also allows for investigation of additional decision epochs and an arbitrary number of enforcement points. To this end, let $\mathbb{R}_+ = [0, \infty)$, the non-negative real numbers, K the number of enforcement points, and N the number of decision epochs. Further, let X_n denote the random variable recording the noise load realization in period or stage n (between decision epoch n and $n+1$), and let

$i = (i_1, \ldots, i_K) \in \mathbb{R}_+^K$: the state with a noise load realization i_k in enforcement point k, $k = 1, \ldots, K$,

$S_n = \{i : i \in \mathbb{R}_+^K\}$: the set of noise load realizations at decision epoch n, $n = 1, \ldots, N$,

D_n : the set of preference lists available at decision epoch n, $n = 1, \ldots, N$,

$P_{n,d}(X_n) = P(X_n \leq x | d)$: the probability distribution of the noise load contribution in stage n when preference list d is selected at stage n,

$p_{n,d}(x)$: its density,

$f_n(i)$: the minimal expected probability of exceeding the noise load limit at the end of period N when the noise load realization at stage n is i.

The function $f_n(i)$ satisfies the following recursion

$$f_n(i) = \min_{d \in D_n} \left[\int f_{n+1}(x+i) p_{n,d}(x) \mathrm{d}x \right], \quad i \in S_n. \tag{1}$$

To see this, note that $f_{n+1}(x+i)$ is the minimal expected probability of exceeding the noise load limit at the end of period N when the noise load realization at stage $n+1$ is $x+i$ (that is when the optimal decision is selected starting at stage $n+1$), and that, for given $d \in D_n$, $p_{n,d}(x)$ is the density of the noise load contribution in period n, and therefore the integral $\int f_{n+1}(x+i)p_{n,d}(x)\mathrm{d}x$ is the expected probability of exceeding the noise load limit at the end of period N following decision d taken at stage n. As a consequence, the expected minimal probability of exceeding the noise load limit starting at stage n is obtained by selecting the optimal decision $d \in D_n$. The *optimal control strategy* $\pi = (\pi_1, \ldots, \pi_N)$ is a set of decision rules $\pi_n : S_n \to D_n$ that assigns an optimal decision $d \in D_n$ to each state $i \in S_n$ in stage n, $n = 1, \ldots, N$.

The recursion (1) is clearly a backward recursion: given $f_{n+1}(x)$ is known, the optimal decision that minimizes the integral can be determined. Thus, the recursion requires the starting values for $f_{N+1}(x)$:

$$f_{N+1}(i) = \begin{cases} 0 & \text{if } i \leq L_{max} \\ 1 & \text{if } i \nleq L_{max} \end{cases} \qquad i \in S_{N+1} \tag{2}$$

where the inequalities are componentwise, that is $i \leq L_{max}$ if and only if $i_k \leq L_{max,k}$ for all enforcement points k, and $i \nleq L_{max}$ when there is some k for which the noise load restriction is violated. The minimum expected probability of exceeding the noise load limit in a year is $f_1(0)$, and once $f_1(0)$ is determined, also the optimal control strategy is determined.

Determining the optimal strategy requires the transition probabilities $P_{n,d}(x)$. Here the formulation of our optimization problem involves the assumption that the contributions of the noise load in subsequent months are independent random variables. This is clearly an assumption, since the weather conditions today are perhaps the best ingredients for a forecast for the weather tomorrow. However, on a monthly scale these effects of dependence are marginal. Thus, we have identified all ingredients for determining the optimal control strategy $\pi = (\pi_1, \ldots, \pi_N)$.

Some numerical issues remain. For example, determining the empirical distribution $P_{n,d}(x)$ or its density $p_{n,d}(x)$ is far from obvious. Below, we will introduce a discretized approach. This discretized approach also induces a discretized version of the optimization problem (1), that is the topic of the next section.

4 Implementation

Multi-dimensional continuous-state dynamic programming problems are a huge challenge, in spite of the growth in computing power. The number of enforcement points and decision epochs prohibits an exact solution within a reasonable amount of computing time. Therefore, we propose a discrete approximation. This approximation involves discretization of the state space, that in turn also calls for a discretization of the transition probabilities.

4.1 Transition probabilities

The monthly noise load contribution consists of a large number of small contributions by different aircraft. These noise load contributions depend on the weather conditions, which are highly unpredictable. The distribution further depends on the preference list d on and the month (since supply of traffic and weather conditions differ between seasons) represented by stage n. This brings us clearly in a setting that allows us to invoke the Central Limit Theorem, implying that the monthly noise load contribution X_n has a multivariate normal distribution with K variates (enforcement points). When preference list d is selected, it has probability density function

$$p_{n,d}(x) = \frac{1}{(2\pi)^{K/2}|\Sigma_{n,d}|^{1/2}} \cdot e^{-\frac{1}{2}(x-\mu_{n,d})^T \Sigma_{n,d}^{-1}(x-\mu_{n,d})} \tag{3}$$

with $\mu_{n,d} \in \mathbb{R}^K$ the expected values of the K variates (the expected noise load contribution at the enforcement points), $\Sigma_{n,d} \in \mathbb{R}^{K \times K}$ their covariance matrix and $|\Sigma|$ its determinant.

The input parameters are estimated from data generated in DAISY [10]. DAISY is an airport environment toolkit developed by Frontier Information Technologies BV that produces values for noise load in enforcement points given a volume of traffic, a preference list, a period, and weather conditions. The effect of a preference list in different stages varies. Simulations for all combinations of N stages with D preference lists were performed using the three decades of recorded meteorological data. From these simulations we have obtained an estimate for the mean $\mu_{n,d}$ and correlation matrix $\Sigma_{n,d}$ for all combinations of stage n and preference list d.

4.2 Discretization

To facilitate numerical evaluation of the risks of exceeding, we have discretized the state spaces S_n, $n = 1, \ldots, N$, by forming a grid with distance ϵ among grid points in each dimension, i.e., we have divided the noise load in intervals of width ϵ. Selecting ϵ we have to balance between sufficient accuracy (small ϵ) and computational efficiency (large ϵ). To this end, we have selected a discretization step of 2%, that is, for enforcement point k the noise load interval $(0, L_{max,k})$ is divided in 50 intervals of equal width. The appeal of this discretization is that it reduces an infinite-instance problem to a finite-instance problem with a finite number of calculations that can be solved numerically and approximates the solution of the SDP.

We will take the grid points to be the center of the interval. A grid point $\hat{i} = (\hat{i}_1, \ldots, \hat{i}_K) \in \mathbb{N}_+^K$ corresponds to a noise load realization in the hypercube $(\hat{i}_k\epsilon - \epsilon/2, \hat{i}_k\epsilon + \epsilon/2)$, $k = 1, \ldots, K$. The state space $\hat{S}_n = \{\hat{i}\cdot\epsilon : \hat{i} \in \mathbb{N}_+^K\}$ is the set of noise load realizations at decision epoch n. All states with noise load realization exceeding the limit at some enforcement point may be lumped into a single state, since the probability of exceeding is 1 in such states irrespective of the number of enforcement points exceeding the limit, and the amount of overshoot.

Discrete transition probabilities are obtained by integrating the transition density $p_{n,d}(x)$ over the discretization increment:

$$\hat{P}_{n,d}(x) = \int\limits_{x-\frac{\epsilon}{2}}^{x+\frac{\epsilon}{2}} p_{n,d}(\xi)\mathrm{d}\xi. \tag{4}$$

Notice that this is a K-dimensional integral, that can numerically readily be evaluated via Monte-Carlo summation, see e.g. [3].

Let $\hat{f}_n(i)$ be the the minimal expected probability of exceeding the noise load limit at the end of period N when the noise load realization at stage n is i in the discretized setting. Clearly, $\hat{f}_n(i)$ satisfies the discretized equivalent to (1):

$$\hat{f}_n(i) = \min_{d\in D_n} \left[\sum_{\left\{x \middle| x+i\in\hat{S}_{n+1}\right\}} \hat{f}_{n+1}(x+i)\cdot\hat{P}_{n,d}(x) \right] \qquad i \in \hat{S}_n \tag{5}$$

The recursion requires starting values $\hat{f}_{N+1}(i)$ by analogy with those of the continuous state problem.

5 Feasibility study

This section presents a feasibility study of our optimization approach with actual traffic and weather data, and a comparison to the heuristic currently in use. In addition, we investigate the possible benefit of an increased number of decision epochs: from a monthly to a bi-weekly schedule. The examples used in this section are for illustration purposes only. Consequently, results presented in this section cannot be used for other purposes like commercialization and decision-making.

As the examples are illustrative, we will only present a series of experiments that closely resemble the actual behavior of the system. The weather data in these experiments corresponds to that estimated from the weather database since 1971. Parameters for the transition probabilities were estimated based on a year of uninterrupted operation (hence, no runway closure due to maintenance or other restrictions), with a traffic supply scenario equivalent of that of 2006 (436,731 flight movements) and current runway and route configurations.

Research leading to this paper has been intended as a feasibility study for an improved preference list selection process. As a consequence, an extensive and optimized numerical program has not been developed. We have implemented our algorithm in the numerical computing environment Matlab [11]. This does not allow for a complete evaluation of all enforcement points within reasonable computing time. Our program is implemented on a 1.7 GHz PC, resulting in running times of a couple of days for 4 enforcement points with a discretization interval of 2% (noise load relative to its limit). Given these limitations, our results indicate that our approach is indeed able to obtain preference lists with low probabilities of exceeding noise load limits.

5.1 Feasibility study

The theoretical framework outlined in Section 3 has been implemented to obtain optimal preference lists in a number of cases. The running time of our algorithm is exponential in the number of enforcement points. Therefore, we have restricted our analysis to sub-models containing 3 enforcement points. Data for these sub-models can readily be extracted from the parameters specified in Section 4. In particular, the correlation matrix merely consists of the rows and columns corresponding to the 3 selected enforcement points.

A selection of 3 enforcement points cannot provide an optimal control strategy for all enforcement points. For example, for a given set of 3 enforcement points, the optimal preference list may completely avoid these points, since noise load at non-modeled enforcement points is neither monitored nor enforced. To counter this effect, experiments are carried out for 8 different representative sets of enforcement points. Each set consists of enforcement points that turn out to be most sensitive for noise load excess. A second condition invoked in our study is that subsets consist of combinations of enforcement points that cover different directions relative to Schiphol. A third restriction that we have incorporated is reflected in the set of preference lists. This set is reduced to a set of preference lists of which the resulting operations are known to contribute to the noise load in the modeled enforcement points. This set consists of 8 preference lists shown in Table 1: list 1 to 4 are expected to contribute relatively more in northern points and 5 to 8 relatively more in southern points.

Without the restrictions mentioned above our algorithm would produce a strategy that results in excess in the non-modeled enforcement points. With the representative subsets the surroundings of Schiphol are taken into account.

For the set of preference lists of Table 1, we have tested our algorithm on the sets of enforcement points indicated in Figure 2. Table 2 below provides for each set of enforcement points the probability of exceeding the noise load restriction, and the preference list in month 1 (November). Note that a complete description of the optimal strategy π is rather involved, since it requires for each realized noise load and each month a specification of the preference list.

As a sanity check, for each optimal strategy (that is the strategy corresponding to the set of enforcement points), we have also considered the noise load contribution at the remaining 32 enforcement points when implementing the optimal strategy obtained for 3 enforcement points. This has shown a slight excess in some non-modeled enforcement points, which leads us to believe that our subsets were well-chosen in accordance with the actual behavior of our system. Moreover, our algorithm seems to capture the behavior of the noise load problem and yields a good policy.

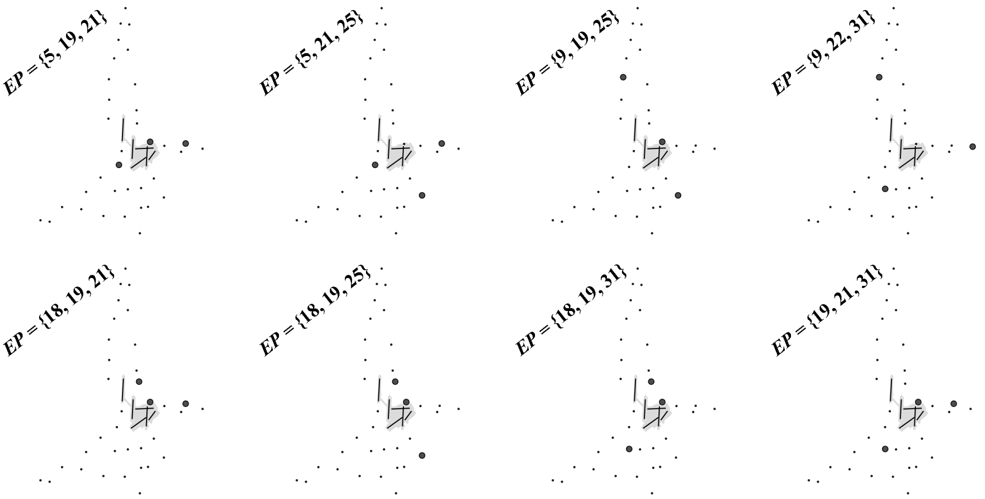

Figure 2: Modeled enforcement point

5.2 Comparison with current heuristic

This section provides a comparison between our algorithm and the current heuristic in a typical situation occurring towards the end of an aviation year. Typically, a small number of enforcement points is at risk of exceeding the noise load limits, while the other points are far from reaching that limit. We will first provide some insight in the difference between the strategies, then compare our strategy with the current heuristic, and finally investigate some possible trade-offs in sub-optimal preference list selection to allow for a selection of preference lists that better matches the requests from e.g. air lines.

The current heuristic provides an optimal selection from the set of preference lists, but takes into account the expected noise load only. Given the current noise load realization, the heuristic produces a decision path π', say, for the remainder of the year, and cannot incorporate weather conditions except for its mean noise load contribution. Let $f_n^{\pi'}(i)$ denote the minimal expected probability of exceeding the noise load limit at the end of period N when the noise load realization at stage n is i under strategy π'. Let e_n be the expected noise load level at decision epoch n, $n = 1, \ldots, N$. Clearly, $e_n \in S_n$. The optimal decision of the current heuristic can also be obtained from our algorithm when we reduce the state spaces S_n to only contain the mean noise load realization. As a consequence, π' is obtained as the optimal solution of a restricted problem: $\pi'_n = \pi_n(e_n) \quad \forall n$. Our algorithm takes

Noise load management with $N = 12$				
enforcement points			Probability of exceeding	Preference list month 1
5	19	21	0.1010	1
5	21	25	0.0368	5
9	19	25	0.0151	3
9	22	31	0.0847	5
18	19	21	0.1626	1
18	19	25	0.0267	3
18	19	31	0.0234	3
19	21	31	0.1519	1

Table 2: Results for optimal strategy

into account all possible future noise load realizations. The resulting strategy π with probability of exceeding $f_n^{\pi}(i)$ outperforms the heuristic, that is $f_n^{\pi}(i) \leq f_n^{\pi'}(i)$. Notice that it may be that $\pi = \pi'$.

Now consider a scenario with $N = 4$, and a set of noise load limits in 3 modeled enforcement points. This corresponds to a situation 4 months prior to the end of the decision year with a given realized noise load and the 'certainty' that all other enforcement points will not exceed. The set of modeled enforcement points for this scenario consists points 9, 22 and 31, which is one of the sets considered in the feasibility study. These points have all consumed 68% of their noise load limits at the time of month 8, leaving 32% to be allocated in the remaining 4 months. For the preference lists from Table 1, we tested the algorithm and compared it to the current heuristic.

Table 3 below gives the optimal control strategy π (note that $\pi_1 = 5$), and its probability of exceeding. The control strategy for the current heuristic is constructed as described above: $\pi' = (5, 1, 5, 1)$. Some variants of π' are considered, since $\pi_n(e_n) \neq \pi_n(e_n \pm \epsilon)$ for some n, indicating that different decisions might occur within the discretization interval of e_n. Variants π'' consist of combinations of two times preference list 1 and two times list 5, as was indicated by π'. Resulting probabilities of exceeding are also shown in Table 3. It can be observed that all control strategies π' and π'' produce considerably higher values for probability of exceeding. The decision variable for the current heuristic is therefore overestimated.

In addition to obtaining an optimal control strategy and corresponding

Noise load management with $N = 4$					
control strategy					Probability of exceeding
π	5	π_2	π_3	π_4	0.1113
π'	5	1	5	1	0.2705
π''	5	5	1	1	0.2747
π''	5	1	1	5	0.2412
π''	1	1	5	5	0.2433
π''	1	5	1	5	0.2428
π''	1	5	5	1	0.2773

Table 3: Results for optimal strategy and current heuristic

probability of exceeding the noise load restriction, our algorithm also allows for fast evaluation of proposed changes to the optimal strategy. Deliberate sub-optimal decisions can be made to satisfy interests and demands of other aviation parties, that yield an increase in the probability of exceeding. Table 4 below provides the probability of exceeding when preference list 2 is forced for a number of months. As can be observed from the table, using preference list 2 for one month results in a doubling of the probability of exceeding the noise load limits. Our algorithm allows for a fast trade-off of the results of deviating from the optimal strategy.

Noise load management with $N = 4$					
control strategy					Probability of exceeding
π	π_1	π_2	π_3	π_4	0.1113
π'	2	π_2	π_3	π_4	0.2037
π'	2	2	π_3	π_4	0.4518
π'	2	2	2	π_4	0.7340
π'	2	2	2	2	0.9744
π'	π_1	2	2	2	0.8391
π'	π_1	π_2	2	2	0.4856
π'	π_1	π_2	π_3	2	0.2567

Table 4: Effect of forcing a decision for a number of months

5.3 Increasing the number of decision epochs

Currently, preference lists are adjusted bimonthly when judged necessary. It may be beneficial to increase the frequency of preference list updates when this considerably affects the probability of exceeding. A trade-off has to be made between the overhead of preference list modification, and the reduction in the probability of exceeding. This trade-off is beyond the scope of this paper. Here, we report on a study into the benefit of doubling the number of decision epochs. The set of enforcement points is that of Section 5.1. As we see from Table 5, the probability of exceeding is significantly lowered by doubling the number of decision epochs.

Noise load management with $N = 24$				
enforcement points			Probability of exceeding	Improvement w.r.t. $N = 12$
5	19	21	0.0397	61 %
5	21	25	0.0249	32 %
9	19	25	0.0042	72 %
9	22	31	0.0293	65 %
18	19	21	0.0141	13 %
18	19	25	0.0221	17 %
18	19	31	0.0198	15 %
19	21	31	0.1298	15 %

Table 5: Effect of doubling the number of decision epochs

6 Concluding remarks

The optimal selection of preference lists is of utmost importance for efficient allocation of Schiphol's capacity considering noise load restrictions. The current heuristic takes into account the expected noise load only. Given the current noise load realization, the heuristic cannot incorporate weather conditions except for its mean noise load contribution. In addition, it does not take into account the possible update of the control strategy in response to realized noise load. This results in an inaccurate control strategy, hence leading to a sub-optimal distribution of noise load over the surroundings of Schiphol. This calls for an improved monthly preference list selection process, that takes into account the probabilistic nature of the weather and resulting possibly different preference lists used at subsequent decision epochs.

In support of this need, this paper has introduced a mathematical framework that each month generates an optimal preference list based on the realized noise load, flight schedules, and available runway combinations, taking into account uncertain weather conditions. The mathematical framework is that of Stochastic Dynamic Programming, which is directly mapped to the decision process.

Results from our feasibility study indicate that our algorithm yields an adequate preference list selection strategy that outperforms the current strategy. In addition, the proposed noise load management scheme allows for fast discrimination among different control strategies. As such, the effect of decisions deviating from the optimal strategy has been investigated.

Our numerical analysis of this feasibility study revealed that optimization is rather slow due to the size of the problem, which is mainly due to the large number of enforcement points. This is partly due to the implementation in Matlab on a regular PC. In addition, concepts from the theory of huge Markov chains may be invoked to improve the efficiency. Our study has shown that our Stochastic Dynamic Programming based algorithm allows for optimal preference list selection taking into account uncertain weather conditions.

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Appendix

Preference list 1										
Peak:	Inbound			Outbound			Off		Night	
nr.	dep	arr		dep		arr	dep	arr	dep	arr
1	36L	06	36R	36L	36C	06	36L	06	36L	06
2	24	18R	18C	24	18L	18R	24	18R	24	18R
3	18L	18R	18C	18L	18C	18R	18C	18R	18C	18R
4	36L	36R	36C	36L	36C	36R	36L	36C	36L	36C
5	24	27	18R	36L	09	06	09	18R	06	06
6	24	18R	22	24	36L	27	09	06	24	18C
7	18L	18R	22	24	18L	27	24	27	24	27
8	09	06	09	24	27	27	36L	27	36L	27
9	36L	06		36L		06	24	24	24	24
10	24	18R		24		18R	27	27	09	09
11	18L	18R		18L		18R	09	09	06	06
12	36L	36R		36L		36R	24	22	09	18R
13	09	18R		09		18R	06	06		
14	09	06		09		06				
15	24	27		24		27				

Preference list 2

Peak:	Inbound			Outbound			Off		Night	
nr.	dep	arr		dep		arr	dep	arr	dep	arr
1	36L	06	36R	36L	09	06	36L	06	36L	06
2	24	18R	18C	36L	36C	06	24	18R	24	18R
3	18L	18R	18C	24	18L	18R	18C	18R	18C	18R
4	36L	36R	36C	18L	18C	18R	36L	36C	36L	36C
5	24	27	18R	36L	36C	36R	09	18R	06	06
6	24	18R	22	24	36L	27	09	06	24	18C
7	18L	18R	22	24	18L	27	24	27	24	27
8	09	06	09	24	27	27	36L	27	36L	27
9	36L	06		36L		06	24	24	24	24
10	24	18R		24		18R	27	27	09	09
11	18L	18R		18L		18R	09	09	06	06
12	36L	36R		36L		36R	24	22	09	18R
13	09	18R		09		18R	06	06		
14	09	06		09		06				
15	24	27		24		27				