Designing Allocation Mechanisms for Carrier Alliances

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Abstract

When cargo carriers form an alliance, a key issue to resolve is how to provide incentive for carriers to make decisions that are optimal for the alliance as a whole. We propose a mechanism that utilizes capacity exchange prices to influence carrier behavior, and analyze two approaches to modeling the impact of these prices on the behavior of an individual carrier. We find that the model used can significantly impact alliance recommendations; one proposed model always finds capacity exchange prices that yield an allocation that is budget-balanced and stable, yet cannot guarantee centralized feasibility. The second model uses more realistic control parameters and does in fact guarantee centralized feasibility. Finally, experimental results for two and three-carrier alliances are analyzed; it is determined that the benefit associated with collaborating increases with network size and fleet capacity, and depending on the characteristics of demand, fleet capacity is a more important factor.

1 Introduction

Consider a group of independent cargo carriers (for example, in the air cargo, sea cargo, or trucking industries) who each wish to improve their own profitability. They may choose to integrate some portion of their transportation networks in order to make better use of their capacity by delivering more-valuable cargo loads. A group of carriers working together in such a manner is referred to as an *alliance*. It is reasonable to assume that carriers considering forming an alliance are interested in designing that alliance to function as well as possible, from the standpoint of both profitability and sustainability over time. The challenge in achieving these goals lies in the tradeoff between decisions that are good for the alliance versus decisions that are good for an individual carrier: decisions that are good for the alliance are not always good for an individual carrier within the alliance, and vice versa. In order for an alliance to operate in an optimal manner, this discrepancy must be resolved. In this paper we propose a method to manage the interactions of carriers such that individual carriers are encouraged to make decisions that are optimal for the alliance as a whole. This is accomplished through careful allocation of alliance resources and profits.

1.1 Air Cargo Alliances

Our motivating application is the air cargo industry. Air cargo is assumed to be any freight, excluding mail and passenger baggage, transported using aircraft. More specifically, we focus on combination carriers, which are those carriers transporting cargo using passenger aircraft. As carriers take steps to improve the profitability of their cargo business, they are increasingly considering collaborations for cargo that are independent of those already established for the passenger industry. The first cargo alliance, SkyTeam Cargo, formed in 2000 and was comprised of the cargo components of Aeromexico, Air France, Delta, and Korean Air [21]. These four airlines were already part of the SkyTeam passenger alliance, but SkyTeam Cargo was formed as an independent strategic cargo alliance. Similarly, the WOW Alliance formed in 2002 with the cargo businesses of Lufthansa, SAS, and Singapore Airlines. Again, these carriers were already partners in the passenger industry, under the Star Alliance. However, a carrier outside the Star Alliance, Japan Airlines, was added later in 2002 [25]. Cargo alliances among carriers that are not already partners in the passenger business are likely to become more common, since carriers compatible for passenger alliances may not be compatible for a cargo alliance. This is due to differences in flow patterns: passengers typically complete a round trip, resulting in balanced flow, while cargo flow follows unbalanced trade patterns [29].

Similar to code sharing in the passenger industry, key decisions involved in the cargo setting include how to share space and revenue among members. An additional consideration in the cargo setting, however, is that of route selection. In contrast to passengers, cargo is relatively insensitive to routing decisions; therefore the decision of how to route cargo through the alliance network becomes a relevant factor in considering collaborations among air cargo carriers. Determining the overall most profitable set of cargo to deliver, and how this cargo should be routed through the combined network, requires a centralized perspective. We discuss a centralized model to solve the acceptance and routing problem for the alliance as a whole in Section 3. A centralized solution is necessary in order for the alliance to achieve the maximum benefit from collaborating, but full centralization is generally not an option given the technical and legal challenges associated with integrating the information systems of autonomous carriers. Thus the maximum benefit will only be attained if the participating carriers can be encouraged to make their own acceptance and routing decisions in accordance with the centralized optimal decision. Finding suitable incentives that will influence the behavior of the carriers in an appropriate manner is dependent upon understanding and modeling the decision process of the carrier within the collaborative system. In Section 4 we discuss two approaches for modeling the behavior of an individual carrier.

Because an alliance is comprised of a collection of autonomous carriers, we seek an allocation method that is implementable without an assumed centralized distributor. We describe a mechanism that transfers revenue through a collection of prices paid and received as capacity is exchanged; Section 4 includes a description of a methodology used for determining these prices, henceforth referred to as *capacity exchange prices*, based on the underlying behavioral models discussed. The differences in allocations obtained from mechanisms based on the behavioral models are characterized in Section 4. Section 5 includes experimental results for alliances comprised of carriers of various network sizes and fleet capacities. We present conclusions and insights in Section 6.

1.2 Motivating Example

The simple case demonstrated in Figure 1 illustrates some questions that must be addressed when an alliance among carriers is considered. In this simplified air cargo system, a time-expanded network with two cities and four time periods is depicted. There are four loads, each of unit size, that need to be accepted or rejected for transport; all loads originate in city 1 and are shown at their earliest available departure time, and the destination of every load is city 2. The *ground edges* are fictitious edges that represent the ability of a load to wait in a location over time. Operating independently, carrier A can earn \$4, while carriers B and C must reject their loads and can earn no revenue. If the three carriers collaborate, then the two higher value loads associated with carriers B and C can be accepted, for a total revenue of \$9. Clearly there is benefit to be gained by collaborating, if carrier A can be convinced to make capacity available to carriers B and C.

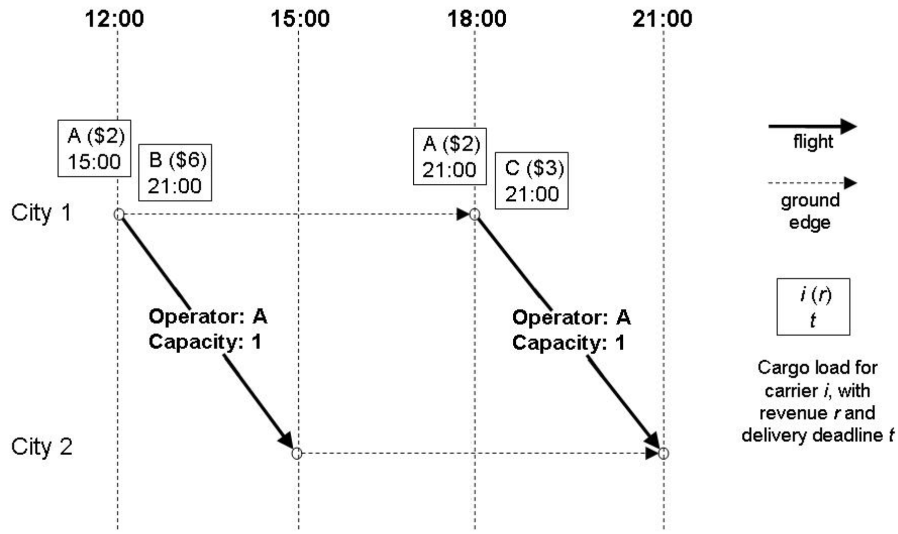

Figure 1: Air Cargo Example with 3 Carriers and 4 Loads

In this paper an allocation mechanism will be established that addresses the following challenges:

- How can resources be utilized such that the overall system profit is maximized?
- Given that this utilization will not necessarily be optimal for individual carriers in the collaboration, what incentives are necessary to encourage carriers to not only participate, but make decisions that lead to system optimal performance?
- How can these incentives be incorporated into a mechanism that manages interactions among carriers, rather than assuming distribution of incentives by a centralized decision-maker after the carriers act as dictated by this decision-maker?

1.3 Related Literature on Air Cargo Alliances

There is very little available in the literature relating to air cargo alliances, most likely since alliances among air cargo carriers are a very recent development. Most literature concerning air cargo is related to dedicated cargo carriers, cargo operations, or the relationship between the cargo and passenger industries. For example, the network design problem for dedicated cargo carriers is addressed by [11] and [14], and short-term capacity planning is studied in [9]. Analysis of airline alliances in the passenger industry is more prevalent, but no existing literature uses a similar methodology or addresses the same questions as in this work. [20] investigates the impact of international alliances on the passenger market by comparing alliances comprised of airlines with complementary and parallel networks; it is predicted that an alliance that joins complementary networks will be more profitable. In response to a concern that alliances would lead to a situation where major carriers would have a monopoly, [17] finds instead that alliances have merely allowed carriers to preserve, not increase, their narrow profit margins through an increase in load factors and productivity. [6] in fact finds that consumers benefit from the formation of passenger alliances; in the two domestic alliances that were studied fares decreased on the markets impacted by the alliance, in part due to increased competition from rivals competing with the alliance. [1] analyzes potential international alliances among carriers, applying non-cooperative game theory to determine the profitability of an alliance under a given level of competition. The primary issue addressed is the selection of international hubs to maximize the profit of merging airlines.

There is also limited research available on the impact that an alliance in one industry (air cargo or passenger) can have on the other. [16] studied a passenger alliance between KLM and Northwest and found that, ultimately, the effects on cargo service were positive. From the other perspective, [28] investigates the effect of an air cargo alliance on the passenger market, finding that cargo service integration can increase outputs in both the cargo and passenger markets.

A widely studied topic in the passenger airline industry that is only recently being applied in the alliance setting is that of revenue management. Literature in this field seeks to maximize revenue through management of seat capacity. [15] provides a review of revenue management literature, but is focused primarily on revenue management implemented by a single carrier. [7] describes the technical challenges associated with alliance revenue management; in addition to addressing challenges, [24] discusses how coordination of seat pricing and capacity planning are currently executed in the alliance setting. [26] provides a more formal analysis of alliance revenue management mechanisms; a free sale scheme and three types of dynamic trading schemes are discussed. The mechanisms are analyzed to determine their effect on the equilibrium behavior of alliance members and the potential for the mechanism to maximize alliance revenue. Revenue management applied to the air cargo industry is even more limited; differences between the cargo revenue management problem and the passenger yield management problem are discussed in [13], as well as complexities in developing additional models to facilitate cargo revenue management.

Outside the airline industry, carrier collaboration has also been studied in the ocean liner shipping industry. [2] addresses issues related to the formation of alliances in the sea cargo industry; in addition to the distribution of alliance revenue, design of the alliance network is of critical importance. [22] demonstrates that alliances among sea cargo carriers lead to increased service frequency and ship size, as well as increased similarity of service routes among carriers. [23] provides a conceptual framework for the application of game theory to alliances in the liner shipping industry. The ability to explain the instability of strategic alliances using cooperative game theory is discussed, as well as the practical limitations of applying game theory to the industry. For an overview of issues related to carrier alliances, including alliances in both the ocean liner and air cargo industries, we refer the reader to [4].

To the best of our knowledge, our approach to designing a profit allocation mechanism for carrier alliances through capacity exchange prices–an approach focused on how differences in modeling the behavior of an individual carrier within the alliance impact the allocations obtained–is unique.

2 A Centralized Model for Accept/Reject and Routing Decisions

An important motivation for the formation of an alliance among carriers is the recognition by those carriers that the alliance will yield benefit beyond what each carrier can accomplish individually. Given that increasing the revenue earned by the alliance increases the benefit that can be distributed among the participating members, it is reasonable to attempt to determine the set of cargo loads to deliver, and the optimal routing of these loads, that will maximize the alliance profit. This information is obtained by solving a network flow problem from the centralized, or system, perspective; that is, the network and demand from each participating carrier are integrated to create one large pseudo-carrier. The network of a carrier is determined according to the amount cargo capacity available on each flight leg operated by that carrier; the demand associated with each carrier is presumed to be a set of loads that the carrier must accept or reject for delivery.

As the focus of this work is on developing a methodology to manage the interactions among carriers such that alliance-optimal behavior is achieved, we make several simplifying assumptions. First, it is assumed that both cargo loads and flight capacity have single dimension units and are deterministic. Second, it is assumed that origins and destinations for loads correspond to airports, which implies that door-to-door pick-up and delivery services are not considered. responsibility of carriers is limited to transportation by air only. Third, we assume that the flight schedule for an individual carrier is motivated by the passenger industry and is therefore fixed, hence we do not consider costs incurred by operating the network. Finally, we permit load splitting in order to obtain a standard multi-commodity flow linear program that can be easily solved.

The length and units of the time horizon considered are intentionally not specified; these can be determined according to the needs and preferences of the alliance. For example, in order to determine the long-term compatibility of a group of carriers or other strategic decisions, it may be appropriate to consider a longer time horizon with larger units of time, as exact flight schedules and fleet assignments are only known for the immediate future. On the other hand, a shorter time horizon with more exact flight information (and hence shorter units of time) is required to effectively determine capacity exchange prices, routing, and other operational decisions; these types of decisions should therefore be made at appropriate intervals on a rolling time horizon. Note that a freight forwarder can be incorporated into this modeling framework by introducing a carrier with a set of associated loads, but no network capacity.

Let N denote the set of carriers, and E^i the set of legs operated by each carrier $i \in N$. The set A contains all airports covered by the legs in E. Given a planning horizon of T time periods, let V denote the set of nodes (a, t) for each $a \in A$ and $t = 1..T$. Each leg $e \in E$ has capacity k_e. Each carrier has a load set L^i in which an individual load (o, d, i) is characterized by an origin o and destination d. The size and per unit revenue of load (o, d, i) is $d^{(o,d,i)}$ and $r^{(o,d,i)}$, respectively.

The centralized goal is to find a flow of loads f such that the system revenue is maximized, which is accomplished by solving the following multi-commodity flow problem:

$$(C): \quad \max \sum_{(o,d,i)\in L} r^{(o,d,i)} f^{(o,d,i)}_{(d,o,i)} \tag{1}$$

$$s.t. \quad \sum_{(u,v)\in E} f^{(o,d,i)}_{(u,v)} - \sum_{(v,w)\in E} f^{(o,d,i)}_{(v,w)} \leq 0 \qquad \forall v \in V, \forall (o,d,i) \in L \tag{2}$$

$$\sum_{(o,d,i)\in L} f^{(o,d,i)}_{e} \leq k_e \qquad \forall e \in E \tag{3}$$

$$f^{(o,d,i)}_{(d,o,i)} \leq d^{(o,d,i)} \quad \forall (o,d,i) \in L \tag{4}$$

$$f^{(o,d,i)}_{e} \geq 0.$$

(1) reflects the centralized goal of maximizing the amount of revenue earned from delivering loads; the flow variable $f^{(o,d,i)}_{(d,o,i)}$ represents flow on a fictitious edge from the destination d to the origin o of load i, which is introduced to account for the amount of load (o,d,i) that is delivered. (2) are flow balance constraints, enforcing that every unit accepted for shipment must be appropriately routed through the network. (3) are capacity constraints for each flight, while (4) ensure that the amount of a load delivered does not exceed its size. Let f^* be the optimal solution to C; from f^* we obtain the optimal accept-reject decision for each load, as well as the optimal routing for the set of accepted loads.

3 Modeling the Individual Perspective

In order for the alliance to earn as much profit as possible, carriers must make their accept-reject and routing decisions in accordance with f^*. We seek to provide a structure to encourage the exchange of capacity among carriers, as carriers will clearly need incentive to allow their capacity to be used by other carriers. A natural way to provide this incentive is by establishing a system in which carriers receive payments in exchange for capacity used by other carriers. Recall that we refer to these payments as *capacity exchange prices*. If c_e is the capacity exchange price on leg e, the net profit from capacity exchanges for carrier i is then given by:

$$s^i = \sum_{e\in E^i} c_e \Big(\sum_{(o,d,k)\notin L^i} f^{(o,d,k)}_{e} \Big) - \sum_{e\notin E^i} c_e \Big(\sum_{(o,d,i)\in L^i} f^{(o,d,i)}_{e} \Big). \tag{5}$$

s^i can be thought of as a side payment provided to carrier i to compensate i for the value of capacity being used by other carriers. If s^i is negative, then carrier i can be thought of as a net consumer of capacity value.

Let q^i be the revenue carrier i earns by delivering loads in accordance with f^*:

$$q^i = \sum_{(o,d,i)\in L^i} r^{(o,d,i)} f^{*(o,d,i)}_{(d,o,i)}. \tag{6}$$

Then the net profit x^i earned by carrier i is given by $x^i = q^i + s^i$. We say that x^i is carrier i's *allocation*.

Ensuring that x^i is sufficient to encourage carrier i to participate, and that the capacity exchange prices are such that carrier i will abide by the centralized solution, requires an understanding of how capacity exchange prices impact the decisions of individual carriers. In this section we discuss two models for the behavior of an individual carrier. The goal in establishing these models is not to analyze the decision process of a carrier, but to gain insight into how to select capacity exchange prices that encourage carriers to make centrally optimal accept-reject and routing decisions. A critical consideration is the fact that a carrier does not operate in isolation; a carrier must consider the use of capacity by other carriers when making routing decisions. The two models presented differ in how this consideration is incorporated.

3.1 Strict Control Model

The first model presented is a model utilized by Agarwal and Ergun in their work in the liner shipping industry [3]. The model is applied to the air cargo industry without modification. In this model, the flow variables for all the loads in the system, including loads associated with other carriers, are included in the model for carrier i. Thus the use of capacity by other carriers is acknowledged explicitly through their associated flow variables. The second term of the objective function (7) reflects the capacity exchange prices received by carrier i as other carriers use capacity operated by carrier i; therefore when capacity exchange prices are high enough, carrier i is encouraged to leave capacity open for use by other carriers.

As the model for each carrier includes the entire set of flow variables present in the Centralized model C, the flow balance, capacity, demand, and non-negativity constraints for each carrier's model are exactly as in C. The objective function for carrier i is as follows:

$$(Strict^i): \quad \max \sum_{(o,d,i)\in L^i} r^{(o,d,i)} f^{(o,d,i)}_{(d,o,i)} + \sum_{e\in E^i} (c_e \sum_{(o,d,i)\notin L^i} f_e^{(o,d,i)}) - \sum_{e\notin E^i} (c_e \sum_{(o,d,i)\in L^i} f_e^{(o,d,i)}). \tag{7}$$

We refer to this model as the *Strict Control* model ($Strict^i$) because it implies mathematically that a single carrier has full control over the decisions of other carriers. While this is not a realistic interpretation of the behavior of an individual carrier, this model leads to allocations with some desirable properties, as we will describe in Section 4.

Given a model to represent the behavior of an individual within the alliance, we seek capacity exchange prices c_e such that the optimal solution to the individual model for carrier i will correspond to the centralized optimal solution f^*. This can be accomplished using inverse optimization. In traditional optimization, optimal values for variables are identified based on a given set of model parameters, whereas in inverse optimization one seeks a set of model parameters that will make a particular feasible solution optimal [5]. As a solution f^* must be optimal when it satisfies primal feasibility, dual feasibility, and complementary slackness conditions, we formulate the inverse problem for a carrier using the dual of his individual problem, making modifications to the constraints to ensure that a feasible dual solution will satisfy complementary slackness conditions with f^*. The dual of the Strict Control model for carrier i is as follows:

$$(D - Strict^i): \qquad \min \sum_{(u,v)\in E} k_e \alpha^i_{(u,v)} + \sum_{(o,d,i)\in L} d^{(o,d,i)} \beta^{i,(o,d,i)}$$

$$
\begin{aligned}
s.t. \qquad \pi_v^{i,(o,d,i)} - \pi_u^{i,(o,d,i)} + \alpha^i_{(u,v)} &\geq 0 \quad \forall (o,d,i) \in L^i, (u,v) \in E^i & (8)\\
\pi_v^{i,(o,d,i)} - \pi_u^{i,(o,d,i)} + \alpha^i_{(u,v)} &\geq -c_{(u,v)} \quad \forall (o,d,i) \in L^i, (u,v) \notin E^i & (9)\\
\pi_v^{i,(o,d,j)} - \pi_u^{i,(o,d,j)} + \alpha^i_{(u,v)} &\geq c_{(u,v)} \quad \forall (o,d,j) \notin L^i, (u,v) \in E^i & (10)\\
\pi_v^{i,(o,d,j)} - \pi_u^{i,(o,d,j)} + \alpha^i_{(u,v)} &\geq 0 \quad \forall (o,d,j) \notin L^i, (u,v) \notin E^i & (11)\\
\pi_o^{i,(o,d,i)} - \pi_d^{i,(o,d,i)} + \beta^{i,(o,d,i)} &\geq r^{(o,d,i)} \quad \forall (o,d,i) \in L^i & (12)\\
\pi_o^{i,(o,d,j)} - \pi_d^{i,(o,d,j)} + \beta^{i,(o,d,j)} &\geq 0 \quad \forall (o,d,j) \notin L^i & (13)\\
\pi_v^{i,(o,d,i)} &\geq 0 \quad \forall v \in V, \forall (o,d,i) \in L & (14)\\
\alpha^i_{(u,v)} &\geq 0 \quad \forall (u,v) \in E & (15)\\
\beta^{i,(o,d,i)} &\geq 0 \quad \forall (o,d,i) \in L & (16)
\end{aligned}
$$

where π^i, α^i, and β^i are the dual variables associated with the flow balance constraints, capacity constraints, and demand constraints, respectively, for carrier i. Constraints (8)-(11) correspond to each flow variable $f^{(o,d,i)}_{(u,v)}$. When carrier i operates leg (u,v), he is not required to pay to use capacity for transporting load (o,d,i) on (u,v); hence the right-hand sides of (8) are 0. When carrier i does not operate leg (u,v), he must pay the capacity exchange price $c_{(u,v)}$ for each unit of load (o,d,i) transported on (u,v); therefore the right-hand sides of (9) are $-c_{(u,v)}$. Similarly, when carrier i operates leg (u,v) he receives capacity exchange price $c_{(u,v)}$ for each unit of load (o,d,j) transported on (u,v), since carrier j must pay for the use of carrier i's capacity. As a result, the right-hand sides of (10) are $c_{(u,v)}$. When carrier i does not operate leg (u,v), he neither receives nor pays for the transportation of load (o,d,j); correspondingly, the right-hand sides of (11) are 0. Constraints (12)-(13) correspond to the variables $f^{(o,d,i)}_{(d,o,i)}$. For each unit of load (o,d,i) delivered, carrier i earns $r^{(o,d,i)}$ in revenue. For this reason, the right-hand sides of (12) are $r^{(o,d,i)}$. The right-hand sides for (13) reflect that carrier i receives no direct revenue from delivery of load (o,d,j).

The inverse problem for carrier i based on the Strict Control model, $InvStrict^i$, is formed by modifying the constraints of $D-Strict^i$ in order to ensure that complementary slackness conditions will be satisfied. For each variable that is positive in the centralized optimal solution f^*, the corresponding dual constraint must hold with equality. In addition, the following constraints are included in $InvStrict^i$:

$$\alpha^i_{(u,v)} = 0 \quad \forall (u,v) \in E : \sum_{(o,d,i)\in L} f^{*(o,d,i)}_{(u,v)} < k_{(u,v)} \tag{17}$$

$$\beta^{i,(o,d,i)} = 0 \quad \forall (o,d,i) \in L : f^{*(o,d,i)}_{(d,o,i)} < d^{(o,d,i)}. \tag{18}$$

(17) invokes the complementary slackness condition for (3) in C; when a leg is not utilized at full capacity, the dual variable corresponding to that leg must equal 0. Similarly, (18) reflects the

complementary slackness condition for constraint (4) in the Centralized model; when a load is not fully delivered, the dual variable corresponding to that load must equal 0.

As our parameters of interest, the capacity exchange prices c_e, appear only in the constraints of the inverse problem ($InvStrict^i$), it follows that our interest in the inverse problem is in finding a set of prices c_e and dual variables π, α, and β that will make the set of constraints in the inverse problem feasible. Any vector c_e that, together with (π, α, β), satisfies the constraints of $InvStrict^i$ will make f^* optimal for $Strict^i$. Thus, to ensure f^* is optimal for every carrier, we must find one common vector c_e and (π, α, β) that satisfies $InvStrict^i$ for every carrier i. Let $InvStrict$ be the constraint set created by combining the constraints of $InvStrict^i$ over all carriers i.

Theorem 1. *A feasible solution (π, α, β, and c_e) to $InvStrict$ is guaranteed to exist.*

Proof. See [3]. □

Intuitively, it makes sense to restrict the capacity exchange prices $c_{(u,v)}$ to non-negative values. The proof of Theorem 1 in fact assumes that capacity exchange prices are non-negative; we therefore can introduce non-negativity constraints on $c_{(u,v)}$ without compromising feasibility.

3.2 Limited Control Model

The Strict Control model recognizes capacity used by other carriers by including flow variables for all carriers and loads in the model for carrier i. In the Limited Control model for carrier i, we include only the flow variables for loads associated with carrier i. The use of capacity by other carriers is acknowledged in this alternative model by limiting carrier i's use of capacity on each flight. Pre-determining capacity allotments is a realistic approach given current industry practice; a carrier typically dedicates space on each flight to specific partnering carriers and freight forwarders. Intuitively, we can partition the capacity available on a flight according to the centralized solution f^*; if carrier i uses k_e^i units of capacity on leg e in f^*, then the individual model for carrier i will restrict carrier i to k_e^i units of capacity on leg e. More specifically, we use the following rules to determine the amount of capacity allotted to each carrier:

- For each edge e utilized at full capacity in f^* ($\sum_{(o,d,i)\in L} f_e^{*(o,d,i)} = k_e$), allot $\sum_{(o,d,i)\in L^i} f_e^{*(o,d,i)}$ to each carrier i.
- For an edge e that is not utilized at full capacity ($\sum_{(o,d,i)\in L} f_e^{*(o,d,i)} < k_e$), allot $\sum_{(o,d,i)\in L^i} f_e^{*(o,d,i)}$ to each carrier i such that $e \notin E^i$. Allot $k_e - \sum_{(o,d,i)\notin L^k} f_e^{*(o,d,i)}$ to the operating carrier k.
- Ground edges are not subject to capacity allotments, as they are assumed to have infinite capacity.

Making a priori capacity allocations in this manner is the premise of the *Limited Control Model.* We are assured that the aggregate solution to all individual carrier problems will be feasible, since the allocation for carrier i is devised so as to not interfere with the use of capacity by other carriers. Given an allotment of capacity k_e^i on every leg $e \in E$, the Limited Control model for carrier i is as follows:

$$LC^i : \max \sum_{(o,d,i) \in L^i} r^{(o,d,i)} f^{(o,d,i)}_{(d,o,i)} - \sum_{e \notin E^i} c_e \Big(\sum_{(o,d,i) \in L^i} f_e^{(o,d,i)} \Big) \tag{19}$$

subject to:

$$\sum_{(u,v) \in E} f^{(o,d,i)}_{(u,v)} - \sum_{(v,w) \in E} f^{(o,d,i)}_{(v,w)} \le 0 \qquad \forall v \in V, \forall (o,d,i) \in L^i \tag{20}$$

$$\sum_{(o,d,i) \in L^i} f_e^{(o,d,i)} \le k_e^i \qquad \forall e \in E \tag{21}$$

$$f^{(o,d,i)}_{(d,o,i)} \le d^{(o,d,i)} \quad \forall (o,d,i) \in L^i \tag{22}$$

$$f_e^{(o,d,i)} \ge 0. \tag{23}$$

As with the Centralized and Strict Control models, the Limited Control model is a multi-commodity flow LP. The value of (19) is equal to the total revenue earned from delivered loads minus the sum of capacity exchange prices paid. Note that this value is a lower bound on x^i, the actual profit allocated to carrier i, because it excludes exchange prices that will be paid to carrier i.

As in the Strict Control model, we find capacity exchange prices under the Limited Control model by employing inverse optimization. The inverse problem for the Limited Control model for carrier i, $InvLC^i$, can be obtained from $InvStrict^i$ by eliminating constraints (10), (11), and (13). $\pi_v^{i,(o,d,i)}$ and $\beta^{i,(o,d,i)}$ are constrained only over $(o,d,i) \in L^i$, rather than over all loads in the alliance as in (14) and (16). Finally, the capacity value $k_{(u,v)}$ in constraint (18) becomes $k^i_{(u,v)}$ due to the carrier specific capacity restrictions in the Limited Control model. As in the Strict Control case, we combine the constraints $InvLC^i$ over all carriers and search for a feasible set of capacity exchange prices. Such a set will guarantee that f^* is optimal for the individual problems $LC(i)$.

Theorem 2. *A feasible solution (π, α, β, and c) to $InvLC$ is guaranteed to exist.*

Proof. Associate dual variables $f^{(o,d,i)}_{(u,v)}$ and $y^{(o,d,i)}$ with constraints (8)-(9) and (12), respectively. Now consider the dual of $InvLC$ when an objective function of $\min 0(c, \pi, \alpha, \beta)$ is added:

$$\max \sum_{(o,d,i) \in L} r^{(o,d,i)} y^{(o,d,i)} \tag{24}$$

subject to:

$$\sum_{(u,v)\in E} f_{(u,v)}^{(o,d,i)} - \sum_{(v,w)\in E} f_{(v,w)}^{(o,d,i)} \leq 0 \quad \forall (o,d,i) \in L, v \in V : v \notin \{o,d\} \tag{25}$$

$$y^{(o,d,i)} - \sum_{(o,w)\in E} f_{(o,w)}^{(o,d,i)} \leq 0 \quad \forall (o,d,i) \in L \tag{26}$$

$$\sum_{(v,d)\in E} f_{(v,d)}^{(o,d,i)} - y^{(o,d,i)} \leq 0 \quad \forall (o,d,i) \in L \tag{27}$$

$$\sum_{(o,d,i)\in L} f_{(u,v)}^{(o,d,i)} \leq 0 \quad \forall (u,v) \in E, \forall i \in N \tag{28}$$

$$y^{(o,d,i)} \leq 0 \quad \forall (o,d,i) \in L \tag{29}$$

$$\sum_{i\in N:(u,v)\notin E^i} \sum_{(o,d,j)\in L^j} f_{(u,v)}^{(o,d,j)} \leq 0 \quad \forall (u,v) \in E \tag{30}$$

$$f_{(u,v)}^{(o,d,i)} \geq 0 \quad \forall (o,d,i) \in L, (u,v) \in E : f_{(u,v)}^{*(o,d,i)} = 0 \tag{31}$$

$$f_{(u,v)}^{(o,d,i)} \quad unr. \quad \forall (o,d,i) \in L, (u,v) \in E : f_{(u,v)}^{*(o,d,i)} > 0 \tag{32}$$

$$y^{(o,d,i)} \geq 0 \quad \forall (o,d,i) \in L : f_{(d,o,i)}^{*(o,d,i)} = 0 \tag{33}$$

$$y^{(o,d,i)} \quad unr. \quad \forall (o,d,i) \in L : f_{(d,o,i)}^{*(o,d,i)} > 0. \tag{34}$$

Equations (25)-(27) are associated with $\pi_v^{(o,d,i)}$, (28) with $\alpha_{(u,v)}^i$, (29) with $\beta^{(o,d,i)}$, and (30) with $c_{(u,v)}$. Assuming all load revenues $r^{(o,d,i)}$ are non-negative, (29) implies that the objective function (24) is bounded. Furthermore, the solution $\mathbf{y} = \mathbf{f} = \mathbf{0}$ is clearly feasible. As the dual is bounded and feasible, $InvLC$ must also be feasible. □

4 Comparison of Allocations under Strict and Limited Control Models

Having established two distinct models for the behavior of an individual carrier within the alliance and the methodology for obtaining capacity exchange prices using each of the models, we now focus on the characteristics of allocations obtained using each model. What are the advantages and disadvantages of the Strict and Limited Control models? Qualitatively, it can be argued that the Limited Control model offers a more realistic view of the decisions available to an individual carrier. Quantitatively, this section focuses on analyzing the allocations obtained under each model to determine their respective ability to ensure alliance optimal behavior is attained. The concepts of cooperative game theory provide a framework for measuring and comparing the benefits of various allocations.

In a *cooperative game*, a group of selfish agents work to maximize their own benefits as well as a system objective. An alliance in which carriers make and receive payments for the use of capacity fits into the structure of a cooperative game with *transferrable payoffs*, or a game in which participants are allowed to exchange utility among each other; in this case the payoffs take the form of money and are transferred via capacity exchange prices. A solution to a cooperative game consists of an allocation of benefits to each participant; in our case an allocation x^i is comprised of direct revenue from delivering loads plus the net sum of capacity exchange prices paid and received. Of particular interest is the notion of the *core*, which is the set of allocations that are (i) budget-balanced, meaning that all benefits are allocated, and (ii) stable, meaning that no subset of participants can benefit by leaving the alliance. Let $v(S)$ be the total profit that a subset of carriers S can earn on their own; that is, $v(S)$ is the optimal objective function value when the Centralized problem C is solved for the subset S. The core is defined as follows:

$$\sum_{i \in N} x^i = v(N) \tag{35}$$

$$\sum_{i \in S} x^i \geq v(S) \quad \forall S \subset N \tag{36}$$

where (35) is the budget-balance condition and (36) is the stability condition. We call the subset of stability equations (36) in which $|S| = 1$ *rationality* constraints, as they ensure that each individual carrier will earn at least as much in the alliance as they could earn operating alone.

Basic cost allocation methods are discussed in [27]; a more detailed discussion about allocation methods and the core of a cooperative game is available in [19]. Key observations from these works include that the core of a cooperative game is often empty, and the core of a game may contain many allocations. Production games based on linear programming models were studied in [18]; flow games were later specifically considered in [12] and [10]. In [12], networks with a single commodity and capacitated edges owned by players were studied. It was shown that such problems have a nonempty core. [10] extended these results into the multi-commodity flow arena by showing that the result applied to networks with many commodities, but one common source and sink. [3] contributes to the study of multi-commodity flow games by considering multiple sources and sinks. Furthermore, whereas previous studies of flow games have assumed a unique owner for every edge, [3] considers the possibility for multiple owners on an edge.

The properties of a core allocation are clearly desirable for a carrier alliance; we ultimately prove in the following discussion that regardless of the individual behavioral model employed, there is always a feasible set of capacity exchange prices that leads to a core allocation. In fact, the following theorem states that a core allocation is always obtained when the Strict Control model is employed:

Theorem 3. *The set of feasible capacity exchange prices for the Strict Control model leads to a set of allocations that is a strict subset of the core of the carrier alliance game.*

Proof. We have already demonstrated that a feasible solution for $InvStrict$ must exist; [3] further shows that any solution to $InvStrict$ yields a core allocation. In order to demonstrate that the set of allocations that can be obtained using the Strict Control model does not in fact equal the core, consider the example described in Figure 2 and Table 1:

The time-expanded network illustrated in Figure 2 includes two legs, $(1, 3)$ and $(2, 4)$, operated by carrier A; the capacity on each of these legs is one. The ground edges $(1, 2)$ and $(3, 4)$ have unlimited capacity. The loads are described in Table 1. For example, demand $(1, 3, A)$ represents a load of carrier A with ready time and origin location corresponding to node 1, and delivery deadline and destination corresponding to node 3.

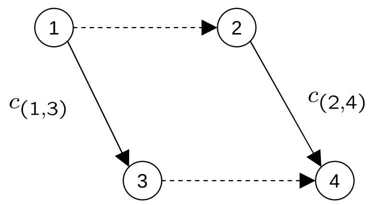

Figure 2: System Network for Theorem 3 Example

Table 1: Loads for Theorem 3 Example

Demand	Per-Unit Revenue ($r^{(o,d,k)}$)	Size ($d^{(o,d,k)}$)
$(1,3,A)$	2	1
$(2,4,A)$	2	1
$(1,4,B)$	6	1
$(2,4,C)$	3	1

The side payment for carrier B is as follows: $s^B = -c_{(1,3)}f^{(1,4,B)}_{(1,3)} - c_{(2,4)}f^{(1,4,B)}_{(2,4)}$. Since $d^{(1,4,B)} = 1$, it must be true that $f^{(1,4,B)}_{(1,3)} + f^{(1,4,B)}_{(2,4)} \leq 1$, which implies $s^B \geq \min\{-c_{(1,3)}, -c_{(2,4)}\} = -\max\{c_{(1,3)}, c_{(2,4)}\}$. Solving *InvStrict* with an objective function of $\max c_{(1,3)}$, an optimal objective function value of 3 is attained. Solving *InvStrict* with an objective function of $\max c_{(2,4)}$ also yields an optimal objective function value of 3. As the maximum feasible value of either leg's capacity exchange price is 3, the minimum value of s^B under the Strict Control model is -3.

It can be easily verified that the allocation $x^A = 7, x^B = 1, x^C = 1$ is a core allocation, since it satisfies equations (35) and (36). As $v^A = 0, v^B = 6$, and $v^C = 3$, in order to obtain this allocation the capacity exchange prices must lead to the following side payments: $s^A = 7, s^B = -5$, and $s^C = -2$. This contradicts the minimum attainable value of s^B, implying that this core allocation cannot be obtained using the Strict Control model. □

Thus far we have shown that using the Strict Control model, one is not only guaranteed to be able to find a feasible set of capacity exchange prices, but that these prices will lead to a core allocation. However, not all core allocations may be obtained. We would like to characterize the relationship between the core of the carrier alliance game and allocations obtained using the Limited Control model as well. An important step in doing this is establishing the relationship of allocations obtained using the Limited Control model to those obtained using the Strict Control model, which is in fact described in the following theorem:

Theorem 4. *The set of allocations that may be obtained using the Strict Control model is a subset of the set of allocations that may be obtained using the Limited Control model.*

Proof. We first show that any set of capacity exchange prices obtained using the Strict Control model can also be obtained using the Limited Control model. Let $|N| = n$, and consider $f^{(o,d,i)}_{(u,v)}$. Assume, without loss of generality, that leg (u,v) is operated by carrier $j \neq i$. In *InvStrict*, there are exactly n constraints corresponding to $f_{(u,v)}^{(o,d,i)}$:

$$\text{(contained in } \mathit{InvStrict}^i) \qquad \pi_v^{i,(o,d,i)} - \pi_u^{i,(o,d,i)} + \alpha_{(u,v)}^i \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} -c_{(u,v)} \tag{37}$$

$$\text{(contained in } \mathit{InvStrict}^j) \qquad \pi_v^{j,(o,d,i)} - \pi_u^{j,(o,d,i)} + \alpha_{(u,v)}^j \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} c_{(u,v)} \tag{38}$$

$$\text{(contained in } \mathit{InvStrict}^k,\ k \notin \{i,j\}) \qquad \pi_v^{k,(o,d,i)} - \pi_u^{k,(o,d,i)} + \alpha_{(u,v)}^k \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} 0 \tag{39}$$

where each equation holds with equality if $f_{(u,v)}^{(o,d,i)} > 0$. In InvLC, there is exactly one constraint corresponding to $f_{(u,v)}^{(o,d,i)}$, which is constraint (37). Similarly, consider $f_{(d,o,i)}^{*(o,d,i)}$. In $\mathit{InvStrict}$ there are again n constraints corresponding to $f_{(d,o,i)}^{*(o,d,i)}$:

$$\text{(contained in } \mathit{InvStrict}^i) \qquad \pi_o^{i,(o,d,i)} - \pi_d^{i,(o,d,i)} + \beta^{i,(o,d,i)} \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} r^{(o,d,i)} \tag{40}$$

$$\text{(contained in } \mathit{InvStrict}^k, k \neq i) \qquad \pi_o^{k,(o,d,i)} - \pi_d^{k,(o,d,i)} + \beta^{k,(o,d,i)} \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} 0 \tag{41}$$

where each equation holds with equality if $f_{(d,o,i)}^{*(o,d,i)} > 0$. In InvLC there is only one constraint corresponding to $f_{(d,o,i)}^{*(o,d,i)}$, which is (40). We conclude that the constraint set InvLC is a subset of the constraint set $\mathit{InvStrict}$, which implies that any solution that is feasible for $\mathit{InvStrict}$ must also be feasible for InvLC. It follows directly that any allocation obtained under $\mathit{InvStrict}$ can also be obtained under InvLC.

To demonstrate that the set of allocations that may be obtained using the Limited Control model contains allocations that cannot be obtained using the Strict Control model, we return to the example used in the proof of Theorem 3. The set of capacity exchange prices $c_{(1,3)} = 5, c_{(2,4)} = 2$ is feasible for InvLC, and results in the allocation $x^A = 7, x^B = 1, x^C = 1$. However, it was demonstrated in the proof of Theorem 3 that this particular allocation cannot be obtained using the Strict Control model. It follows that the set of allocations obtained using the Strict Control model must be a subset of the set of allocations obtained using the Limited Control model. □

Because of Theorem 4 we know that it is possible to obtain a core allocation using the Limited Control model. But are we guaranteed a core allocation? We in fact show in the next theorem that non-core allocations may be obtained when using the Limited Control model.

Theorem 5. *The set of feasible capacity exchange prices for the Limited Control model leads to a set of allocations that may contain allocations outside the core.*

Proof. Returning once again to the example discussed in Theorem 3, the capacity exchange prices $c_{(1,3)} = c_{(2,4)} = 0$ are feasible for InvLC. $c_{(1,3)} = c_{(2,4)} = 0$ implies $s^A = 0, s^B = 0, s^C = 0$. The resulting allocation, $x^A = 0, x^B = 6, x^C = 3$, clearly does not satisfy the set of stability equations 36, as $v(A) = 4$, and is therefore not contained in the core. □

That we may obtain an allocation outside the core when employing the Limited Control model is at first disconcerting, as an allocation in which some subset of members is actually receiving less profit than they could earn on their own is clearly undesirable. However, we can ensure that such an allocation is not obtained by the addition of stability constraints to $InvLC$. The number of stability constraints of type (36) required is $2^{|N|} - 1$. Based on the relatively small number of carriers participating in an air cargo alliance (for example, the SkyTeam Cargo alliance is currently comprised of 8 carriers, while the WOW alliance is comprised of 4 carriers), the total number of stability constraints will not be prohibitively large. However, if the enumeration of all stability constraints does become a concern, one possibility is to incorporate stability constraints for subsets of size m or smaller, where $m < |N|$. This is reasonable under the assumption that carriers have limited information about other carriers participating in the alliance. An important consequence of Theorem 4 is the following corollary, which implies that it is possible to guarantee a core allocation when using the Limited Control model.

Corollary 6. *InvLC remains feasible when enhanced with stability constraints.*

There are some instances, however, for which the set of allocations that may be obtained using the Limited Control model is in fact a subset of the core of the carrier alliance game. Theorem 7 below characterizes conditions that are necessary in order for this to be the case; these conditions are especially interesting because they imply that in order for the Limited Control model to produce an allocation that is guaranteed to be in the core of the carrier alliance game with transferrable payoffs, the carrier alliance game with non-transferrable payoffs must have a non-empty core as well. (A game with non-transferrable payoffs corresponds to an alliance in which the allocation for carrier i is equal to the direct revenue earned by carrier i, or $x^i = q^i \ \forall i \in N$.) This is the case because, as is proven below, a solution in which all capacity exchange prices are zero is always feasible for $InvLC$.

Theorem 7. *Given a set of carriers N, the set of feasible solutions for InvLC leads to a set of allocations that is a subset of core only if* $v(S) \leq \sum_{i \in S} v^i \ \forall S \subset N$.

Proof. Assume that $\mathbf{c} = 0$ is a feasible solution to $InvLC$. The allocation x^i received by carrier i is then equal to v^i, the amount of revenue carrier i receives by delivering loads in accordance with f^*. If there exists a subset $S \subset N$ such that $v(S) > \sum_{i \in S} v^i$, then it must also be true that $v(S) > \sum_{i \in S} x^i$ and $\mathbf{x}$ cannot be a core allocation since it violates (Reference here to core stability equation).

It remains to show that $\mathbf{c} = 0$ is a feasible solution to $InvLC$. Consider the Limited Control model for carrier i :

$$LC^i : \max \sum_{(o,d,i) \in L^i} r^{(o,d,i)} f_{(d,o,i)}^{(o,d,i)} - \sum_{e \notin E^i} c_e \Big(\sum_{(o,d,i) \in L^i} f_e^{(o,d,i)} \Big) \tag{42}$$

subject to:

$$\sum_{(u,v)\in E} f_{(u,v)}^{(o,d,i)} - \sum_{(v,w)\in E} f_{(v,w)}^{(o,d,i)} \leq 0 \qquad \forall v \in V, \forall (o,d,i) \in L^i \tag{43}$$

$$\sum_{(o,d,i)\in L^i} f_e^{(o,d,i)} \leq k_e^i \qquad \forall e \in E \tag{44}$$

$$f_{(d,o,i)}^{(o,d,i)} \leq d^{(o,d,i)} \quad \forall (o,d,i) \in L^i \tag{45}$$

$$f_e^{(o,d,i)} \geq 0. \tag{46}$$

Let f^{*i} be the vector of components of f^* pertaining to the loads of carrier i. That is, f^{*i} is comprised of $f_{(d,o,i)}^{*(o,d,i)}$ and $f_e^{*(o,d,i)}$, $\forall e \in E$. We know that f^{*i} is a feasible solution to LC^i, since the capacity limits k_e^i were constructed in a manner that ensures $\sum_{(o,d,i)\in L^i} f_e^{*(o,d,i)} \leq k_e^i$. Let $\widehat{f^i}$ be an optimal solution to $InvLC^i$ when $\mathbf{c} = 0$, and assume f^{*i} is not optimal for carrier i when $\mathbf{c} = 0$. $f = \widehat{f^i} \cup \bigcup_{j\in N, j\neq i} f^{*j}$ must be a feasible solution to the centralized problem C, since the capacity limits k_e^i were also constructed to ensure that $\sum_{i\in N} k_e^i \leq k_e \ \forall e \in E$. Furthermore, $\sum_{(o,d,i)\in L^i} r^{(o,d,i)} \widehat{f}_{(d,o,i)}^{(o,d,i)} > \sum_{(o,d,i)\in L^i} r^{(o,d,i)} f_{(d,o,i)}^{*(o,d,i)}$ (which must be true due to the optimality of $\widehat{f^i}$) implies that $\sum_{(o,d,i)\in L^i} r^{(o,d,i)} f_{(d,o,i)}^{(o,d,i)} + \sum_{j\in N, j\neq i} \sum_{(o,d,j)\in L^j} r^{(o,d,j)} f_{(d,o,j)}^{*(o,d,j)} > \sum_{i\in N} \sum_{(o,d,i)\in L} r^{(o,d,i)} f_{(d,o,i)}^{*(o,d,i)}$, which contradicts the optimality of f^*. We conclude that f^{*i} must be optimal for LC^i when $\mathbf{c} = 0$.

Now consider the dual of LC^i:

$$DLC^i: \qquad \min \sum_{(u,v)\in E} k_e \alpha_{(u,v)}^i + \sum_{(o,d,i)\in L} d^{(o,d,i)} \beta^{i,(o,d,i)} \tag{47}$$

subject to:

$$\pi_v^{(o,d,i)} - \pi_u^{(o,d,i)} + \alpha_{(u,v)}^i \geq 0 \quad \forall (o,d,i) \in L^i, (u,v) \in E^i \tag{48}$$

$$\pi_v^{(o,d,i)} - \pi_u^{(o,d,i)} + \alpha_{(u,v)}^i \geq -c_{(u,v)} \quad \forall (o,d,i) \in L^i, (u,v) \notin E^i \tag{49}$$

$$\pi_o^{(o,d,i)} - \pi_d^{(o,d,i)} + \beta^{(o,d,i)} \geq r^{(o,d,i)} \quad \forall (o,d,i) \in L^i \tag{50}$$

$$\pi_v^{(o,d,i)} \geq 0 \quad \forall v \in V, \forall (o,d,i) \in L \tag{51}$$

$$\alpha_{(u,v)}^i \geq 0 \quad \forall (u,v) \in E \tag{52}$$

$$\beta^{(o,d,i)} \geq 0 \quad \forall (o,d,i) \in L \tag{53}$$

Because LC^i has an optimal solution when $\mathbf{c} = \mathbf{0}$ (namely, f^{*i}), DLC^i must also have an optimal solution when $\mathbf{c} = 0$. Let $(\pi^{*i}, \alpha^{*i}, \beta^{*i})$ be optimal for DLC^i when $\mathbf{c} = 0$. f^{*i} and $(\pi^{*i}, \alpha^{*i}, \beta^{*i})$ must satisfy complementary slackness conditions for LC^i and DLC^i; it therefore follows that $(\pi^{*i}, \alpha^{*i}, \beta^{*i}, \mathbf{c} = 0)$ must be feasible for $InvLC^i$. (Recall that $InvLC^i$ is constructed by modifying the dual of LC^i to ensure that complementary slackness conditions are satisfied with f^{*i}.)

We have shown that $\mathbf{c} = 0$ must be feasible for $InvLC^i$. Because $InvLC$ is constructed by combining the constraints of $InvLC^i$ for all $i \in N$, and the exchange prices c are the only components common among the constraints $InvLC^i$ and $InvLC^j$, $\bigcup_{i\in N}(\pi^{*i}, \alpha^{*i}, \beta^{*i}), \mathbf{c} = 0$ must be feasible for $InvLC$. □

If one can obtain a core allocation using either behavioral model, then is there an advantage to using one particular model over another? We demonstrate in the following example that when using the Strict Control model, the aggregated individual solutions may be suboptimal or even infeasible from the centralized perspective. The inability to ensure feasibility of the aggregation of the individual solutions obtained when each carrier solves $Strict^i$ for a given set of capacity exchange prices is an obvious limitation of the Strict Control model. It is perhaps counterintuitive, as one might expect that as the level of control represented in a given model increases, the ability to produce the desired result (in this case, behavior consistent with the alliance optimal solution) would also increase. Instead we find that it is exactly this increased control on the part of an individual carrier that leads to behavior inconsistent with the centralized solution.

Remark 1. *Given a set of capacity exchange prices feasible for $InvStrict$, it is possible that multiple optimal solutions exist for $Strict^i$ that, in the centralized setting, are suboptimal or create infeasibility.*

Consider a simple system in which carrier A operates a leg with origin o, destination d, and capacity 2. There are two loads in the system, also with origin o and destination d; one load is associated with carrier A and one load is associated with carrier B. The per unit revenue associated with loads (o, d, A) and (o, d, B) are 2 and 1, respectively. The size of loads (o, d, A) and (o, d, B) are 1 and 2, respectively.

The centralized optimal solution is to deliver one unit of each load. That is, $f^{*(o,d,A)}_{(d,o,A)} = f^{*(o,d,A)}_{(o,d)} = f^{*(o,d,B)}_{(d,o,B)} = f^{*(o,d,B)}_{(o,d)} = 1$. The only value of $c_{(o,d)}$ that is feasible for $InvStrict$ is $c_{(o,d)} = 1$. The objective function for $Strict^B$ is $\max f^{(o,d,B)}_{(d,o,B)} - c_{(o,d)} f^{(o,d,B)}_{(o,d)}$, and since $f^{(o,d,B)}_{(d,o,B)} = f^{(o,d,B)}_{(o,d)}$ in any feasible solution to $Strict^B$, all feasible solutions to $Strict^B$ have an objective function value of 0. Consider the following basic solutions:

1. Basic variables: $f^{(o,d,A)}_{(d,o,A)} = f^{(o,d,A)}_{(o,d)} = f^{(o,d,B)}_{(d,o,B)} = f^{(o,d,B)}_{(o,d)} = 1, s^7 = 1, s^1 = s^4 = 0$
   Non-basic variables: $s^2 = s^3 = s^5 = s^6 = 0$

2. Basic variables: $s^1 = s^2 = s^3 = s^4 = 0, s^5 = s^7 = 2, s^6 = 1$
   Non-basic variables: $f^{(o,d,A)}_{(d,o,A)} = f^{(o,d,A)}_{(o,d)} = f^{(o,d,B)}_{(d,o,B)} = f^{(o,d,B)}_{(o,d)} = 0$

3. Basic variables: $f^{(o,d,B)}_{(d,o,B)} = f^{(o,d,B)}_{(o,d)} = 2, s^1 = s^2 = s^3 = s^4 = 0, s^6 = 1$
   Non-basic variables: $f^{(o,d,A)}_{(d,o,A)} = f^{(o,d,A)}_{(o,d)} = 0, s^5 = s^7 = 0$

Each of these solutions is in fact a basic feasible solution. It follows that each of these solutions is an optimal solution that may be obtained by using a standard solver such as CPLEX.

Solution 1 is in fact the centralized optimal solution (or more specifically, a component thereof). Solution 2, when aggregated with the solution obtained from $Strict^A$, which is $f^{(o,d,A)}_{(d,o,A)} = f^{(o,d,A)}_{(o,d)} = f^{(o,d,B)}_{(d,o,B)} = f^{(o,d,B)}_{(o,d)} = 1$, is not infeasible from the centralized perspective (assuming that the optimal value of $f^{(o,d,i)}$ is retained from $Strict^i$, while the optimal value of $f^{(o,d,j)}$ from $Strict^i$ is ignored). However, this aggregate solution is suboptimal. Solution 3, when aggregated with the solution obtained from $Strict^A$, is in fact infeasible from the centralized perspective.

Due to the possibility of multiple optimal solutions for $Strict^i$, there is no guarantee that carrier i will behave in accordance with the centralized solution, even when there is a single set of exchange prices $\mathbf{c}$ that is feasible for $InvStrict$. Using a set of capacity exchange prices feasible for $InvLC$, the possibility of multiple optimal solutions for LC^i also exists. Given a choice between rejecting a load and delivering all or part of that load for no profit (and no loss), we can reasonably assume that a carrier will choose to deliver as much of the load as possible in order to maintain or improve customer service. We are therefore concerned primarily about the instances in which a carrier's decision leads to infeasibility from the centralized perspective. As the Limited Control model employs capacity restrictions k_e^i for each carrier i and leg e that are constructed to ensure that $\sum_{i \in N} k_e^i \leq k_e$, we observe the following:

Remark 2. *Any solution obtained using the Limited Control model is feasible from the centralized perspective.*

5 Experimental Results

We conducted two sets of experiments in order to investigate the benefit to be gained by collaborating, and how this benefit changes as the number and size of the carriers participating in the alliance changes. In the first set of experiments, all alliances are comprised of two carriers, while in the second set of experiments each alliance has three carriers. Some instance classes have "carriers" with no legs; these carriers represent freight forwarders and allow us to investigate the impact of freight forwarders on alliances among carriers. In this section we describe the experimental procedure, present results, and discuss insights obtained from each set of experiments.

5.1 Data Generation

Using data publicly available from the Bureau of Transportation Statistics [8], we identified 5 classes of combination carriers based on network size and capacity of fleet. Network size is approximated by the number of (origin, destination) pairs served by a carrier, while fleet capacity is approximated by the average number of passengers per departure. For each classification listed in Tables 2(a) and 2(b), the actual range represented by the classification is given. Using these classifications, we obtain the 5 general classes of carriers described in Table 2(c).

Table 2: Class Descriptions

(a) Network Size

Network Size	# of (o,d) Pairs
large	400 - 775
medium	131-366
small	16-100

(b) Fleet Capacity

Fleet Capacity	# of Passengers per Departure
large	95-150
small	15-90

(c) Carrier Classifications

Class	Network Size	Fleet Capacity	# of Carriers in Class
C1	large	large	4
C2	medium	large	5
C3	medium	small	8
C4	small	large	5
C5	small	small	3

These carrier classes were used to generate carriers for the experiments described in the following sections. The network size and fleet capacity were scaled to reflect, approximately, the relative size relationships among the classes. If the number of (origin, destination) pairs served by a carrier in class C4 or C5 is n, then the number of pairs served by a carrier in class C2 or C3 is $5n$, and the number of pairs served by a carrier in class C1 is $12n$. A small fleet capacity is represented by a generated carrier with a capacity of 2 on each leg, while a large fleet capacity is represented by a carrier with a capacity of 5 on each leg. Finally, two classes of freight forwarders are used in the following experiments. A large freight forwarder is represented by class F1, and is associated with $12n$ loads. A small freight forwarder is represented by class F2, and is associated with $5n$ loads.

In both sets of experiments, each carrier operates a pure hub-and-spoke network. The networks of the carriers are completely integrated, meaning that there is a leg from each hub of carrier i to each of hub of carrier j for all pairs of carriers i and j. In order to simplify analysis, the network is generated such that the decisions about whether to accept a load and how to route that load are dependent solely on network geography and capacity, and not on time. (This is accomplished by orienting all spoke legs from spoke to hub, and then setting the origin and destination time of every spoke-to-hub leg as 0 and 1, respectively; every inter-hub leg as 1 and 2, respectively, and every load as 0 and 2, respectively.)

In a particular instance class, the number of carriers, number of "spoke" legs for each carrier, and the capacity of each spoke leg are specified according to the carrier class of the carrier. In addition, the number of hubs depends on the size of the carrier; carriers with a large, medium, and small network size operate 3, 2, and 1 hubs, respectively. The number of spoke legs operated by a carrier in class C4 or C5 is 5 ($n = 5$). The origins for spoke legs operated by carrier i are approximately equally distributed among the hubs operated by carrier i, while every spoke leg has a unique destination. As described above, an inter-hub leg exists between every pair of hubs. Figure 3 depicts the system network for an alliance comprised of one carrier with a large network and one carrier with a medium network. In this example, carrier A operates all inter-hub leg originating from H_1^A, H_2^A or H_3^A, while carrier B operates all inter-hub legs originating from H_1^B or H_2^B.

The capacity of spoke legs is set according to the fleet capacity of the carrier class. The capacity of the inter-hub legs is equal to C, where C is a very large number. This large capacity, while not realistic, ensures that limited capacity on inter-hub routes does not restrict the benefit of collaborating. In this pure hub-and-spoke system, it can easily be seen that the benefit associated with collaborating increases as the capacity on inter-hub legs increases (until inter-hub capacity reaches a high-enough level), because any load associated with carrier i that has a destination outside the network of carrier i must travel on an inter-hub leg. We therefore assume that carriers participating in an alliance will increase inter-hub capacity to a level that ensures sufficient benefit.

The number of loads associated with a carrier is equal to the number of spoke legs operated by that carrier, which approximates a proportional relationship between the size of a carrier's network and the number of cargo loads booked by that carrier. As any load originating at a spoke must be transported to the hub of that spoke before it can be transported anywhere else, we simplify the system by generating the origin of a load associated with carrier i randomly from the set of carrier i's hubs. The destination of a load associated with carrier i is generated within the network of carrier i with some probability p; all spoke destinations within the network of carrier i are equally

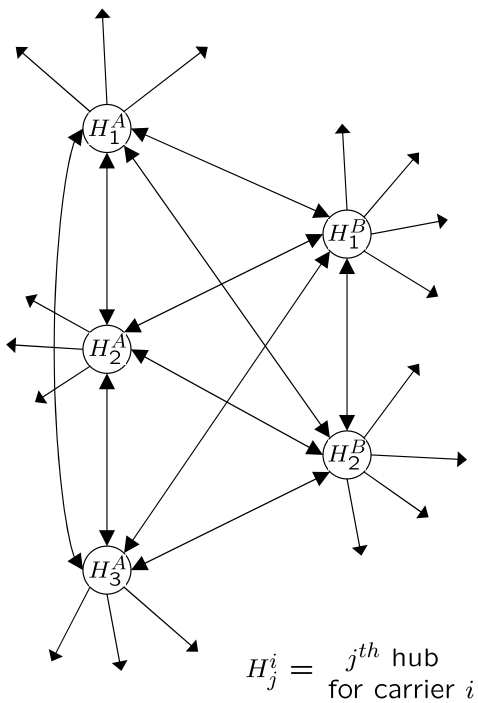

Figure 3: Integrated Hub-and-Spoke Network

likely with probability $\frac{p}{spokes^i}$ where $spokes^i$ is the number of spoke legs operated by carrier i. The probability p that the destination of a load associated with carrier i is within the market served by carrier i is calculated for each instance k in one of the following ways:

1. $p^i = \frac{a^i}{a^i+1}$, where a^i is the size of carrier i's network relative to the smallest carrier in the data ($a^i = \frac{legs^i}{\min_{j \in N} legs^j}$). In alliances where only one carrier operates legs (because the remaining partners are forwarders), $p^i = 1$ for this carrier.

2. $p^i = \frac{legs^i}{\sum_{j \in N^k} legs^j}$ where N^k is the group of carriers in instance k.

For a load associated with carrier i, all destinations outside the network of carrier i have an equal probability of being selected; this probability is $\frac{1-p^i}{\sum_{j \neq i} spokes^j}$. (Consequently, loads associated with freight forwarders have destinations that are uniformly distributed throughout the network.) Obtaining accurate demand distributions is very difficult; for this reason we test our model using two different distributions. In both distributions the proportion of loads that fall within a carrier's own network increases with the size of a carrier, but when the first measure is used a significantly higher proportion of a carrier's loads fall within his network.

For each carrier, the maximum size of a load, S^i, is equal to the capacity of that carrier's legs, and the maximum per-unit revenue of each load is 3. ($S^{F1} = 5$, and $S^{F2} = 2$.) The size and per-unit revenue of each load associated with carrier i are generated according to a uniform distribution over the ranges [1, S^i] and [1, 3], respectively.

5.2 Results and Insights from 2-Carrier Experiments

For each instance class, the results reported in Tables 3 and 4 represent the average from 30 instances generated with the same class parameters. Information pertaining to the alliance optimal solution is contained in Table 3. The "Carriers" column indicates the class from which each carrier is selected; in instance class 1, for example, both carriers in the alliance are carriers from class C1. The increase in system revenue is calculated as the total revenue earned by the alliance minus the sum of the revenue each carrier earns by working independently. The percent increase in accepted loads measures the percent difference in the number of loads that can be completely delivered in the local (independent) solution for a carrier and the number of loads associated with that carrier that are completely delivered in the centralized alliance solution.

Table 3: System Revenue and Accepted Loads for Two-Carrier Alliances

		Demand Distribution 1				**Demand distribution 2**			
Instance Class	**Carriers (A,B)**	**Chg. in System Revenue**		**Chg. in Loads Accepted**		**Chg. in System Revenue**		**Chg. in Loads Accepted**	
		Actual	**%**	**A**	**B**	**Actual**	**%**	**A**	**B**
1	C1,C1	40.1	7.2%	7.2%	5.9%	266.1	81.6%	72.4%	71.2%
2	C1,C2	33.0	8.4%	6.7%	16.7%	167.2	65.3%	29.7%	242.0%
3	C1,C3	19.6	5.9%	2.2%	12.8%	84.7	34.5%	8.5%	205.0%
4	C1,C4	30.6	10.2%	6.2%	62.7%	42.3	14.7%	5.8%	1018.2%
5	C1,C5	14.6	4.9%	1.9%	44.9%	24.1	8.3%	1.7%	861.5%
6	C1,F1	206.4	68.0%	-24.2%	N/A	206.4	68.0%	-24.2%	N/A
7	C1,F2	55.1	18.2%	-6.5%	N/A	55.1	18.2%	-6.5%	N/A
8	C2,C2	33.6	15.3%	14.5%	14.7%	113.4	79.7%	64.9%	81.0%
9	C2,C3	17.8	11.3%	3.6%	18.1%	60.8	62.8%	18.3%	91.3%
10	C2,C4	33.6	28.9%	14.2%	88.5%	42.3	39.0%	13.5%	568.4%
11	C2,C5	15.3	13.2%	4.4%	86.4%	19.9	17.7%	3.8%	535.0%
12	C2,F1	157.9	124.8%	-44.7%	N/A	157.9	124.8%	-44.7%	N/A
13	C2,F2	54.1	43.9%	-16.2%	N/A	54.1	43.9%	-16.2%	N/A
14	C3,C3	16.3	15.8%	15.8%	14.5%	54.5	82.9%	72.5%	73.7%
15	C3,C4	18.7	29.3%	20.2%	26.8%	22.6	41.1%	18.7%	126.1%
16	C3,C5	13.0	22.3%	15.1%	36.6%	18.6	35.0%	14.1%	409.5%
17	C3,F1	79.6	134.9%	-56.8%	N/A	79.6	134.9%	-56.8%	N/A
18	C3,F2	35.0	60.7%	-28.0%	N/A	35.0	60.7%	-28.0%	N/A
19	C4,C4	23.3	89.7%	54.4%	78.0%	23.3	89.7%	54.4%	78.0%
20	C4,C5	11.1	52.0%	10.4%	55.7%	11.1	52.0%	10.4%	55.7%
21	C4,F1	48.0	185.7%	-95.5%	N/A	48.0	185.7%	-95.5%	N/A
22	C4,F2	33.2	116.9%	-59.8%	N/A	33.2	116.9%	-59.8%	N/A
23	C5,C5	9.8	72.9%	66.7%	65.6%	9.8	72.9%	66.7%	65.6%
24	C5,F1	17.7	147.4%	-98.2%	N/A	17.7	147.4%	-98.2%	N/A
25	C5,F2	16.9	159.4%	-88.2%	N/A	16.9	159.4%	-88.2%	N/A

Table 4 contains results pertaining to the allocations received by each carrier. Specifically, the table shows the benefit each carrier receives by joining the alliance, calculated as the difference between the allocation for carrier i and the revenue carrier i could earn operating alone, or $x^i - v(i)$.

Table 4: Benefit Experienced by Joining Two Carrier Alliance

Instance Class	Carriers (A,B)	Demand Distribution 1						Demand distribution 2					
		Strict Control		Limited Control		Limited w/ Rationality		Strict Control		Limited Control		Limited w/ Rationality	
		A	B	A	B	A	B	A	B	A	B	A	B
1	C1,C1	21	19	20	20	15	25	122	145	132	134	61	205
2	C1,C2	16	17	16	17	15	18	83	84	76	92	70	98
3	C1,C3	7	12	12	8	5	15	29	56	43	42	43	41
4	C1,C4	17	14	18	13	20	10	25	17	17	25	39	3
5	C1,C5	6	9	12	3	7	7	9	16	12	12	20	4
6	C1,F1	108	99	-54	261	206	0	108	99	-54	261	206	0
7	C1,F2	31	24	-10	65	53	2	31	24	-10	65	53	2
8	C2,C2	16	17	17	17	14	20	56	58	54	59	15	98
9	C2,C3	5	13	10	7	9	9	17	44	31	30	15	46
10	C2,C4	18	16	19	15	18	15	23	19	18	25	29	13
11	C2,C5	7	8	10	6	10	5	8	12	9	11	16	4
12	C2,F1	59	99	-43	201	158	0	59	99	-43	201	158	0
13	C2,F2	24	30	-10	64	54	0	24	30	-10	64	54	0
14	C3,C3	8	8	8	8	6	10	22	32	27	27	6	49
15	C3,C4	17	2	10	9	11	8	19	3	9	13	16	6
16	C3,C5	6	7	9	4	8	5	10	9	8	10	11	7
17	C3,F1	25	54	-28	108	80	0	25	54	-28	108	80	0
18	C3,F2	15	20	-11	46	35	0	15	20	-11	46	35	0
19	C4,C4	11	12	11	12	11	13	11	12	11	12	11	13
20	C4,C5	3	8	6	5	6	5	3	8	6	5	6	5
21	C4,F1	13	35	-25	73	48	0	13	35	-25	73	48	0
22	C4,F2	10	24	-14	47	33	0	10	24	-14	47	33	0
23	C5,C5	5	5	5	5	4	6	5	5	5	5	4	6
24	C5,F1	4	14	-12	30	18	0	4	14	-12	30	18	0
25	C5,F2	7	10	-9	26	17	0	7	10	-9	26	17	0

Analyzing the results in Tables 3 and 4, we obtain several observations and insights:

- Given that an allocation in which both carriers receive non-negative benefit is a core allocation, the mechanism is behaving as expected with regard to the results of Section 4. Namely, every allocation under the Strict Control model is a core allocation, while for some instances, the Limited Control model yields an allocation outside the core. Adding rationality constraints to the Limited Control model results in a core allocation for a two-carrier alliance.
- Not surprisingly, the benefit associated with collaborating increases as the probability that a load can be served by its associated carrier decreases. Note that when the benefit under distribution 2 is not higher than the benefit under distribution 1, it is an instance in which

the probability that a load can be served by its associated carrier is the same under both distributions.

- The benefit associated with collaborating, measured by the increase in system revenue, increases with the size of the network and fleet capacity. Under demand distribution 1 there are slightly diminishing returns, as the percentage increase in profit declines as network size and fleet capacity increase, while under demand distribution 2, the percentage increase in profit increases with network and fleet size. Thus we conclude that the marginal benefit associated with increasing network and fleet sizes in collaborating partners increases as the proportion of loads that a carrier can serve using only his network decreases.

- Under demand distribution 1, fleet capacity has more impact than network size on the benefit associated with collaborating. Furthermore, a carrier with large fleet capacity does not experience a significant increase in the number of loads completely accepted for delivery when collaborating with a carrier with small fleet capacity. These observations lead to an interesting insight: consider the relationship between a large national carrier and a smaller subsidiary. If the subsidiary carrier can serve a high proportion of its own demand (as is the case under demand distribution 1), the parent carrier stands to benefit more by increasing the fleet size of its subsidiary than by increasing the size of the subsidiary network.

- The benefit associated with collaborating, measured both by the percentage increase in the number of loads completely accepted for delivery as well as by an improvement over $v(i)$, is strictly positive when a carrier collaborates with another carrier. The number of loads completely accepted strictly decreases, however, for a carrier collaborating with a freight forwarder. This result suggests that carriers may want to negotiate rules regarding priority of the carrier's loads relative to the forwarder's loads in order that the carrier's customer service level does not decline as a result of entering into collaboration with a forwarder.

5.3 Results and Insights from 3-Carrier Experiments

In the second set of experiments, alliances with three carriers were examined. Once again, 30 instances with the same class parameters were generated for each instance class. A total of 80 instance classes were tested; all possible combinations of three carriers were represented, excluding alliances comprised only of freight forwarders. Analyzing data similar to that reported in Tables 3 and 4, we observe the following:

- Under demand distribution 1, the benefit associated with adding a third carrier to an existing (or potential) two-carrier alliance varies greatly. For example, given an alliance comprised of two C1 carriers, adding a third carrier yields no benefit to the original two C1 carriers. Given an alliance of C1 and C4, adding a third carrier helps C4 (but not C1) if the third carrier has a large fleet capacity. When two carriers of type C5 collaborate, both carriers are helped by the addition of a third carrier with large fleet capacity.

- Under demand distribution 2, it is in general beneficial to all carriers to grow the alliance.

- As in the two carrier experiments, we observe that under demand distribution 1, higher fleet capacity seems to yield higher benefits from collaborating than does size of network.
- While the number of loads each carrier accepts does not always decrease when two carriers collaborate with a freight forwarder, we still observe a dramatic decline in the number of carrier loads accepted as compared to when two carriers collaborate with a third carrier.

We also see even more pronounced in the 3-carrier experiments that collaborating yields much higher benefits as the proportion of loads that can be served entirely by their associated carrier decreases. This effect, in addition to the first two observations discusses above, imply that the properties of demand experienced by carriers can greatly impact how much the carriers can benefit by collaborating. While it is true that under both distributions studied the benefit experienced by any carrier is strictly positive when collaborating with other carriers, it is an important observation that some pairs of carriers are in fact better off (in terms of the number of loads completely accepted) by not adding a third carrier. Before evaluating the benefits of a potential alliance or additional partner, therefore, it is important to consider the characteristics of the demand associated with each carrier.

6 Summary and Insights

In this paper we have examined a mechanism that utilizes capacity exchange prices in order to achieve optimal alliance behavior as well as an implementable distribution of alliance revenues. The methodology underlying the mechanism employs inverse optimization to determine capacity exchange prices that encourage carriers to behave in a manner that is optimal for the alliance; this tool is dependent on the mathematical model of the behavior of an individual carrier within the alliance. We studied two ways of modeling this behavior, and examined the capacity exchange prices, and therefore profit allocations, resulting from each model. The models studied differed in how the capacity used by other carrier's loads is acknowledged in an individual carrier's model.

We found that using a Strict Control behavioral model, in which the flow of other carrier's loads is recognized by including the corresponding decision variables in both the objective function and constraints, we obtain a set of allocations that is a subset of the core. That is, every allocation obtained using this model is budget-balanced and stable, meaning that no subset of alliance members can earn more profit by leaving the alliance. In the Limited Control behavioral model, the knowledge that other carriers are utilizing capacity is incorporated by appropriately restricting the amount of capacity available to the individual carrier. This model yields a larger range of capacity exchange prices; in addition to every allocation obtained under the Strict Control model, more allocations, some of which may lie outside the core, become feasible under this model. Furthermore, the Limited Control model guarantees centralized feasibility. The key insight obtained from the comparison of the two models is that the behavioral model used can significantly impact the alliance recommendations, in this case, capacity exchange prices.

Finally, experimental analysis was conducted to determine the benefit to be gained by collaborating, and how this benefit changes as the number and characteristics of the carriers participating in the alliance changes. In general, the benefit a carrier experiences by collaborating increases with the network size and fleet capacity of the partnering carriers. The characteristics of the demand distribution significantly impact the benefit a carrier experiences by participating, however. When a carrier must utilize the networks of partner carriers to deliver a high proportion of his loads, he will always benefit as the alliance grows, while this is not necessarily the case for a carrier who can deliver a high proportion of his loads using only his own network. An interesting insight gained from the experiments is that a carrier may benefit more by increasing the fleet capacity of a subsidiary carrier than by increasing the number of (origin, destination) pairs served. Finally, the analysis confirmed that the allocation mechanism operates as expected with regard to the underlying behavioral model selected.

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