Integrated Airline Fleet and Crew Robust Planning

Chunhua Gao* Ellis Johnson* Barry Smith#

* H. Milton Stewart School of Industrial and Systems Engineering
Georgia Institute of Technology
765 Ferst Drive NW, Atlanta, GA 30332
cgao@isye.gatech.edu, ejohnson@isye.gatech.edu
\# Sabre Holdings
3150 Sabre Drive, Southlake, TX 76092
Barry.Smith@sabre-holdings.com

Abstract

The airline fleet assignment problem involves assigning aircrafts to flights to maximize profit. Different fleet assignment solutions cause dramatically different performance in subsequent crew planning and operational processes. We have developed an integrated fleet and crew robust planning method to provide fleet assignment solutions that are both friendly to crew planning and robust to real time operations. The three challenges of this work are 1) to understand the influence of fleet assignment on crew scheduling; 2) to address crew scheduling in a tractable way in the integrated model; and 3) to achieve robustness. We address these challenges by developing a new approach that integrates crew connections within the fleet assignment model and imposes station purity by limiting the number of fleet types and crew bases allowed to serve each airport. Computational results demonstrate that the proposed approach can reduce crew planning cost, improve robustness, and solve industrial size problems with good computational efficiency.

1 Introduction

The airline planning process has evolved itself into a sequence of decision-making phases with the decision from one phase used as input for the subsequent phases. Specifically, the airline planning process starts with schedule construction and fleet planning that are succeeded by aircraft maintenance routing and crew scheduling. Aircraft maintenance routing and crew scheduling are two concurrent procedures both working on decomposed schedules of separate fleets. By making decisions sequentially, there is limited information shared and little interaction between the phases. Different fleet assignment solutions could cause dramatically different performance in the subsequent crew scheduling phase. During the day of operation, the schedule is executed as planned if no disruptions occur. However, in reality, delays and disruptions caused by adverse weather, mechanical failures, crew sickness, and air traffic control are unavoidable. Airline planning solutions that fail to account for recovery flexibilities would result in high operational cost. Therefore, it has been a major challenge to integrate both planning functions and operational considerations in the planning process.

To achieve the goal of integration over the planning functions and the timeline between planning and operations, researchers have started to work on airline integrated planning and robust planning. Integrated planning is intended to integrate the functional phases at the planning stage and robust planning is intended to make decisions at the planning stage that are beneficial to the operations. Since a comprehensive model integrating all functions or integrating over both functions and the timeline is still not computationally attainable, alternative integrating strategies are investigated to achieve partial integration. These integrating strategies include: (1) integrating schedule design and fleet assignment (Rexing et al. 2000, Lohatepanont and Barnhart 2004); (2) integrating fleet assignment and aircraft maintenance routing (Clarke et al. 1996, Barnhart et al. 1998b); (3) integrating aircraft routing and crew pairing (Klabjan et al. 2002, Cohn and Barnhart 2003, Cordeau et al. 2001, 2005); (4) integrating fleet assignment and crew scheduling (Clarke et al. 1996, Barnhart et al. 1998a, Sandhu and Klabjan 2004). In parallel, researches on robust planning include: (1) robust fleet assignment (Ageeva 2000, Rosenberger 2004, Smith and Johnson 2006); (2) robust aircraft routing (Kang and Clarke 2003, Lan et al. 2003); (3) robust crew scheduling (Yen and Birge 2006, Schaefer et al. 2005, Ehrgott and Ryan 2002, Chebalov and Klabjan 2006).

The objective of this paper is to integrate fleet assignment and crew scheduling and to also provide robust solutions to real-time operations. There are three challenges in this work: 1) To understand the influence of fleet assignment on the performance of crew scheduling. This includes what can lead to poor crew solutions, for example, crew double overnight rests. 2) To address the crew scheduling problem in the integrated model. The crew problem becomes much harder in the integrated model, since it is necessary to work on the complete schedule, instead of the sub-schedules for different fleet types. Thus, computational tractability is an important issue to be considered. 3) To achieve robustness in an integrated framework, so that the resulting plans are robust for both aircraft and crew recovery in operations.

In regards to integrating fleet and crew planning, (Clarke et al. 1996) was the first attempt to address crew scheduling issues in the fleet assignment model (FAM). In their work, a cost is added on each lonely double overnight and the optimization model is then used to balance the costs between lonely double overnights and fleeting. Barnhart et al. (1998a) propose an integrated approximate model for fleet assignment and crew pairing optimization which combines the basic FAM and a duty-based model for the crew pairing problem. (Sandhu and Klabjan 2004) propose an integrated planning model which integrates fleeting, aircraft routing and crew pairing simultaneously. Crew pairings are modeled explicitly and the aircraft rotation problem is captured by the plane count constraints. The integrated planning problem is truly computationally intensive. A computing environment consisting of a cluster of 27 dual 900 MHz Itanium 2 processors is adopted to conduct computational experiments in their work.

Previous research on robust airline crew scheduling includes stochastic and deterministic methods. (Yen and Birge 2006) propose a stochastic crew scheduling model and devise a solution methodology for integrating disruptions in the evaluation of crew schedules. (Schaefer et al 2005) propose a stochastic extension to the deterministic crew scheduling problem. They modify the coefficient vector of the objective function to reflect the expected cost of each decision variable rather than deterministic cost. Monte Carlo simulation is used to estimate the operational cost. (Ehrgott and Ryan 2002) propose a bi-criteria optimization model. The measure called "non-robustness" is evaluated for each pairing based on the effect of potential delays within the pairing. The non-robustness measure is then treated as a second objective. (Chebalov and Klabjan 2006) present a deterministic method that addresses robustness by considering crews that can be swapped in operations. They add a second objective for maximizing the number of move-up crew to the traditional crew scheduling model.

(Smith and Johnson 2006) propose a robust fleet assignment model imposing station purity which can also be called fleet purity. Fleet purity ensures that the number of fleet types serving a given station does not exceed a specified limit. Adding fleet purity can greatly reduce planned crew costs, maintenance costs, and improve robustness. However, it has significant negative impact on the computational efficiency of FAM. To improve the computational efficiency, a column generation solution approach called station decomposition was proposed.

In this paper, an integrated fleet and crew robust planning approach is developed. In this approach, to avoid the curse of dimensionality, crew connections rather than explicit crew pairings or duties are modeled to represent the crew scheduling problem within the integrated model. To achieve robustness, we extend the station purity idea (which originally refers to fleet purity only) to both fleet purity and crew base purity so that the numbers of both fleet types and crew bases allowed to serve each airport are limited. A move-up crew is a crew that is ready to fly a different flight which means that it is qualified to operate the specific aircraft and it is from the same crew base and it has the same number of days until the end of the pairing (Chebalov and Klabjan 2006). Imposing crew base purity can increase the opportunities of finding a move-up crew in crew recovery. Furthermore, it has the benefit of improving computational efficiency and quality of the FAM solutions.

The contributions of this paper include:

- developing an integrated fleet and crew robust planning approach which provides fleet assignment solutions both friendly to crew planning and robust to real time operations. Experiments are conducted using data from a major U.S. airline.
- studying station purity schemes including both fleet purity and crew base purity and investigating the impact of station purity on FAM profit, computational efficiency, robustness, and performance of the crew solution. In addition to the savings in maintenance and operational costs, we demonstrate that for the industrial size testing schedule 1) adding crew base purity can avoid locked rotations in FAM solution and reduce CPU time by 10~100 times without significantly reducing the FAM profit; 2) adding fleet purity can improve the crew solution by 2~3% in pay-and-credit which amounts to crew scheduling cost reduction of 5~8 million dollars per year.

The following is an overview of the remainder of this paper. In Section 2, we investigate the impact of fleeting solutions on the subsequent planning and operational processes and explore opportunities for improving the fleeting solution. Extended station purity schemes including both crew base purity and fleet purity are proposed in Section 3. We study the impact of station purity on FAM profit, computational efficiency, robustness and crew planning in Section 4. Imposing station purity, an integrated fleet and crew robust planning model is proposed in Section 5. Solution algorithm approach and computational results are presented in Section 6. Finally, we summarize our work in Section 7.

2 Airline Fleet Assignment Modeling

Given a predefined flight schedule, the fleet assignment problem determines which aircraft fleet type is assigned to a given flight segment to maximize profit. The calculation of profit is based on estimates of operating cost and revenue for each possible fleet/flight assignment. In this section, we introduce the basic FAM formulation (Section 2.1) and investigate the impact of fleeting solutions on the subsequent planning and operational processes (Section 2.2) for the sake of exploring opportunities to improve the fleeting model and solutions. In Section 2.3, the robust FAM model and the concept of fleet purity proposed in (Smith and Johnson 2006) are briefly introduced, which provide a basis for the work in this paper.

2.1 Basic FAM model

The basic FAM formulation maximizes operating profit: revenue minus operating cost. The fleet assignment model can be classified as a large multicommodity flow problem with side constraints. A time line network is defined for each station and fleet type combination. Ground arcs are used to track the number of planes on the ground. There are two sets of variables: flight-fleet assignment variables and ground arc variables. The three sets of constraints are:

Cover constraints – every flight must be assigned to one and only one fleet type;

Balance constraints – ensure flow conservation in the time line network;

Plane count constraints – the total number of planes in the air and on the ground can not exceed the available fleet size.

Thus, the basic FAM is formulated as:

Maximize:
$$\sum_{f \in F} \sum_{i \in L} (R_{f,i} - C_{f,i}) x_{f,i} \tag{1}$$

Subject to:
$$\sum_{f \in F} x_{f,i} = 1, \forall i \in L \tag{2}$$

$$y_{f,s,t^-} + \sum_{i \in I(f,s,t), i \in L} x_{f,i} - y_{f,s,t^+} - \sum_{i \in O(f,s,t), i \in L} x_{f,i} = 0, \forall f, s, t \tag{3}$$

$$\sum_{s \in S} y_{f,s,t_m} + \sum_{i \in CL(f)} x_{f,i} \le N_f, \forall f \in F \tag{4}$$

$$x_{f,i} \in \{0,1\}, \forall f \in F, \forall i \in L \tag{5}$$

$$y_{f,s,t} \geq 0, \forall f, s, t \tag{6}$$

The definitions of the sets appeared in the formulation are:

S:	Set of stations, indexed by s.
F:	Set of fleet types, indexed by f.
L:	Set of flight legs, indexed by i.
CL(f):	Set of flight legs crossing the counting line flown by fleet f.
I(f,s,t):	Set of flight legs inbound to {f,s,t}.
O(f,s,t):	Set of flight legs outbound from {f,s,t}.

The decision variables are:

$$x_{f,i} = \begin{cases} 1, \text{if leg i} \in L \text{ is assigned fleet f} . \\ 0, \text{otherwise.} \end{cases}$$

y_{f,s,t^-}:	The number of aircraft on the ground for fleet type f, at station s, on the ground arc just prior to time t.
y_{f,s,t^+}:	The number of aircraft on the ground for fleet type f, at station s, on the ground arc just following time t.

The parameters defined in the model include:

$R_{f,i}$:	Revenue for flight leg i if it is assigned fleet type f.
$C_{f,i}$:	Cost for flight leg i if it is assigned fleet type f.
N_f :	The number of aircraft available of fleet type f.

This basic FAM model is described thoroughly in Hane et al. (1995).

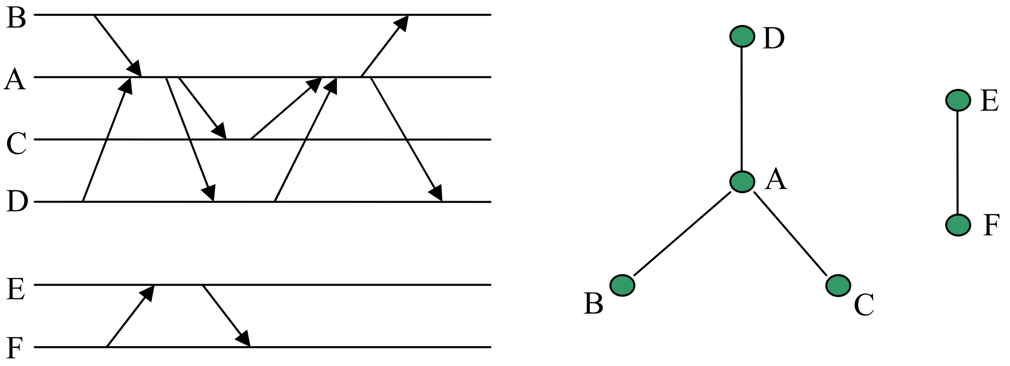
2.2 Impact of fleeting solutions on subsequent processes

FAM solutions ignoring the subsequent processes could easily lose its savings in fleet assignment to excessive crew cost and high operational cost. Moreover, FAM solutions that do not satisfy maintenance constraints or FAM solutions that cause aircraft locked rotations are unacceptable to the airline. The goal of our work is to provide modeling devices that incorporate aspects of aircraft routing, maintenance, crew scheduling and operational issues into the fleet assignment model while retaining its computational efficiency. Thus, it is necessary to analyze the impact of fleet solutions on the subsequent processes.

2.2.1 Aircraft routing

Airlines must meet maintenance requirements in making aircraft rotations. In order to ensure air travel safety, the Federal Aviation Administration (FAA) requires airlines to perform four types of aircraft maintenance called A-, B-, C-, and D-checks. Airlines have their self-imposed inspection and maintenance requirements which are more stringent than FAA requirements. The four types of maintenance vary in scope, duration of time, and frequency. Among them, C-, and D-check requirements do not affect daily scheduling and routing since the aircraft is taken out of the schedule. A-checks involve a routine visual inspection of all major systems which require 4 hours. It is common for airlines to perform A-checks in every 35 hours of flying. B-checks include a thorough visual inspection plus lubrication of all moving parts and are performed every 300 to 600 flying hours. The B-checks require a 10 to 15 hour long stay at the maintenance hangar. (Clarke et al. 1996) incorporates B-check requirements into the fleet assignment problem, and (Clarke et al. 1997) incorporates A-check requirements into the aircraft rotation problem.

Aircraft rotation is the routing of individual airplanes, which is determined after fleet assignment. Typically, the airlines require that each aircraft in a fleet fly an identical route consisting of all the legs assigned to that fleet, which implies that the route must be a cycle. When the flights covered by aircraft are separate from the flights covered by another aircraft in the fleet, it is called a "broken rotation." A "locked rotation" is a broken rotation where the separate sequences do not have any locations in common (Clarke et al. 1997). Locked rotations are unacceptable to almost all airlines. To fix a locked rotation requires a new solution from fleet assignment. In Section 4.2 of this paper, we will show adding crew base purity can help avoid locked rotations caused by FAM solution. Figure 1 illustrates a flight network and its adjacency graph flown by a certain fleet type. The nodes in the adjacency graph represent stations, the arcs represent that there is at least one flight between the connecting two stations. It is apparent that the existence of unconnected sub-graphs in the adjacency graph results in locked rotations. The rotations that cover the flights between stations A-B, A-C, and A-D can not have any locations in common with the rotations that cover the flights between stations E and F. The locked rotations can also cause crew scheduling problem infeasible unless crews are deadheaded to the separated network which implies extra crew flying time and cost.



(a) time-space flight network (b) adjacency graph

Figure 1: Locked rotations caused by unconnected adjacency graph

2.2.2 Crew scheduling

The crew scheduling problem is to find a minimum cost assignment of flight crews to a given flight schedule. The flight schedule considered includes all flight legs assigned to a crew compatible fleet. The crew scheduling problem is typically broken into two sequentially solved subproblems, the crew pairing problem and the crew rostering problem. The crew pairing problem generates minimum cost pairings that cover the collection of flight legs in the schedule, while the crew rostering problem combines the pairings into month-long crew schedules and assigns the month-long plans to individual crew members.

A crew pairing is a sequence of flights that starts at a crew base and ends at that same crew base. It can span several days in which crew members will rest, usually overnight, at some location other than where they reside. The periods between rests in a pairing are called duties. Throughout this paper, we call the two subsequent flights within a duty as **crew day connection**, and the two subsequent flights connecting one duty with another via an overnight rest as **crew night connection**. The cost structure of duties and pairings are defined in Equations (7) and (8) respectively. Usually the overall quality of a crew scheduling solution is evaluated by pay and credit of the crew pairing solution, which is the excess cost of crew beyond the required flying hours, see Equation (9).

$$\text{duty cost} = \max\{\sum blocktime, \text{ coeff1*elapsed time, min duty guarantee}\}. \tag{7}$$

$$\text{pairing cost} = \max\{\sum duty\cos t, \text{ coeff2*TAFB, \#duties*average duty guarantee}\}. \tag{8}$$

$$\text{pay\&credit} = \frac{\sum pairing\cos t - total\ blocktime}{total\ blocktime} \tag{9}$$

Anbil et al. (1992) discuss three main causes of pay and credit for pairings: (1) long or frequent sits within a duty; (2) long overnight rests between duties, and (3) “deadheading.” Long overnight rests and long sits result in high TAFB (Time-Away-From-Base) penalty which is the second item in the pairing cost. Figure 2 illustrates an example of long overnight caused by fleet assignment. A **lonely double overnight** occurs when a crew arrives late at night at a station that is not its base, and the aircraft it arrived on leaves before the crew has had sufficient rest and there is no other departure of that fleet that day (Clarke et al. 1996). In the case shown in Figure 2, the crew arriving via leg L_1 at 11pm cannot leave via leg L_2 until after a 31 hour layover. However, if a pair of midday flights (L_3 and L_4) are also assigned to the same fleet, the crew arriving via leg L_1 can leave in the middle of the next day via L_4 and the crew arriving via leg L_3 can leave the next morning via L_2. This option is called **midday breakouts**.

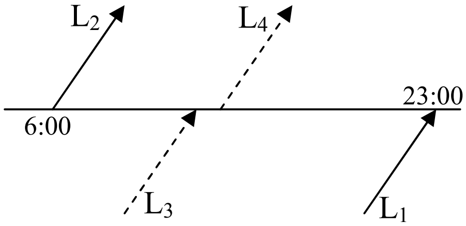


Figure 2: Lonely double overnights caused by fleet assignment

In (Gao and Johnson 2006), we propose schedule analysis methods for crew friendliness of a given schedule when the third items dominate the pairing cost and demonstrate that an unbalanced fleet assignment could cause more duties required in the pairing solution for certain fleets and result in high pay and credit because of the cost incurred by average duty guarantee.

2.2.3 Operational recovery

Airline operations are frequently disrupted by inclement weather, mechanical problems, and air traffic control and ground delay programs of FAA. FAM solution can significantly affect the time and cost required to return to the planned operations since aircraft and crew recovery are all predetermined by fleet assignment. (Ageeva 2000) modify the fleet assignment problem to reward opportunities for swapping planes. Such opportunities can help to localize the impact of a disruption caused by an unavailable aircraft. (Rosenberger et al. 2004) develop a FAM formulation to increase operational robustness by reducing hub connectivity. They show through simulation that decreased hub connectivity results in fewer cancellations and delays. (Smith and Johnson 2006) develop fleet assignment solutions that increase planning flexibility and reduce cost by imposing fleet purity therefore limiting the number of fleet types allowed to serve each airport in the schedule. Imposing fleet purity on the fleet assignment model can limit aircraft dispersion in the network and make solutions more robust relative to crew planning, maintenance planning and operations. In this paper, we further extend the notion of station purity to include both crew base purity and fleet purity. In the following subsection, the robust FAM model and the concept of fleet purity in (Smith and Johnson 2006) will be briefly introduced as a basis of our work.

2.3 Robust fleet assignment imposing station purity

(Smith and Johnson 2006) address the crew, maintenance, and operational issues simultaneously through station purity. Station purity ensures that the number of fleet types serving a given station does not exceed a specified limit. This limit on the number of fleets serving a station is the station's purity level. The purity level can be defined on crew-compatible fleet families which ensures opportunities to swap aircraft or crews for either operational or profitability reasons. To implement fleet purity, an auxiliary variable $w_{f,s}$ is defined to indicate whether fleet or fleet family f serves station s in the FAM solution. The following new constraints are added to the basic FAM formulation:

$$w_{f,s} \geq I_{s,i} x_{f,i}, \forall i \in L, f \in F, s \in S \tag{10}$$

$$\sum_{f \in F} w_{f,s} \leq SP_s, \forall s \in S \tag{11}$$

$$w_{f,s} \in \{0,1\}, \forall f \in F, s \in S \tag{12}$$

$$I_{s,i} = \begin{cases} 1, \text{if flight i serves station s.} \\ 0, \text{otherwise.} \end{cases} \tag{13}$$

The parameter SP_s defines the fleet/family purity level.

(Smith and Johnson 2006) demonstrate that imposing station purity on the fleet assignment model can limit aircraft dispersion in the network and make solutions more robust relative to crew planning, maintenance planning and operations. It is also found that station purity can significantly degrade computational efficiency. A column generation solution approach, station decomposition, and a primal-dual method are developed to improve solution quality and computational efficiency. It is estimated that the annual net benefit of station purity because of reduced maintenance and crew scheduling costs is greater than \$100 million for a major U.S. domestic airline.

3 Extended Station Purity

A move-up crew is not only required to operate a compatible fleet family, but also required to come from the same crew base. To further improve robustness in crew scheduling, also to facilitate conducting integrated planning, we extend the notion of station purity to crew base purity. Crew base purity implies that the number of crew bases serving each station is limited. The nature of the hub-and-spoke network ensures that crew base purity is rational and feasible. In fact, most spoke stations are only connected to a few crew bases nearby. It is not recommended to send crews from crew bases far away to visit the spoke stations. The benefit is quick recovery from disruptions by finding a move-up crew or deadheading crew home fast. In this section, we first define crew base purity and discuss how to implement crew base purity. Then we show via an example how to define extended station purity scenarios by combining fleet purity and crew base purity.

3.1 Crew base purity

Crew base purity is determined by the adjacency graph of the flight network. In a hub-and- spoke network, most spoke stations are only connected to a few crew bases nearby. We define **naturally pure spoke** and **mixed spoke** depending on if a spoke is connected to only one crew base or more. For a naturally pure spoke, its crew base purity level is restricted as 1, and the specific crew base which it connects to is assigned to this spoke. For a mixed spoke, we usually need to assign all the connected crew bases to it. Otherwise, we may possibly lose feasible crew scheduling solutions. For a crew base station, we allow other crew bases within distances of 2 in the adjacency graph (i.e., they are connected through non-stop or 1-stop flights) to have day visits at this station.

In addition to the crew base purity level defined on stations, we can further restrict the crew base assignment to flights at mixed spokes. **Leg pure** is one of the strategies which means if a flight segment connects a spoke with a crew base, we assign this fixed crew base to this flight. If a schedule does not have crewbase-to-crewbase or spoke-to-spoke flights, a complete implementation of leg pure plan implies that the schedule is divided into separated subsets and different crew bases will be assigned to the corresponding subsets of the schedule. Leg pure plan provides a way to simplify the crew scheduling problem.

3.2 Experimental schedule

A flight schedule with an industrial problem size is used to test the impact of station purity as well as the integrated fleet and crew planning model. This flight schedule is based on a daily schedule of a major U.S. domestic carrier. The scenario flight network is constructed by deleting parts of the schedule that are quite isolated from the rest of the network. Consequently, there are 1388 daily flights and six crew bases in this schedule. The fleet consists of 3 fleet families JET-1, JET-2, and TURBO. Table 1 shows the size of the fleets. JET-1 is the largest fleet and includes three crew-compatible sub-fleets. One of the main characteristics of this flight network is that the fleets are divided up among crew bases, resulting in 15 different fleet types in the problem. It is noted that quite a few regional airlines or commuter airlines operate with this characteristic, i.e., fleets are divided up among hubs or crew bases and crews serve the aircrafts from their own base.

Table 1: Fleet family and size

Fleet family	Size
JET-1	165
JET-2	24
TURBO	12

In our tests, large spokes are defined as having more than 40 daily operations, medium spokes as having from 21 to 40 daily operations, and small spokes as having less than 20 daily operations. Based on these criteria, 98 out of the total 100 spokes are small and medium spokes. The number of naturally pure spokes is 66 and most of the mixed spokes only connect to two crew bases. Table 2 lists the statistics of the experimental flight network, and Figure 3 shows the adjacency graph of the flight network from the experimental schedule.

Table 2: Statistics of the experimental flight network

	Cities	106
	Flights	1,388
	Fleet types	15
	Fleet families	3
	Crew bases	6
	Large spokes	2
	Medium spokes	13
	Small spokes	85
	Naturally pure spokes	66
Mixed spokes	Connect to 2 crew bases	27
	Connect to 3 or more crew bases	7

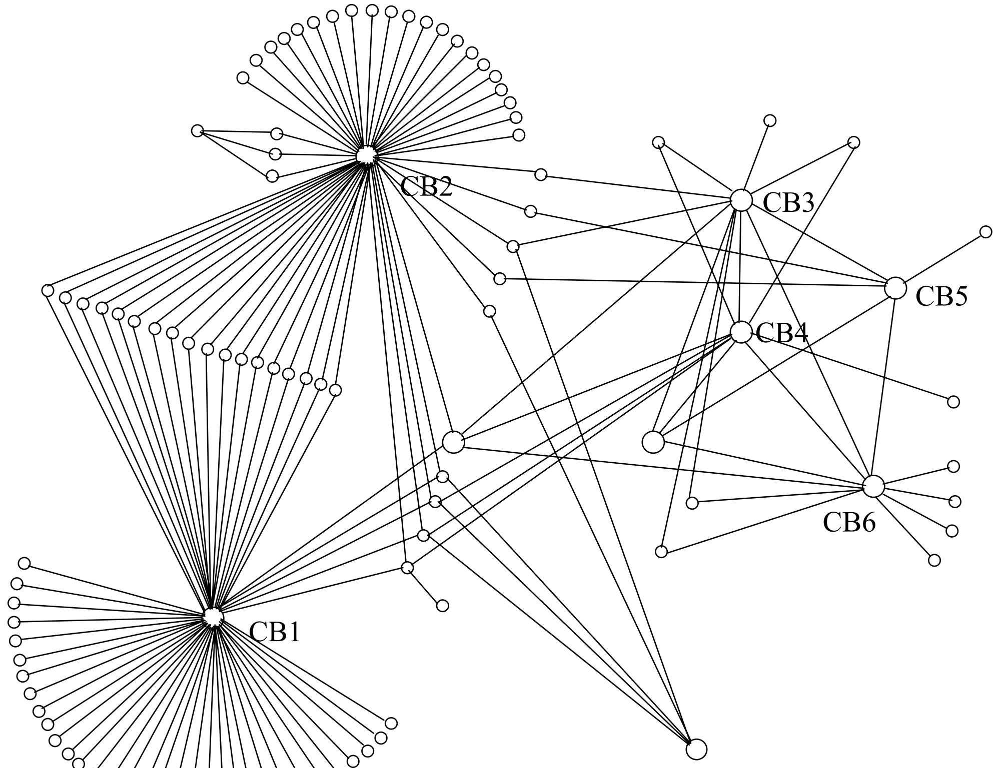


Figure 3: Adjacency graph of the experimental flight network

3.3 Station purity scenarios

The extended station purity proposed in this paper includes both fleet purity and crew base purity. Nine station purity scenarios are designed to study the impact of station purities. Fleet purity is related to both the size of the station and the crew base purity. For fleet purity, three cases are considered. In the first case, no purity is imposed on fleet type or fleet family as in the basic FAM formulation. The second case is called "Family purity" in which (1) for crew bases or large spokes, instead of imposing hard constraints on station fleet/family purity level, penalties on fleet family and station combinations are defined in the objective function; (2) for small or medium sized spokes, hard constraints on fleet family purity at these stations are defined to be 1. The third case is called "Fleet purity" in which (1) for crew bases or large spokes, penalties on fleet type (instead of fleet family) and station combinations are defined in the objective function; (2) for a small or medium sized spoke, depending on whether it is a naturally pure spoke or mixed spoke, hard constraint on fleet type purity or fleet family purity is defined to be 1 respectively at this station.

Similarly, for crew base purity, three cases are considered. In the first case, no crew base purity is imposed. The second case is called “CB purity 1” in which crew base purity level as introduced in Section 3.1 is imposed on all naturally pure spokes, mixed spokes, and crew bases. Besides, leg pure plan is imposed on those mixed spokes connecting to more than two crew bases, and a bonus is defined in the objective function to encourage leg pure plan at other stations. The third case is referred to “CB purity 2” which is based on the second case. The only difference is that in the third case, leg pure plan is also imposed on the mixed spokes between crew bases CB1 and CB2.

Consequently, the combination of different settings of fleet purity and crew base purity defines nine station purity scenarios. In the next section, we examine the impact of the extended station purity on FAM profit, computational efficiency, robustness, and crew scheduling using the experimental schedule.

4 Impact of the Extended Station Purity

The impact of the extended station purity is tested using the robust FAM model introduced in Section 2.3. The crew base purity is implemented by restricting the legal flight-fleet assignments. In the experimental schedule, fleets are distributed among crew bases, and crews are encouraged to fly their own fleets. Therefore, crew base purity implies that some flight-fleet assignments are disabled. For instance, the flights connecting to a naturally pure spoke of one crew base should not be assigned fleet types distributed at other crew bases. Our proposed method can also be applied to general problems without the characteristic that fleets are divided up among the crew bases, as long as the plane count is restricted on the real fleet types instead of the fleets distributed at the crew bases. To implement fleet purity, the two constraints defined in Equations (10) and (11) are included in the robust FAM model. In this section, the impact of the extended station purity on FAM profit, computational efficiency, robustness, and crew scheduling is investigated using the experimental schedule.

4.1 Impact on FAM profit and computational time

Table 3 gives the problem sizes of the robust FAM model under different station purity scenarios. It is shown that adding crew base purity can reduce the size of the problem and adding “Fleet purity” has a larger problem size than adding “Family purity”.

Table 4 shows the robust FAM results for different station purity scenarios. By adding “Family purity”, FAM profit decreases by 2.9% on average while CPU time increases by 300 times on average. By adding “Fleet purity”, FAM profit decreases by 3.6% on average while CPU time increases by 900 times on average. However, by adding crew base purity, FAM profit only decreases by 0.6% on average. The most notable benefits of adding crew base purity is the improvement of computational efficiency. By adding “CB purity 1”, the CPU time is reduced by 16 times on average. By adding “CB purity 2”, the CPU time is reduced by 10 ~ 100 times. Figure 4 and Figure 5 clearly illustrate the impact of station purity on CPU time as well as on FAM profit.

Table 3: Problem sizes

	No CB purity				CB purity 1				CB purity 2			
	No	Combo*	Rows	Cols	No	Combo	Rows	Cols	No	Combo	Rows	Cols
No fleet purity	1	9,026	7,589	15,212	4	6,684	6,352	11,576	7	5,530	5,644	9,732
Family purity	2	9,026	25,741	16,803	5	6,684	19,827	13,184	8	5,530	16,804	11,322
Fleet purity	3	9,026	43,793	17,473	6	6,684	33,195	13,851	9	5,530	27,864	11,988

*Combo refers to legal flight-fleet assignments

Table 4: Robust FAM results

	No CB purity			CB purity 1			CB purity 2		
	No.	Time*	Profit**	No.	Time	Profit	No.	Time	Profit
No fleet purity	1	44.3	6.54	4	3.3	6.48	7	4.03	6.476
Family purity	2	15,463	6.33	5	548	6.30	8	1,683.6	6.30
Fleet purity	3	38,756	6.29	6	6,573	6.26	9	232.2	6.25

* in CPU seconds

** in million dollars

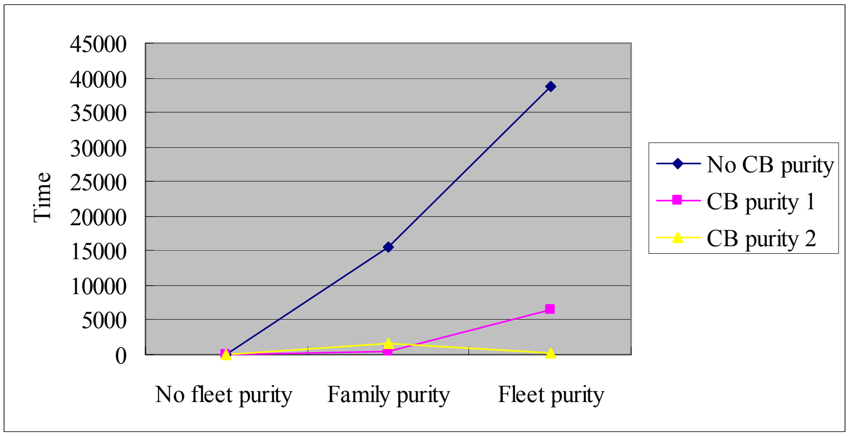


Figure 4: Impact of station purity on runtime

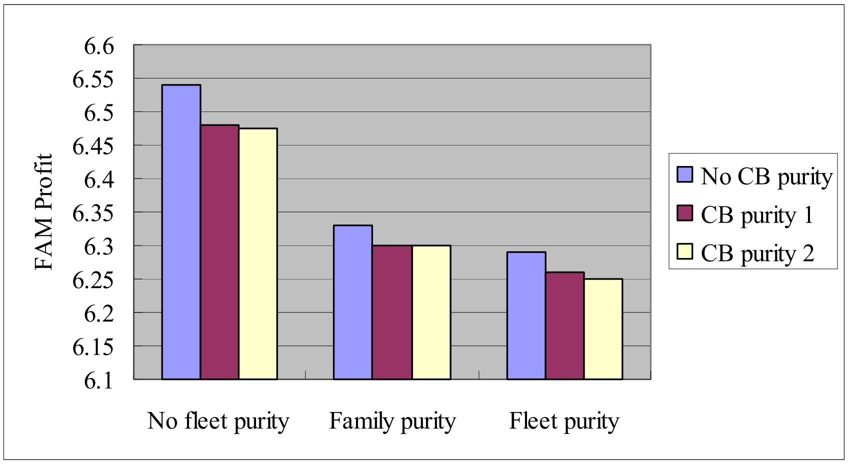


Figure 5: Impact of station purity on FAM profit

In this problem, there is one large fleet family and two small fleet families. The reason for FAM profit loss is that by setting the fleet/family purity level as 1, only a small number of stations can be served by the two small fleet families. The fleet/family purity constraints can be relaxed by setting the fleet/family purity level as 2. Further computational experiments show that FAM profit only decreases by 0.09% with family purity level set as 2 for the cases of "Family purity"; and FAM profit decreases by 0.13% with family and fleet type purity level set as 2 for the cases of "Fleet purity". In practice, compromise would be necessary for balancing the benefits of robustness and the loss of FAM profit by adjusting the fleet purity level.

4.2 Impact on crew scheduling and robustness

Taking the FAM solution from the robust FAM model, crew scheduling problems are solved for each fleet. It is found that without imposing crew base purity, the FAM solution tends to produce locked rotations, which means that the flights covered by some aircrafts are separate from the flights covered by other aircrafts in the fleet. Locked rotations are unacceptable to most airlines. The crew scheduling problem for a FAM solution with locked rotations is not feasible. Deadheads are required to lead crews to those separate routes. Adding crew base purity is an effective way to avoid the locked rotations since it encourages crew and fleet not to fly far from its base station. The definition of crew base purity on naturally pure spokes and mixed spokes, as well as the hard or soft restrictions on leg pure plan, can altogether limit the stations that a fleet can visit within distances of 2 to its base station in the adjacency graph. This means the stations that a fleet can visit are connected by a non-stop or 1-stop itinerary to its base station. The resulting sub-network for each fleet tends to be connected.

The crew scheduling problems are solved for all three scenarios of "CB purity 1." Table 5 shows crew solution performance under different fleet purity in terms of actual pairing cost in minutes, excess cost in Pay & credit, total number of duties in the pairing solution, and the maximum sit time needed to produce a feasible crew solution. In order to examine the performance of crew scheduling, we did not add a limit to the maximum sit time when solving the crew pairing problem. The purpose is to make the crew pairing problem feasible without using deadhead. The objective function of the crew pairing model could enforce the solution looking for small sit time crew connections. The maximum sit time existed in the pairing solution in turn demonstrates the solution quality and computational complexity of the crew pairing problem, since long sit time implies high Pay & credit, and larger size of the crew pairing model. Table 5 shows that Solution 6 (Fleet purity) has the best crew results. Solution 5 (Family purity) has much better crew results than Solution 4, but is not as good as Solution 6.

Table 5: Crew solution results

	Solution No.	Pairing cost*	Pay & credit	Number of duties	Maximum Sit time*
No fleet purity	4	130,038.45	4.95%	378	506
Family purity	5	127,413.8	2.83%	394	293
Fleet purity	6	126,653.35	2.22%	392	250

* in minutes

Table 6 compares the number of crew double overnights influenced by different fleet purities. By adding fleet purity, the total number of fleet(family)-station combinations and family-station singletons (the number of family-station combinations with only one arrival and departure for a family) are reduced. Therefore, the crew lonely double overnights can also be reduced which has a significant positive effect on the crew performance. In fact, crew lonely double overnight is influenced by both family-station singletons and crewbase-station singletons (the number of crewbase-station combinations with only one arrival and departure served a crew base). Analysis is conducted on the station activities of the original schedule and it is found that the original schedule includes 3 double overnight rests intrinsically. Table

6 shows that Solution 6 (Fleet purity) has the least number of double overnights. Solution 5 (Family purity) has three more double overnights than Solution 6. Adding fleet purity, Family purity or Fleet purity, can greatly reduce the existence of crew double overnights, which is one of the major reasons that cause high crew cost. Note that Solution 6 has better crew performance than Solution 5. This is because, in the case of "Family purity", there are more chances for lonely crew base visits than in the case of "Fleet purity", which may cause more crew double overnight rests. The lonely crew base visits at stations are not preferable in the operational process.

Table 6: Impact of fleet purity on crew lonely double overnights

	Sol No.	Family station pairs	Fleet Station pairs	Family-station (FS) singletons	Crew lonely double ovn caused by FS singletons	Crewbase-station singletons	Double ovn in crew solution
No fleet purity	4	160	308	33	9	15	21
Family purity	5	110	241	9	0	16	9
Fleet purity	6	111	184	9	0	12	6

Robustness is improved when each fleet and crew base serving a station operates at relatively high frequency into and out of that station. The number of Family-station pairs, Fleet-station pairs, Family-station singletons, and Crewbase-station singletons in Table 6 provide a measure of robustness. Table 6 shows that imposing station purity can greatly reduce the lonely fleets and lonely crew bases serving a station and also create more opportunities for aircraft and crew swapping to cover operational disruptions.

To summarize, the impact and value of station purity are as follows.

(1) Adding crew base purity can avoid locked rotations in FAM solution. It will not degrade the FAM profit significantly. Most importantly, the CPU time needed to solve the robust FAM model can be reduced by 10~100 times.

(2) By adding fleet/family purity, the crew solution is improved by 2~3% in pay-and-credit which gives a saving of crew scheduling cost by up to 5~8 million dollars per year.

The value of purity also includes the savings of $500,000 per year for each family-station pair (Smith and Johnson 2006) and the savings in aircraft maintenance cost and operational costs.

5 Integrated Fleet and Crew Robust Planning Model

Imposing station purity to FAM formulation can improve crew scheduling because FAM solutions with fleets serving a smaller number of stations with greater frequency provide more flexibility for crew assignment to reduce crew cost. In the computational experiments conducted in Section 4, the best crew solution (Solution 6) obtained for the experimental schedule has six double overnights and a maximum sit time of 250 minutes. In addition to the three intrinsic double overnights of the schedule, three more double overnights were caused by fleet assignment. There were also some long sit times in the crew solution. The maximum sit time is preferably less than 180 minutes. These findings suggest that further research on integrated fleet and crew planning is necessary.

Addressing the crew scheduling problem in the integrated model is perhaps the most challenging task in integrated planning. Crew scheduling for separate fleets is already computationally expensive. In an integrated model, the crew problem becomes much harder to solve since it is necessary to work on the complete schedule. In the experimental schedule, the number of duties enumerated on the complete schedule exceeds two millions. It is extremely computationally expensive to model pairings or duties explicitly in the integrated model. Therefore, we integrate fleet assignment with crew connections in our integrated planning model.

Crew connections, instead of explicit pairings or duties, are modeled to represent the routings of crew for the sake of retaining a computational tractable integrated model. Crew lonely double overnights and day connections with long sit time are the major reasons that cause poor crew solution. Adding crew connection factors in the fleet assignment model can help produce crew-friendly FAM solutions. Day and night connections are enumerated a priori based on the minimum and maximum sit or rest time, which constitute the connection variables. At spokes, a flight has either a day connection or a night connection, while at crew bases, a flight can either be the beginning/ending of a pairing or have day connections. The solution of this model provides fleet assignment solution and a pseudo crew pairing solution. Based on crew base and fleet division provided by the solution, legal pairings can be obtained by solving the decomposed sub-problems.

This model maintains the characteristics and benefits of station purity. Different from the basic and robust FAM models, in this integrated model, each flight is assigned both a fleet type and a crew base. As to connection variables, the two flights that constitute a connection should be assigned with the same crew base and crew-compatible fleet types since they represent how crew rotations are formed. Next we introduce the mathematical formulation of this integrated fleet and crew robust planning model which will be called in brief "the integrated model" hereafter in this paper.

5.1 Sets

L: Set of legs in the schedule, indexed by i.

S: Set of stations, indexed by s.

CB: Set of stations that are crew bases.

F: Set of fleet types, indexed by f.

CF: Set of compatible crew base and fleet pairs, indexed by (c,f).

$CL(f)$: Set of flight legs crossing the counting line flown by fleet f.

$I(f,s,t)$: Set of flight legs inbound to {f,s,t}.

$O(f,s,t)$: Set of flight legs outbound from {f,s,t}.

5.2 Decision variables

$$x_i^{(c,f)} = \begin{cases} 1, \text{if leg i} \in L \text{ is assigned fleet f and crew base c.} \\ 0, \text{otherwise.} \end{cases}$$

y_{f,s,t^-}: The number of aircraft on the ground for fleet type f, at station s, on the ground arc just prior to time t.

y_{f,s,t^+}: The number of aircraft on the ground for fleet type f, at station s, on the ground arc just following time t.

$$w_{f,s} = \begin{cases} 1, \text{if fleet f} \in \text{F serves station s} \in \text{S in the solution.} \\ 0, \text{otherwise.} \end{cases}$$

fs: Total number of fleet-station combinations in the solution.

$$x_{day(j,i)}^{(c,f_j,f_i)} = \begin{cases} 1, \text{if day connection (leg j, leg i) is assigned crew base c, fleet type } f_j,f_i. \\ 0, \text{otherwise.} \end{cases},$$

$$x_{nite(j,i)}^{(c,f_j,f_i)} = \begin{cases} 1, \text{if night connection (leg j, leg i) is assigned crew base c, fleet type } f_j,f_i. \\ 0, \text{otherwise.} \end{cases}$$

$$x_{start,i}^{(c)} = \begin{cases} 1, \text{if departure leg i at crew base c is the begining of a pairing.} \\ 0, \text{otherwise.} \end{cases}$$

$$x_{end,i}^{(c)} = \begin{cases} 1, \text{if arrival leg i at crew base c is the end of a pairing.} \\ 0, \text{otherwise.} \end{cases}$$

5.3 Parameters and data

$R_{f,i}$: Revenue for flight leg i if it is assigned fleet type f.

$C_{f,i}$: Cost for flight leg i if it is assigned fleet type f.

$B_{f,i}$: Bonus for flight leg i if it is assigned fleet type f.

$Ctim_{day(i,j)}$: Connection time for day connection day(i,j), in minutes.

$Ctim_{nite(i,j)}$: Connection time for night connection nite(i,j), in minutes..

N_f : The number of aircraft available of fleet type f.

$I_{s,i}$ $= \begin{cases} 1, \text{if flight i serves station s.} \\ 0, \text{otherwise.} \end{cases}$

SP_s : Fleet purity level at station s.

5.4 Formulation

Maximize:

$$\sum_{(c,f)}\sum_{i\in L}(R_{f,i} - C_{f,i} + B_{f,i})x_i^{(c,f)} - \sum_{day(i,j)} Ctim_{day(i,j)} x_{day(i,j)}^{(c,f_i,f_j)} - \sum_{nite(i,j)} Ctim_{nite(i,j)} x_{nite(i,j)}^{(c,f_i,f_j)} - P*fs \tag{14}$$

Subject to:

$$\sum_{(c,f)} x_i^{(c,f)} = 1, \forall i \in L \tag{15}$$

$$y_{f,s,t^-} + \sum_{(c,f)\ni f}\ \sum_{i\in I(f,s,t), i\in L} x_i^{(c,f)} - y_{f,s,t^+} - \sum_{(c,f)\ni f}\ \sum_{i\in O(f,s,t), i\in L} x_i^{(c,f)} = 0, \forall f, s, t \tag{16}$$

$$\sum_{s\in S} y_{f,s,t_m} + \sum_{(c,f)\ni f}\ \sum_{i\in CL(f), i\in L} x_i^{(c,f)} \le N_f, \forall f \in F \tag{17}$$

$$w_{f,s} \ge I_{s,i} x_i^{(c,f)}, \forall i \in L, \forall (c,f) \tag{18}$$

$$\sum_{f\in F} w_{f,s} \le SP_s, \forall s \in S \tag{19}$$

$$fs = \sum_{f\in F}\sum_{s\in S} w_{f,s} \tag{20}$$

$$x_i^{(c,f)} = \sum_j\sum_{f_j} x_{day(i,j)}^{(c,f,f_j)} + \sum_j\sum_{f_j} x_{nite(i,j)}^{(c,f,f_j)}, \forall (c,f), \forall \text{ incoming } \ i \in L \text{ at station } \ s \notin CB \tag{21}$$

$$x_i^{(c,f)} = \sum_j\sum_{f_j} x_{day(j,i)}^{(c,f_j,f)} + \sum_j\sum_{f_j} x_{nite(j,i)}^{(c,f_j,f)}, \forall (c,f), \forall \text{ outgoing } \ i \in L \text{ at station } \ s \notin CB \tag{22}$$

$$x_i^{(c,f)} = \sum_j\sum_{f_j} x_{day(i,j)}^{(c,f,f_j)} + x_{end,i}^{(c)}, \forall (c,f), \forall \text{ incoming } \ i \in L \text{ at crewbase c} \tag{23}$$

$$x_i^{(c,f)} = \sum_j\sum_{f_j} x_{day(j,i)}^{(c,f_j,f)} + x_{start,i}^{(c)}, \forall (c,f), \forall \text{ outgoing } \ i \in L \text{ at crewbase c} \tag{24}$$

$$x_i^{(c,f)} = \sum_j\sum_{f_j} x_{day(i,j)}^{(c,f,f_j)}, \forall (c,f), \forall \text{ incoming } \ i \in L \ \text{ at station } \ s \in CB \text{, but } \ s \ne c \tag{25}$$

$$x_i^{(c,f)} = \sum_j\sum_{f_j} x_{day(j,i)}^{(c,f_j,f)}, \forall (c,f), \forall \text{ outgoing } \ i \in L \ \text{ at station } \ s \in CB \text{, but } \ s \ne c \tag{26}$$

$$x_i^{(c,f)} \in \{0,1\}, \forall (c,f), \forall i \in L \tag{27}$$

$$y_{f,s,t} \geq 0, \forall f, s, t \tag{28}$$

$$w_{f,s} \in \{0,1\}, \forall f \in F, s \in S \tag{29}$$

$$fs \geq 0 \tag{30}$$

$$x_{day(j,i)}^{(c,f_j,f)}, x_{nite(j,i)}^{(c,f_j,f)}, x_{start,i}^{(c)}, x_{end,i}^{(c)} \in \{0,1\} \tag{31}$$

The objective function includes four items: FAM profit (revenue minus operating cost), bonus defined on leg pure plan, costs incurred by crew connection time, and penalty defined on fleet(family)-station pairs. The cost items on crew connection time in the objective function will make the solutions of this model avoid lonely double overnights and long sit time. The last item in the objective function is penalty defined on fleet(family)-station pairs. At crew base stations or large spokes, it is inappropriate to impose a small fleet purity level; however, a penalty on fleet(family)-station pairs in the objective function can encourage fleet purity as well.

Constraints (15) – (17), corresponding to those in the basic FAM formulation, are cover constraints, station fleet balance constraints, and plane count constraints, respectively. The only difference is that for each flight, not only a fleet type but also a crew base is assigned here. Constraints (18) – (20) are related to fleet purity, which are same as those defined in the robust FAM model (Smith and Johnson 2006). Equation (19) defines hard constraints on fleet/family purity. Variable *fs* indicates the total number of fleet/family station combinations in the solution. Constraints (21) – (26) are related to crew connections. Equations (21)-(22) are applied to stations that are not crew bases. Equation (21) implies that each incoming flight leg to such a station has either a day connection or a night connection. Equation (22) implies that each outgoing flight leg from such a station has either a day connection or a night connection. The constraints (23) to (26) are applied to stations that are crew bases. This means a departure flight at crew base c is either the beginning of a pairing or it has day connections.

Accordingly, an arrival flight at crew base c is either the end of a pairing or it has day connections. If crews from one crew base visit another crew base, they can only have day connections there.

6 Computational Results

6.1 Algorithm and implementation

In preprocessing, legal fleet and crew base assignments are generated for flights and connections based on the scenarios of station purities. For each station, a set of crew base stations is defined to identify which crew bases are allowed to serve this station. Each crew can only fly the fleets from their own base. Once the crew base purity for each station is determined, there are compatible pairs of fleets and crew bases to be assigned to flights. As for connections, by checking the arrival station and the departure station of a connection, only the crew bases in the intersection set of the two corresponding crew base sets are allowed to serve this connection. This scheme allows crew from one crew base to visit another crew base but then return to its own connected stations.

The computational experiments are conducted on a Pentium 4 processor (1.83GHz, 1.5G RAM) using ILOG CPLEX 9.0. The models are formulated in ILOG Concert 2.0. The linear programming (LP) relaxation problem is solved by dual steepest-edge simplex. In order to get integer solution, flight variables and connection variables with value ≥ 0.99 in the LP solution are fixed to 1. As a result, related connection variables with different fleet or crew base assignment can be fixed to 0. The resulting mixed integer programming (MIP) problem is then solved by CPLEX's branching and bound process.

6.2 Results

The integrated fleet assignment and crew connection model is applied to solve the experimental problem described in Section 3.2. The size of the MIP model and the performance of this method are summarized in Table 7. Four purity scenarios are tested. The indices of these scenarios are in subsequent order with the earlier scenarios introduced in Section 4.1. Scenario 10 has “Fleet purity” (same meaning as) defined in Section 3.3, and basic crew base purity including: 1) crew base purity imposed on all naturally pure spokes, mixed spokes, and crew bases; 2) leg pure plan on those mixed spokes connecting to more than two crew bases except the stations that will generate double overnights. Scenario 11 has “Fleet purity”, crew base purities as in scenario 10, and bonus defined in the objective function to encourage leg pure plan at other stations. Scenario 12 has “Fleet purity”, crew base purities as in scenario 11, and leg pure plan on parts of the mixed spokes between crew bases CB1 and CB2. Different from scenario 12, scenario 13 has “Family purity” instead of “Fleet purity”.

Table 7 shows that the CPU time to solve the integrated model is reduced dramatically as the purity scenarios improved. Scenario 13 can be solved in thirteen minutes, which is about 100 times faster than solving scenario 10. Moreover, compared to the results in Table 4, the FAM profit didn’t degrade because of adding crew connection variables and constraints. On the contrary, the CPLEX MIP gap demonstrates that the quality of the integer solution is improved.

Table 7: Results of the integrated model

No.	Fleet purity type	Crew base purity	Rows	Cols	Time**			Profit*	Gap***
					LP	IP	Total		
10	Fleet	base	45,571	195,494	826	74,977	75,803	6.28	2.93
11	Fleet	Bonus on leg pure	45,571	195,494	764	9,089	9,853	6.26	2.30
12	Fleet	Forced leg pure	42,274	183,323	491	1,811	2,302	6.29	1.50
13	Family	Forced leg pure	30,238	182,656	229	554	783	6.34	0.68

* FAM profit in million dollars

** in CPU seconds

***MIP gap (in percent) returned by CPLEX.

The decomposed crew scheduling problems are solved for scenario 12 and scenario 13 respectively. Table 8 shows crew solution performances, and Table 9 compares the number of crew double overnights influenced by different fleet purities. For both scenarios, there are no more double overnights besides the ones intrinsic to the original schedule. Moreover, the maximum sit time is 3 hours as required. "Fleet purity" as in scenario 12 gives better crew solution. In scenario 13, the "Family purity" case, there are more chances for lonely crew base visits. Although these lonely crew base visits are not lonely double overnight rests, it is not as flexible for crew scheduling or crew recovery as the "Fleet purity" case. However, the "Family purity" gives better FAM profit and has better computational efficiency.

Table 8: Crew solution of the integrated model

	Solution No.	Pairing cost	Pay & credit	Number of duties	Maximum Sit time
Fleet purity	12	126,370.35	1.99%	380	[20,180]
Family purity	13	126,772.75	2.114%	392	[20,180]

Table 9: Statistics of the integrated model

	Sol No.	Family station pairs	Fleet Station pairs	Family-station (FS) singletons	Crew lonely double ovn caused by FS singletons	Crewbase-station singletons	Double ovn in crew solution
Fleet purity	12	118	183	10	0	12	3
Family purity	13	118	243	10	0	16	3

6.3 Discussion

Besides the improvement of crew scheduling incurred by adding fleet purity, the integrated model further improves crew solution performance through modeling the crew connections. It maintains the robustness and computational efficiency by imposing appropriate fleet purity and crew base purity.

From the results in Section 4.2 and Section 6.2, we can see that leg pure plan is effective in improving computational efficiency. This is because the strict constraints or bonuses on leg plan help breaking the symmetry within fleet or crew base assignment. However, it must be noted that sometimes a complete leg pure plan is too restrictive to produce any feasible solution. This is especially true when crew base balance constraints are required, that is, the number of hours that crews located at some base spent away from their crew base must be within specified limits (Hoffman and Padberg 1993). Another situation is that at some stations, leg pure plan creates lonely double overnight. As shown in Figure 6, the alphabets in circles represent the arrival or departure stations (which are crew bases in this case) of the flights, leg pure plan implies crew connections of L_1-L_2 and L_3-L_4, in which L_1-L_2 leads to double overnight. However, the crew connections of L_1-L_4 and L_3-L_2 without imposing leg pure plan generate two legal short overnight rests, which is preferred. This example shows that leg pure plan should be applied appropriately in order to avoid generating crew lonely double overnights.

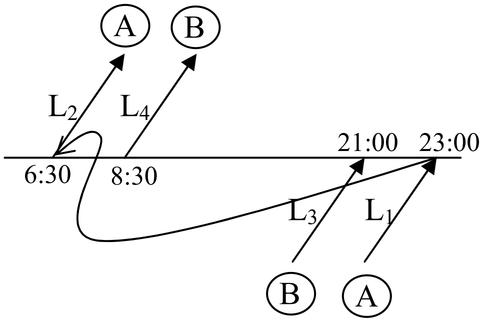


Figure 6: leg pure plan and crew lonely double overnight

Crew base purity and crew connection plan at spokes altogether make the subsequent crew scheduling problem much easier to solve. Shaw (2003) investigates the advantages of hub-to-hub strings in constructing pairings, which include reduction in the amount of storage space and in the time required to do pricing out. Crew connections at spokes are mostly hub-to-hub strings. Furthermore, they provide a feasible spoke plan for crew. Crew base purity further restricts the crew base assignment on crew connections. Hence, crew base purity and crew connection plan remove unfavorable options in crew scheduling so as to improve computational efficiency. When fleet purity is combined with crew base purity, the computational efficiency of the robust FAM model can also be improved. The primary objective of this paper is to obtain integrated and robust planning approach while retaining its computational efficiency through designing effective modeling mechanisms.

Future research direction includes integrating airline maintenance routing with the current integrated model. The interaction between aircraft routing and crew scheduling is due to crew short connect (Cohn and Barnhart 2003, Cordeau et al. 2001, 2005). That is, the minimum sit time for crew is longer than the minimum turn time for aircraft, but the connection time for crew can be less than its minimum sit time as long as crew follows the route of the same aircraft. In our model, issues of crew short connect can be modeled. It would be convenient for the current integrated model to represent crew short connects and identify crew short connects in the solution where same aircraft constraints should be imposed.

7 Summary

The integrated fleet and crew robust planning approach provides modeling devices that incorporate aspects of aircraft routing, maintenance, crew scheduling, and operational issues into the fleet assignment model while retaining its computational efficiency. We study station purity schemes including both fleet purity and crew base purity and investigate the impact of station purity on FAM profit, computational efficiency, robustness, and performance of the crew solution. We conduct computational experiments and demonstrate that, for the industrial size testing schedule, in addition to the savings in maintenance and operational costs: 1) adding crew base purity can avoid locked rotations in FAM solution and reduce CPU time by

10~100 times without significant reduction of the FAM profit; 2) adding fleet purity can improve the crew solution by 2~3% in pay-and-credit, which amounts to a reduction in crew scheduling cost of 5~8 million dollars per year. We developed an integrated fleet and crew robust planning approach which provide fleet assignment solutions both friendly to crew planning and robust to real time operations. Computational results show that the integrated model further improves crew solution performance and maintains the robustness and computational efficiency by imposing appropriate fleet purity and crew base purity.

8 Acknowledgments

We would like to thank the employees of American Airlines for their assistance. We also thank Dr. Michael Clarke from Sabre Holdings for valuable comments and discussions.

9 References

Abara, J., (1989). *Applying integer linear programming to the fleet assignment problem*. Interfaces 19**,** 20-28.

Ageeva, Y., (2000). *Approaches to incorporating robustness into airline scheduling*. PhD Dissertation, Massachusetts Institute of Technology.

Anbil, R., Tanga, R., Johnson, E. L., (1992). *A global approach to crew-pairing optimization*. IBM Systems Journal 31**,** 71-78.

Ball, M., Barnhart, C., Nemhauser, G.L., and Odoni, A., (2006). *Air transportation: irregular operations and control*. In: Transportation, Vol. 14 Handbooks of Operations Research and Management Science, Barnhart, C., and Laporte, G. (Eds.).

Barnhart, C., Johnson, E. L., Anbil, R., Hatay, L., (1994). *A column-generation technique for the long-haul crew-assignment problem*. In: Optimization in industry 2: Mathematical programming and modeling techniques in practice, Ciriani, T. A., Leachman, R. C. (Eds.), John Wiley & Sons Ltd, pp. 7-24.

Barnhart, C., Lu, F., Shenoi, R., (1998a). *Integrated airline schedule planning*. In: Operations Research in the Airline Industry, G. Yu (Eds.), Kruwer Academic Publishers, pp. 384-403.

Barnhart, C., Boland, N. L., Clarke, L. W., Johnson, E. L., Nemhauser, G. L., Shenoi, R. G., (1998b). *Flight string models for aircraft fleeting and routing*. Transportation Science 32**,** 208-220.

Barnhart, C., Cohn, A.M., Johnson, E.L., Klabjan, D., Nemhauser, G., and Vance, P.H., (2002a). *Crew scheduling*. In: Handbook of Transportation Science, 2nd Edition, R.W. Hall (Eds.), pp. 517-560.

Barnhart, C., Belobaba, P., Odoni, A. R., (2003). *Applications of operations research in the air transport industry*. Transportation Science 37**,** 368-391.

Barnhart, C., Cohn, A., (2004). *Airline schedule planning: accomplishments and opportunities*. Manufacturing and Service Operations Management 6**,** 3-22.

Butchers, E. R., Day, P. R., Goldie, A. P., Miller, S., Meyer, J. A., Ryan, D. M., Scott, A. C., Wallace, C. A., (2001). *Optimized crew scheduling at Air New Zealand*. Interfaces 31**,** 30-56.

Chebalov, S., Klabjan, D., (2006). *Robust airline crew scheduling: move-up crews*. Transportation Science 40**,** 300-312.

Chu, H. D., Gelman, E., Johnson, E. L., (1997). *Solving large scale crew scheduling problems*. European Journal of Operational Research 97**,** 260-268.

Clarke, L., Johnson, E., Nemhauser, G., Zhu, Z., (1997). *The aircraft rotation problem*. Annals of Operations Research 69**,** 33-46.

Clarke, L. W., Hane, C. A., Johnson, E. L., Nemhauser, G. L., (1996). *Maintenance and crew considerations in fleet assignment*. Transportation Science 30**,** 249-260.

Clarke, M., Smith, B., (2004). *Impact of Operations Research on the Evolution of the Airline Industry*. Journal of Aircraft 41**,** 62-72.

Cohn, A. M., Barnhart, C., (2003). *Improving crew scheduling by incorporating key maintenance routing decisions*. Operations Research 51**,** 387-396.

Cordeau, J. F., Stojkovic, G., Soumis, F., Desrosiers, J., (2001). *Benders decomposition for simultaneous aircraft routing and crew scheduling*. Transportation Science 35**,** 375-388.

Cordeau, J. F. J., Mercier, A., Soumis, F., (2005). *A computational study of Benders decomposition for the integrated aircraft routing and crew scheduling problem*. Computers & Operations Research 32**,** 1451-1476.

Ehrgott, M., Ryan, D., (2002). *Constructing robust crew schedules with bicriteria optimization*. Journal of Multi-Criteria Decision Analysis 11**,** 139-150.

Gao, C., Johnson, E., (2006). *Rethinking the airline crew scheduling process*. Submitted to AGIFORS Anna Velicek Award Competition 2006.

Gao, C., (2007). *Airline integrated planning and operations*. Ph.D. Dissertation, Georgia Institute of Technology.

Gershkoff, I., (1989). *Optimizing flight crew schedules*. Interfaces 19**,** 29-43.

Gopalan, R., Talluri, K. T., (1998b). *The aircraft maintenance routing problem*. Operations Research 46**,** 260-271.

Hane, C. A., Barnhart, C., Johnson, E. L., Marsten, R. E., Nemhauser, G. L., Sigismondi, G., (1995). *The fleet assignment problem: solving a large-scale integer program*. Mathematical Programming, Series B 70**,** 211-232.

Hoffman, K. L., Padberg, M., (1993). *Solving airline crew scheduling problems by branch-and-cut*. Management Science 39**,** 657-682.

ILOG. *ILOG CPLEX 9.0 User's Manual*, 2003.

Ioachim, I., Desrosiers, J., Soumis, F., Belanger, N., (1999). *Fleet assignment and routing with schedule synchronization constraints*. European Journal of Operational Research 119**,** 75-90.

Kang, L., Clarke, J.-P., (2003). *Degradable airline schedule*. Presented at INFORMS, Atlanta,GA.

Klabjan, D., Johnson, E. L., Nemhauser, G. L., Gelman, E., Ramaswamy, S., (2002). *Airline crew scheduling with time windows and plane-count constraints*. Transportation Science 36**,** 337-348.

Lan, S., Barnhart, C., Clarke, J.-P., (2003). *Robust aircraft maintenance routing and flight schedule retiming*. Presented at INFORMS, Atlanta,GA.

Lohatepanont, M., Barnhart, C., (2004). *Airline schedule planning: integrated models and algorithms for schedule design and fleet assignment*. Transportation Science 38**,** 19-32.

Mercier, A., Cordeau, J. F., Soumis, F., (2005). *A computational study of Benders decomposition for the integrated aircraft routing and crew scheduling problem*. Computers and Operations Research 32**,** 1451-1476.

Rexing, B., Barnhart, C., Kniker, T., Jarrah, A., Krishnamurthy, N., (2000). *Airline fleet assignment with time windows*. Transportation Science 34**,** 1-20.

Rosenberger, J. M., Johnson, E. L., Nemhauser, G. L., (2004). *A robust fleet-assignment model with hub isolation and short cycles*. Transportation Science 38**,** 357-368.

Sandhu, R., Klabjan, D., (2004). *Integrated airline planning*. AGIFORS Symposium 2004, Singapore.

Schaefer, A. J., Johnson, E. L., Kleywegt, A. J., Nemhauser, G. L., (2005). *Airline crew scheduling under uncertainty*. Transportation Science 39**,** 340-348.

Shaw, T.L., (2003). *Hybrid column generation for large network routing problems: with implementations in airline crew scheduling*. Ph.D. Dissertation, Georgia Institute of Technology.

Smith, B., (2004). *Robust airline fleet assignment*. Ph.D. Dissertation, Georgia Institute of Technology.

Smith, B., Johnson, E. L., (2006). *Robust airline fleet assignment: imposing station purity using station decomposition*. Transportation Science 40, 497-516.

Talluri, K. T., (1996). *Swapping applications in a daily airline fleet assignment*. Transportation Science 30**,** 237-248.

Talluri, K. T., (1998). *The four-day aircraft maintenance routing problem*. Transportation Science 32**,** 43-53.

Vance, P. H., Barnhart, C., Johnson, E. L., Nemhauser, G. L., (1997). *Airline crew scheduling: a new formulation and decomposition algorithm*. Operations Research 45**,** 188-200.

Yen, J., Birge, J., (2006). *A stochastic programming approach to the airline crew scheduling problem*. Transportation Science 40**,** 3-14.