

Schedule Design and fleet Assignment with Demand-Supply Interactions: An Alternative Approach

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Abstract

The paper solves schedule design and fleet assignment problem while taking care of various demand-supply interactions. Scope of the paper includes both unconstrained and constrained demand. Paper provides a method to incorporate the change in unconstrained itinerary demand as a function of supply and the approach is less subjective and hence more scalable as compared to existing methods. Paper also discusses a new approach to handle spill and recapture. The resultant model is a mixed integer non-linear model, that has been converted into a mixed integer linear problem. The paper illustrates the utility of approach with the help of examples.

Key words: Demand-Supply Interactions, Airline Scheduling

1 Introduction

Schedule design and fleet assignment are first two stages of airline schedule planning ((Barnhart & Amy, 2004)). Schedule design stage constitutes deciding the itineraries to operate in each market along with their corresponding departure & arrival time and departure & arrival airport. The fleet assignment stage uses set of flights to operate, demand for each flight and number of each fleet type available, to decide the type of fleet assigned to each flight. These two stages are related to each other in terms of number of fleet available, demand for itineraries and set of flights to offer etc.

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The two approaches to solve these problems are - integrated and sequential. Integrated approach tries to combine more than one stage of airline schedule planning in the model. It helps in capturing the interactions across two stages. The sequential approach solves each stage one by one and is useful in terms of reducing the complexity of the model. It is used when the problem size becomes large and it is difficult to handle integrated models. In this paper, we work on integrated models. We use integrated approach, as it helps in capturing the interaction across scheduling stages with respect to demand. However, the concepts and model proposed in this paper can be applied to sequential approach as well.

These two planning stages - schedule design and fleet assignment - are important, as they provide foundation for planning of later stages, impact fleet utilization and assist in some of the most important decisions (more of short-term nature, some might be long-term) for an airline. These stages also affect the market positioning and strategy for an airline. Because of the significance of these stages in airline planning, it becomes important to understand and appropriately incorporate different aspects of the problems in modeling, as it might affect the problem definition and hence can influence the solution of these stages. In this paper, we work on one of the most important aspects for both of these stages, i.e. '*Demand*'. So, while solving these problems, we try to understand the interactions of these stages with demand and incorporate those into our model. Literature contains some references on this class of problems. We discuss the positioning of our work with respect of literature in later sections.

The two sides of these interactions are - Scheduling stages (representing supply side) and demand. Literature addresses various issues related to each of these sides individually. But the work capturing their interactions has received less attention. Though, the work capturing demand-supply interactions in short-term has got comparatively more attention (e.g. Revenue Management).

We briefly discussed the two planning stages in initial paragraphs of this section. We summarize demand forecasting process in the following paragraph. The next paragraph explains the need for studying demand-supply interactions.

Airline demand forecast can be broadly divided into two steps. First one is total market demand forecast for air travel. It is done based on seasonality, day of the week and time of day. The second step, i.e. using demand share models, calculate the fraction of total demand attracted by each itinerary/flight. The two most common used classes of share models are: QSI (Quality Service Index) based and logit based.

Schedule design stage is more closely related to total unconstrained demand

for a market (represented as a function of time-of-day) and itinerary. The fleet assignment is more close to flight level demand and accordingly it assigns the fleet type to each flight. While this shows the role of demand as an input parameter for these stages, another side is the affect of these stages on demand. For example, the total demand in a market and for each itinerary, among other factors, is also a function of itinerary offerings in that market. Similarly, the number of people being denied the service on an itinerary and being recaptured on another flights - depend on the capacity of flight denying service; presence and availability of seats on other flights. So while, demand acts as an input parameter for planning, these stages affect various parts of demand. While there is lots of literature on these planning problems, the work on capturing the demand as a function of supply is much less. The paper addresses such interactions; the specific details of the interactions have been discussed in the following section.

1.1 Demand-Supply Interactions (for Network Design and Fleet Assignment)

This section provides examples of different demand-supply interactions; And explains, in detail, the interactions captured in this paper. Some aspects of such interactions have been addressed in literature. This paper works on unconstrained and constrained demand and contributes to the literature in two ways - First, it provides an approach to incorporate unconstrained demand interaction that makes the planning process more methodical and helps in reducing the subjectivity at various points. Secondly, it also provides a new approach to handle a few problems on constrained side, existing in literature.

The following figure demonstrates some of the demand-supply interactions at various levels:

In figure1, we list some issues on both side (i.e. demand and supply) with reference to interactions of two sides. The total demand in a market is shown to be related to the factors like presence of air travel and other modes of transportation along with their fare levels etc. Attributes of itineraries, present in the market, also play role in determining the total demand. Next, the unconstrained demand for each itinerary is shown to be a function of that itinerary's attributes; And presence and attributes of other itineraries (airline's own itineraries and competitor's itineraries). The number of customers request accepted and denied for each itinerary is affected by capacity assigned and demand of each flight leg of itinerary. Spill criterion (which determines how much to spill from each itinerary in a set of itineraries, where the set shares a flight leg whose demand is more than capacity) of airline plays an important role in this. The amount of spilled demand, recaptured by the airline's own itineraries, depends on presence, availability & attributes of other

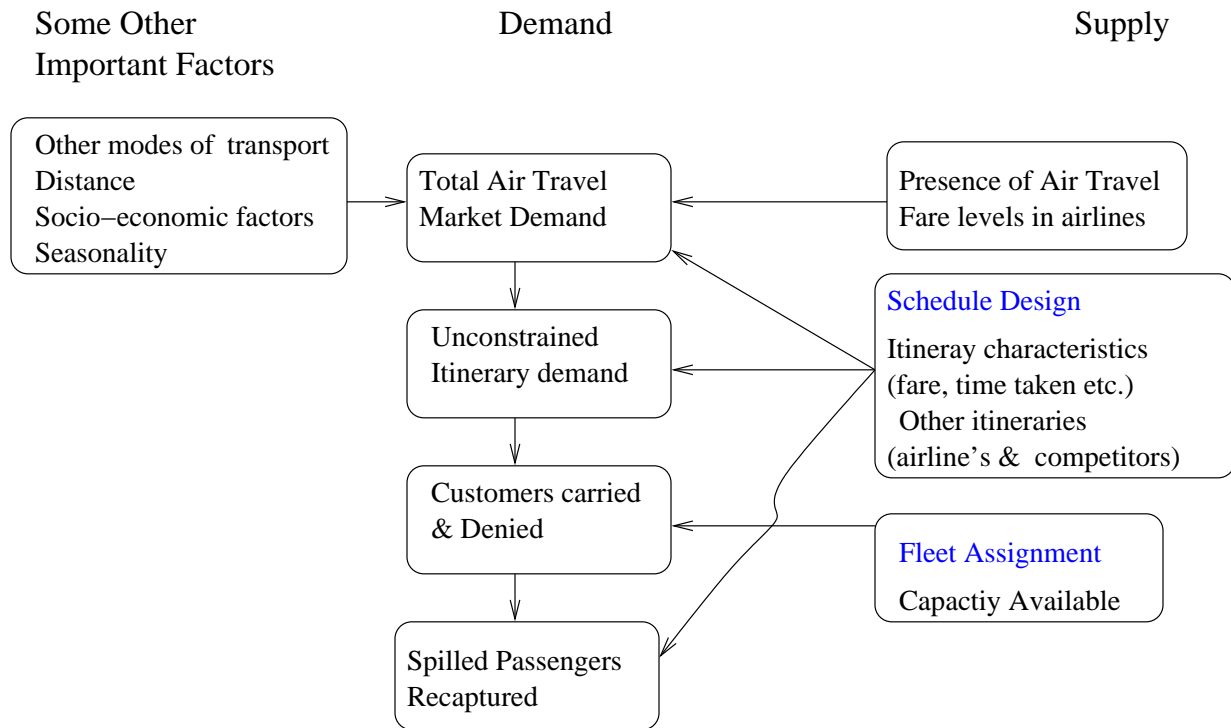


Fig. 1. Demand-Supply Interaction

itineraries.

This paper, works on incorporating interactions of unconstrained and constrained demand with supply. Constrained side works on aspects like spill, recapture, passenger flows and fares etc. For these problems, the flights to offered are fixed and the decisions to be made are - of assigning capacity (fleet assignment) and studying the mix of different passenger classes over different fare class and flights. Sometimes, it also includes making some changes in the existing schedule to better match the supply with demand. Although the set of flights offered is assumed to be known here, it might make a difference as recapture rate of demand spilled from one itinerary would depend on the presence of other itineraries. This in turn, can also affect the optimum set of flights to be offered in market. On unconstrained demand side, the interactions can be of various types. For example, it can be related to total market demand calculation or to some later stages of demand calculation. In this paper, we assume that the total unconstrained demand for each market is fixed and known. We work on incorporating interactions among unconstrained itinerary demand and supply and we also integrate it with the constrained demand. We discuss the interactions captured in this paper and their impact on different aspects of planning to emphasize the need for a more methodical (structured) and scalable approach in this section.

Following are the examples of some of the important interactions between unconstrained demand and the scheduling problems:

(Lohatepanont & Barnhart, 2004) provide the following example to show that the demand for each itinerary changes as a function of presence/absence of other itineraries and the change is non-linear.

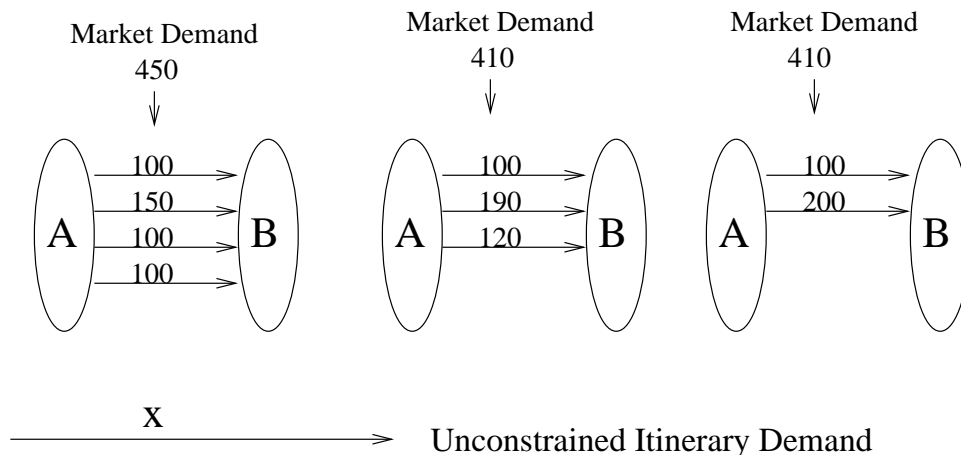


Fig. 2. Demand-Supply Interactions**Barnhart,2004*

In figure2, the first itinerary is assumed to be in the morning and rest three itineraries in the evening. So, when the evening itineraries are deleted, demand changes for evening itineraries, but it does not change for morning itinerary. It is also shown that the effect of deletion of two itineraries is not proportional to the deletion of one. Another scenario might also exist, where the case could have been that the deletion of a single evening itinerary affects morning itinerary as well, though differently from evening itinerary.

The above affect can be explained as following: For given total unconstrained market demand, the demand share for each itinerary is calculated based on the various characteristics of that itinerary and characteristics of the other itineraries in the market. So, the demand for each an itinerary would be affected by presence, characteristics and number of other itineraries. The reason for non-linear change in the demand of itineraries, due to deletion of one itinerary, might be that people attach utility with different attributes (departure time, fare, quality of service etc.) of itineraries. The utility function might not be linear in attributes. Next, there can be different ways of combining these utilities. Two most common ways of combining these utilities are - Quality Service Index (QSI) models and Logit-based models. Because of these factors, the change in demand of other itineraries by deleting one of the itineraries might not be linear in terms of number of itineraries.

As explained above, the demand of each itinerary changes as a function of the set of itineraries in the market. The next step might be to analyse its affect on result of planning problems. Demand is an important parameter for these models. So, more is the estimation error in demand, higher is the probability of deviating from the optimal solution of real existing problem. The problem formulation, with inaccurate estimates of demand, might not represent the

actual problem scenario at hand. This, in result, can affect the solution obtained by network design and fleet assignment models. The solution obtained using these inaccurate values might not be optimal. Following example shows that, while inaccurate estimates of demand might affect the decision within a market and such decisions might also have network affects. The example, illustrating the decision change along with network affects is as following:

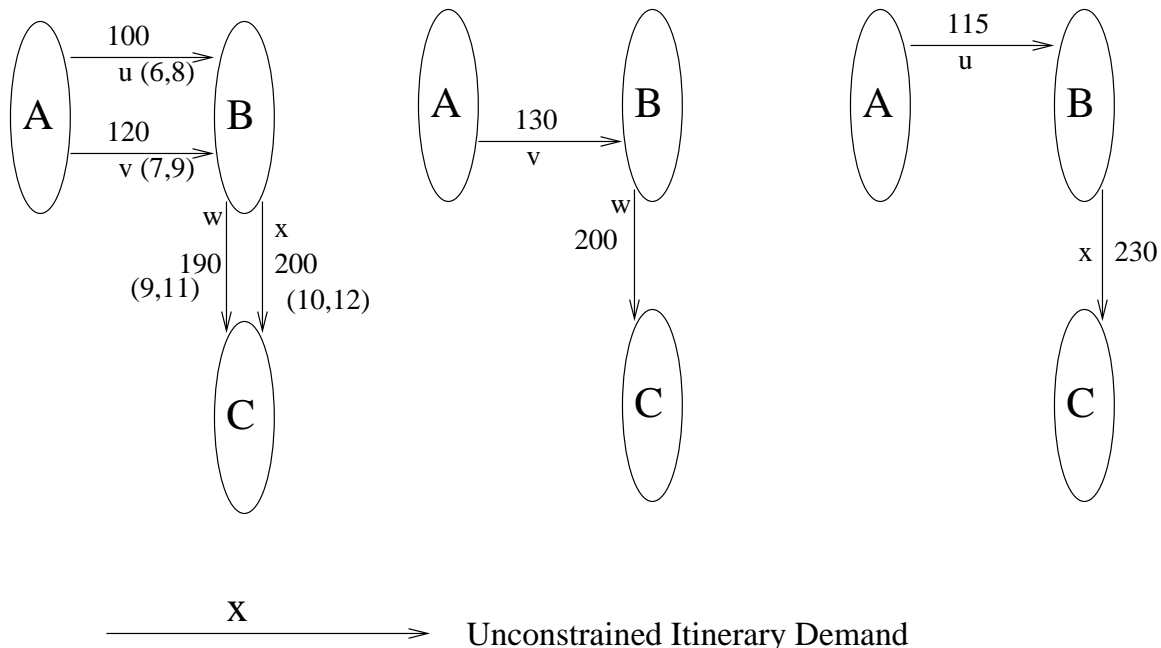


Fig. 3. Network Effects

Here we assume two market A-B and B-C. The numbers on lines show the unconstrained itinerary demands. The pairs in parenthesis show the arrival and departure time for the itinerary. Letters u, v, w, x are itinerary indices. Here, in (i), the demands shown are, the demand for each itinerary when the other itinerary is also flown. We assume that there is only one aircraft available with capacity of 250 and hence only one of the two itineraries in each market can be flown. We assume that there are some competitor's itineraries present in each of these markets. We also assume that all itineraries have same fare. Then if no demand updation is made for the deletion of an itinerary in the market, then the itineraries selected would be as shown in ii. But then the demand for each of these itineraries would not be as shown in i. Rather, it would be as shown in iii. And, actually if the other set of itineraries were flown, the total demand would be more in iii than ii. Also, if we look at the demand for each itinerary in market A – B, the demand for itinerary v is more than demand for itinerary u , but comparing cases ii and iii show that operating v might mean operating w and hence total demand & revenue are less.

The conclusions from the above discussion are:

- Demand changes with the itinerary offerings in the market.

- The change is not linear.
- Accuracy of demand estimates might have influence on decisions in a market and such decisions can have network effects. This in turn can affect the quality of solutions.

So, there is a need for structured, methodical, scalable and comparatively less subjective method that can help in reducing the inaccuracies due to subjectivity, while capturing demand-supply interactions.

Demand calculation models are basically forecasting methods and have inherent error in it like all forecasting models. But assuming, we have confidence in these forecasting models (as planning is based on the expected events about future and as these forecasting models are derived from the population. So, we expect the utility functions and their behavior to follow the trend in future), it would be preferable to utilize the demand forecasts appropriately to improve the solution quality of optimization models. Such a methodical approach to incorporate these demand supply models in planning would help in reducing the subjectivity and also have more confidence in our scheduling models. In this paper, we develop an optimization model for network design and fleet assignment, that can take care of the fact that for given total unconstrained market demand, the unconstrained itinerary demand, spill and recapture are functions of the presence and attributes of other itineraries, in a structured manner.

Some other important demand-supply interactions can be -

- *Revenue Management* concepts to capture the maximum possible demand while maximizing the profitability. When we talk about *revenue management* etc., there is lots of work ((Talluri & Ryzin, 2004); (Talluri, 2001a); (McGill & Ryzin, 1999), (Belobaba, 1989) & (Subramanian, Jr., & Lautenbacher, 1999) - to name a few); that tries to manage available capacity in different fare classes and itineraries by the help of fare with respect to time and other attributes. But most of the work present is generally on the shorter planning horizon. Scheduling stages like network design generally are done for comparatively much longer horizon (It might be 8 or more months before flights operations, in many cases). Though, some attempts have been made to partially combine these two sides of supply ((Glover, Glover, Lorenzo, & McMillan, 1982), (Kniker, 1998)). But capturing RM issues along with these planning stages might result in a very complex (if not intractable). Even then, it would be some probabilistic estimate, as the actual demand would be known only with time. So, it might not be possible to integrate these two supply sides.
- *In Dynamic scheduling* an attempt is made to make use of the fact that as the time for a flight/itinerary comes closer, the demand forecast can be improved and hence this information can be used for decision making pro-

cess. (Talluri, 2001b) works on this concept. The work tries to use re-timing of flights (with in a small interval) and re-fleeting mechanisms to improve profits. Here again, the planning horizon is different from the two planning stages addressed in this paper. Also, the basis of dynamic scheduling is to procure information obtained along time and utilize it to improve the existing schedule. So, it acts as improving the outcome of these two stages due to more information availability. (Berge & Hopperstad, 1993) also works on dynamic fleet assignment.

Following section discusses existing works capturing demand's interactions with planning stages and the need for our alternative approach in the next section. We also discuss the issues of generalizability of approaches etc. The detailed definition of the problem has been provided in later section.

1.2 Existing Literature, Gaps and Scope of the paper

The paper provides an alternative method to develop schedule design and fleet assignment models taking care of their interaction with unconstrained demand and constrained demand. While the literature contains lots of work on these planning stages ((Hane et al., 1995), (Kniker, 1998), (Rexing, Barnhart, Kniker, Jarrah, & Krishnamurthy, 2000) to name a few); the references emphasizing the importance of such interactions and model incorporating them are comparatively less. Recently, such work has been gaining lots of attention from researchers ((Kniker, 1998),(Yan & Tseng, 2002), (Hsu & Wen, 2003) and (Lohatepanont & Barnhart, 2004)). We discuss these works and corresponding gaps in the following paragraphs.

(Yan & Tseng, 2002) tries to integrate demand side with scheduling model. It follows a time-space network based approach for problem solving. The model formulation is an integer multiple commodity network flow problem. But here the demand interaction is mainly concerned with incorporating the cost to customer in the model, along with airline's cost. But our work is more related to capturing change in demand as a function of supply.

(Kniker, 1998) works on fleet assignment stages and tries to obtain an optimum flow of passengers over itineraries and flights using Passenger Mix Model (PMM). PMM requires a fleeted schedule to work on. PMM works on the constrained demand using capacity available, spill, recapture and other information. One of the assumptions, here, is that the amount of spill from one itinerary to other itineraries is at airline's discretion and for each such spilled passenger airline offers exactly one alternative itinerary. Such conditions might be difficult to meet (fulfill) in real cases. We do not make any such assumptions in our model. Constrained demand modeling of our approach also

helps in overcoming few other problems, not addressed in literature. We describe these issues in detail in next section, when we explain spill & recapture modeling of our method.

(Hsu & Wen, 2003) tries to solve schedule design problem while capturing demand-supply interactions using an iterative approach. It does so by using two submodels - one to calculate itinerary (paper refers to it as route) demand for a given schedule (Called Market Share models) and the other one, i.e., supply sub-model obtains the frequencies using these demand estimates. Demand calculation model uses the cost to customer as a criteria to choose flights. The cost to customer has been calculated using fare and conversion of other criterion like schedule delay etc into monetary terms. This cost is used in market share model to calculate route demand. Once the demand has been derived, the supply side determines the flight frequencies between each pair of Origin-Destination using a Mixed Integer Programming formulation. The iterative solution methodology starts with initial estimates of frequency, fare and other parameters. First the demand sub-model is solved to achieve initial approximations of market share. The supply submodel, in next step, provides the frequencies and fares while trying to maximize profit using these demand estimates. Once these steps have been executed, the paper uses a measure called *Relative Change in Airline Profit* to decide whether to continue with next iteration (repeating the above steps, using new values of frequency etc. obtained in this iteration) or stop. If the condition is met in any iteration, the process stops.

Some gaps in the above paper can be listed as below. Converting the criterion like schedule delay to costs would require subjective estimates which might be difficult to obtain. Still, it might not be possible to capture all the factors in terms of cost. The reason is that each flight can have many different attributes like level-of-service, fleet type etc. and such an approach might not be able to incorporate all of those. For example, time-of-day is one of the most important criteria to determine demand and also, the preference of other attributes might be a function of time-of-day. For example, the case might be that early morning flights consists of more business travelers and they are more sensitive to time and quality of service than leisure travelers, who are more sensitive to fare. So, it might require different utility function for different attributes based on departure time. With the approach used in the paper, it might not be easy to incorporate such factors. The model does not include the flow-balance constraint also. So, it might be possible that the resulting solution obtained using this iterative approach, does not even have a feasible routing. This might need changes in solution, which in turn might have network affects. In this process, the utility of capturing demand-supply interactions might get affected. The next problem is convergence of the iterative method. Though the issue has been discussed in the paper, it does not prove the convergence in general conditions. And without the proof, calculating the bound or estimating the

generalizability of the approach might be questionable. Considering the market share models used for demand calculation; the most common demand models used in academic literature and practice are - QSI or logit-based. But, the paper does not use any of these models. So, the consistency with existing models for various other problems might be a problem.

Next, the approach of (Lohatepanont & Barnhart, 2004) can be described as following. The paper tries to solve an integrated schedule design and fleet assignment problem. Here, the incremental scheduling approach has been considered. It means that some of the flights are fixed to be operating and the model needs to optimize among remaining optional flights about which one to operate or not. One of the advantages of using incremental scheduling is that it helps in reducing the complexity. The paper uses an iterative solution methodology and demand correction terms (explained below) to capture change in itinerary demand as a function of supply (itineraries offered and their capacity). The optimization model for schedule design considers demand in two parts - demand for each itinerary as an input parameter and a demand correction term for each ordered pair of itineraries. Input demand for each itinerary is calculated assuming that all the itineraries are being operated. But when the model is solved, only some of the itineraries are operated and not all of them are operated. The demand correction terms are used to capture the change in demand of an itinerary when the other itineraries are deleted. Correction terms are defined for each pair of itineraries in market and each term represents the change in demand of one itinerary when the other itinerary is deleted. The higher order-correction terms, representing affect of deleting more than one itinerary simultaneously, are not considered. The demand correction terms are considered as parameters of the model. For initial values of correction terms, some approximations are made. The correction terms are updated with each iteration of solution method. The iterative solution method, for mathematical formulation of the problem addressed in the paper is described as following. First the model is solved with the initial itinerary demand values, demand correction terms and other parameters. The output obtained provides a list of itineraries to operate and fleet assignment for all operating flights. The next step solves the Passenger Mix Model (PMM ((Kniker, 1998))). PMM tries to optimize passenger flow for a given schedule design and fleet assignment for flights. PMM, here, uses the schedule design and fleet assignment obtained in first step. with this model is compared with the revenue obtained using Passenger Mix Model (PMM, provided by Kniker (1998)). Next, the stopping criteria is checked to decide whether to continue with next iteration or to stop. The stop criterion used in the paper are -

(i) The revenue difference between model revenue and PMM revenue

or

(ii) The revenue difference between two consecutive iterations.

If the stopping criterion is met (i.e. if the difference is below some threshold value), the algorithm stops. Otherwise, correction terms are revised based on which itineraries are showing more of difference and in the new iteration the model is solved with these revised values.

There might be some gaps in this approach. First, one needs to obtain an initial approximation for demand correction terms for all the itineraries pair in a market. There might be subjectivity in obtaining initial approximations. And given the huge number of itineraries and hence even larger number of correction terms, it might not be even possible to estimate the amount of approximation error in determining these values. The number of iterations and the solution quality might depend a lot on the initial values. Subjectivity also comes in the revision of demand correction terms. The solution methodology also has the problems of iterative approach, as discussed with respect to (Hsu & Wen, 2003) paper. Next, If we look at the termination condition for their approach, it is possible that there can be some cases, where the condition might not be as effective as in many other cases. For example, considering the case that it is a demand-driven market and load-factor for many flights is significantly less than 1, PMM might not show much of a difference from the solution obtained in first step of iterative approach in terms of the solution. So, there might not be many iterations of solution methodology. The reason for this, is that PMM works on demand spills for optimizing passenger flow. But in case of low load factors, spill would not be significant and the termination criterion might be met in initial few iterations. In that case the demand correction terms might not be revised. Still, the case can be that the unconstrained itinerary demand estimates have error in them and hence the decision might not be optimal due to inaccurate demand estimates. Such an example has been discussed in the illustration section. This is just an example of a special case scenario. It might be very difficult to analyse all the cases and list the performance of the model due to the lack of a benchmark set of problems. The above problem might be due to the reason that the model tries to capture the change in unconstrained demand (as a function of itineraries presence) by optimizing passenger flow on constrained demand side(i.e. using spill and recapture). But it might now be possible to approximate demand change by constrained side in all cases. Another problem with the approach is that it does not consider the higher-order demand correction terms. As discussed in previous section, the changes in demand with respect to itinerary presence are non-linear. And the situation aggravates even more because in the paper, the initial average unconstrained itinerary demand is calculated assuming all the itineraries are being operated. So the number of itineraries being deleted in each market would be very high. And it might not be possible to estimate the effect of ignoring higher order terms on the resultant value of demand estimates and hence the scheduling output. Another concern is the

problems with PMM (as discussed with respect to (Kniker, 1998), assumes the spill for each pair can be at airline's decision by offering the customer only one choice (if the desired itinerary is not available to customer) as alternative. But it might not be possible, for an airline, to implement it; specially in a competitive environment. The last point is that the model used for demand forecast is Quality Service Index (QSI - based). Though this model is simpler than other demand models like logit and OGEV, but the latter have been empirically proved to perform better ((Coldren, 2005)). So, it might be desirable to explore and utilize logit-based models in scheduling problems.

Most of the work in literature considers demand as an input parameter. While (Kniker, 1998) works on constrained demand concepts like spill, recapture etc; we could find only two reference - (Hsu & Wen, 2003) and (Lohatepanont & Barnhart, 2004) - that work on modeling unconstrained itinerary demand as a function of supply. While both the these works have some gaps, one of the common issues is that both use iterative approaches to capture these interactions. And it is extremely difficult, if not impossible, to quantify the convergence and performance of these approaches. One of the reasons for this is that airlines network, markets, demand and various other factors, combined together can give rise a large number of scenarios. So, proving convergence in general might be difficult. Along with this, lack of benchmark problem set in airlines scheduling makes it even more difficult to obtain bounds or analyze these models in general. Another concern, common to both the works, is that they require initial approximations and these can affect the quality of solution and time taken to solve the problem.

We provide an alternative approach for schedule design and fleet assignment problem, where we take care of demand supply interactions like - modeling demand as a function of supply (i.e. the frequency, departure time, quality of service and capacity) being offered in the market. Demand-supply interactions tries to model the concept that itinerary demand is a function of other itineraries in the market, spill from some itineraries and the corresponding recapture. We integrate it with the modeling to capture spill & recapture mechanisms when a flight's demand exceeds capacity of fleet assigned to it. Our approach to model constrained demand factors is different from existing literature ((Lohatepanont & Barnhart, 2004), (Kniker, 1998)). We describe this approach and difference with respect to corresponding existing literature when we explain the model in next section. We incorporate itinerary demand as function of the schedule being offered. This approach helps to overcome various problems of initial subjective approximates, iterative approaches etc. One of the major advantage of our approach is that, unlike other papers, it tries to capture changes in unconstrained demand using more direct and general manner rather than trying to approximate it using constrained demand or other aspects. We also don't need to design termination conditions, which might be affect the model performance. It helps in excluding the possibility of error in

special cases (For example, we discussed one such case with low load factors for (Lohatepanont & Barnhart, 2004). Another aspect, of our work is the demand model used. We argue in favor of logit-based models instead of Quality Service Index (QSI) based models. There has been lots of work on developing logit-based demand models. But we could not find their use in optimization models for these planning stages. Logit-based models have been used in few other airline problems ((Pels, Nijkamp, & Rietveld, 2000) for example). But there the problems are more close to hub-location and related problems and the corresponding issues are different. We use a variant of the logit based demand share model, provided in (Parker & Walker, 2006) to calculate demand for our optimization model.

Issues	Literature	Scope
Domain Integrated Network Design and Fleet Assignment	Yes	Yes
Dynamic Unconstrained Demand Calculation	No - As parameter approximation; corrected at each iteration	Yes; New Approach (Methodical; Reduces Subjectivity)
Demand Unconstrained & constrained	Unconstrained - approximations updations based on constrained side factors	Both considered
Spill and Recapture	Yes	A new approach (Utility explained later)
Solution Approach	Iterative (convergence issues)	Optimization (one iteration)
Logit-based demand model	Not applied in this problem	Yes
Recapture Metric	Yes (QSI based)	Yes (Logit-based)

Table 1 **Scope of the paper**

2 Model Development

The problem considered here is to model and solve the schedule design and fleet assignment phases of airline scheduling, taking care of the demand-supply interactions into account. It means that we have to decide the following:

1. From a given (possible) set of itineraries, which itineraries to operate. (Schedule Design)
2. Deciding the fleet-type for each flight, based on number of different types

of fleet available and demand for different itineraries. (Fleet Assignment)

The two modeling approaches for schedule design are - Incremental and Full-scale scheduling. Former, means making changes in the existing schedule (By maintaining lists of fixed flights - called mandatory flights and rest of the flights are called optional flights) and accordingly adjusting other values. This approach is popular among practitioners in airlines. Full-scale modeling means designing the whole schedule afresh and not assuming any present structure. This approach has more complexity. But it has got significance in terms of planning when major changes are required. Even when it does not seem very important to make big changes, it might be important to at least study the full network because of following two reasons. The first one is that, such schedule design planning problems have comparatively long planning horizon (6-8 months before implementation) in many cases. So, unless the problem is becoming intractable (In which case, one might need to look for alternative approaches like incremental scheduling), even if the problem takes more time to solve, it might not be a problem. Secondly, as discussed above and also studied by ((Glover et al., 1982), (Kniker, 1998) and (Lohatepanont & Barnhart, 2004)), various planning decisions have network affects. So, it might not be possible to access these affects while deciding which flights to make mandatory and which ones optional. Though, it might be clear for some flights whether to make those mandatory or not, lots of flights might be at boundary of these two classes. We work on full scale modeling and later show that, with slight changes, it can be used for incremental scheduling also.

Various issues need to resolved for the formulation of this problem. Following example demonstrates the various issues related with the problem:

Some of important issues are as following:

- **Fare** is one of the most important itinerary selection attribute. While planning for these stages, we don't consider revenue management (RM) issues and use average fare to calculate itinerary demand. There are two reasons for it. One reason is that such planning is done around 8 or more months in advance, but many of the RM concepts are time-dependent and based on the demand pattern as a function of time. Such information becomes available with time. So, at such early stages of scheduling, we consider average fare. Most of the planning literature also makes use of average fare. Incorporating revenue management issues might also make scheduling model more complex to solve, if not intractable. Various methods are used to calculate average fare. For model validation purposes, we have considered different average fares keeping other parameter fixed to illustrate the utility of model in various scenarios. Our model assumes average fare to be an input parameter.
- **Total Unconstrained Market Demand** is assumed to be fixed and an

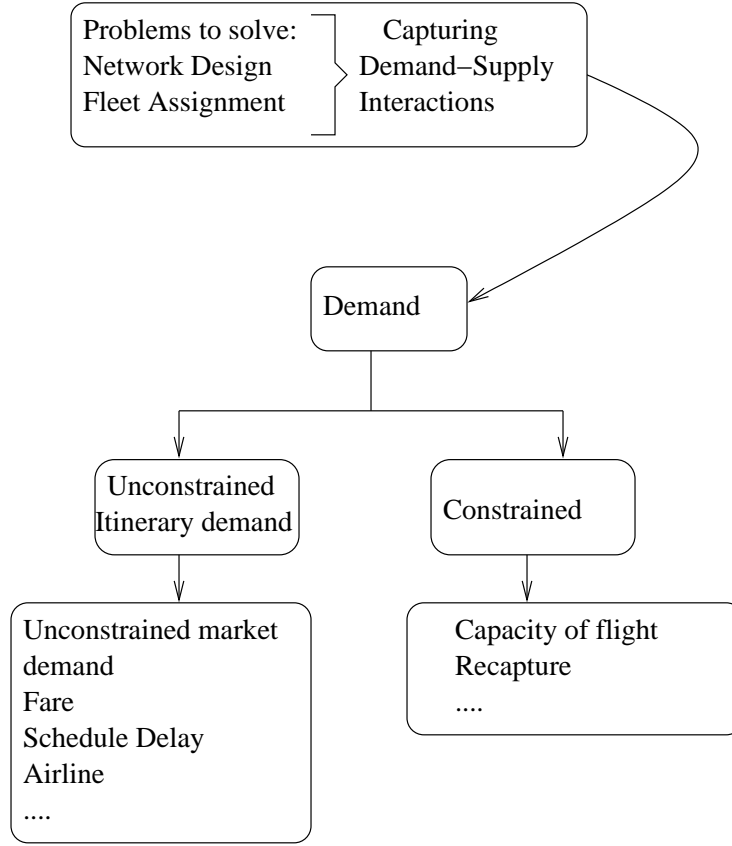


Fig. 4. Formulation Issues

input parameter. Among other factors, it is also a function of the frequency and attributes of itineraries in the market. But, to a large extent, it depends on various socio-economic factors and it might not be possible to incorporate those issues into scheduling models. Other than these factors, the total demand also depends on the season of year, day of week etc. As we are working on a daily scheduling problem, we assume that the total demand considered is for the day of planning, after taking into considerations the factors of seasonality etc. Here we work with deterministic average demand and do not consider the variation around that mean value.

- **Unconstrained Itinerary Demand** denotes the share of total demand received by a particular itinerary. Unconstrained Itinerary Demand is supposed to be a variable of the model. We use logit-based models due to the accuracy provided by them over QSI-based models. Itinerary demand is calculated on the basis of model discussed in (Parker & Walker, 2006). However, for demand calculation they use time as a continuous horizon. We discretize time points in our model. The expression is shown as following:

$$D_i = \sum_h Dem_h \frac{e^{G_h(\tau_h - t_i) + \sum_b \beta_b \cdot U_{i,b}}}{\sum_j e^{G_h(\tau_h - t_i) + \sum_b \beta_b \cdot U_{j,b}}}$$

Here D_i represents unconstrained demand for itinerary i ; Dem_h repre-

sents the demand at hour h in market of itinerary i ; G_h represents the utility function for departure time (schedule delay) and other attributes (b) have been supposed to have linear utility function. $U_{i,b}$ represents the value of attribute b for itinerary i . Various attributes considered in the model are - departure time, schedule delay, fare, airline parameter (denoting the airlines's preference among customers), number of hours for total travel and number of stops. We are not considering code-share or bundling. In code-sharing, airline ties up with other airlines to mutually use each other seats. In bundling, customers buy tickets from different airlines and their itinerary is composed of flight from different airlines. We assume none of these exist.

The above discussed model expression has been used as a functional form to calculate demand in the model. This approach is one of the contributions of the model.

We assume unconstrained itinerary demand consists of unconstrained demand for the itinerary and the customers spilled from other itineraries & are willing to take this itinerary (As it might not be possible to differentiate them from the customers having this itinerary as their first choice).

- **Spill** ((Swan, 2002), (Li & Oum, 2000)) represents the number of customers being denied service because of non-availability of seats. It happens when the demand for a flight is more than the capacity of fleet assigned. Spill related definition has been discussed in appendix. We use the deterministic average demand and capacity of fleets is fixed (It can take a few values based on the fleet type assigned to flight and does not have any variation). Spill from itineraries also does not have variance component and is deterministic.

A flight might be part of more than one itineraries. So, the demand for the flight constitutes the demand for all those itineraries. The flight would be able to accommodate demand maximum upto its capacity and if demand is more that this capacity, rest of the customers would be denied the service. But which itinerary spills how many customers, can be decided on various basis. Some such methods have been discussed in (Kniker, 1998). We use spill from each itinerary as a variable in our model and how much would be spilled from which itinerary would be decided by the model itself based on its total effect on the objective value. It means that the spill would be decided based on its effect on the profitability of whole schedule. Spill can be defined in terms of demand and capacity, as following:

$$Spill = \max((Unconstrained + Rcp - Capacity), 0) \quad (1)$$

Here *Spill* denotes the customers being denied request; *Unconstrained* denotes the unconstrained demand on flight due to all itineraries having the flight as one of the legs; *Rcp* denotes the customers being spilled from other itineraries and getting recaptured by the itineraries having the flight as one of the legs; *Capacity* denotes the capacity of fleet type assigned to

the flight.

In the model, the constraints should ensure that the demand carried by each flight does not exceed the capacity assigned to that flight. They should also ensure that spill occurs only when demand is more than capacity.

The literature models the spill as:

$$Unconstrained + Rcp - Spill \leq Capacity \quad (2)$$

Another constraint used, along with above constraint is that spill is less than the total demand. And in objective function, the spill multiplied by the fare of corresponding itinerary is used (Deletion in case of profit maximization). In this case, we expect $Spill$ to be positive if other components of Left Hand Side add up to more than $Capacity$ and zero otherwise.

But there might be some problems with this approach. These have been discussed below with the help of examples:

Say, the spill from itinerary 1 should be 50 (Based on difference between demand and capacity) and corresponding fare of the itinerary is \$100. Say the recapture rate of itinerary 2 is .6 and fare is \$200. Assuming itinerary 2 has capacity available, then spill from 1 would increase to as much as 2 absorb. So, it will not represent actual scenario.

Similar case might happen in situation where spill is not required, still it happens because the recapture in other itineraries is more profitable.

So, one need to take care of such cases.

One way to handle this situation might be to penalize spill in objective function. But even that might not provide the solution, as explained following:

Say, the two itineraries are 1 and 2. So, we want to penalize spill from each of them, we need a penalty coefficient. But that should be such that it takes care of the fact, that provided all other values are same and if spill has to happen from one of the itineraries, it should happen from the itinerary with lower fare. The relative difference in fare should be used for selection purpose. So, how much ever penalty we add to fare, still the difference between fares would be present and can affect the solution.

In our model, we define two variables $Spill^+$ and $Spill^-$. Here $Spill^+$ takes a positive value if demand is less than capacity. $Spill^-$ takes positive value if demand exceeds capacity. This is done to take care of the fact that term $Unconstrained + Rcp - Capacity$ is unrestricted is sign.

$$spill = spill^- - spill^+ \quad (3)$$

where

$$spill^+, spill^- \geq 0 \quad (4)$$

Next thing we do it model the constraint as an equality constraint as following:

$$Unconstrained + Rcp - Spill^- + Spill^+ = Capacity \quad (5)$$

In case of inequality constraint Spill could happen even if demand was less than capacity. But in case of equality that condition is taken care. Next thing we do is to Add high penalty to both $Spill^+$ and $Spill^-$. We add some big number to the fare of each itinerary and use the resultant number as coefficient of $Spill^+$ and $Spill^-$ in objective function and then subtract the terms from objective function (In case of maximization objective). We also add the constraint that Spill will happen only if itinerary is being operated.

Spill has been considered in Lohatepanont & Barnhart (2004) and other related models. But there, it has been modeled as a combination of variables, one variable for each pair alternative itinerary in the market (As it assumes that alternative itinerary selection is at airline's discretion for the whole system optimization). But our optimization model is based on the assumption that passengers try to maximize their utility in terms of different itinerary attributes. User and system optimization has been discussed in Kniker (1998). We don't assume that the airline has the discretion of spilling passengers to specific alternative itinerary of airline's choice. It might not be possible to do it in real-life. Though, revenue management might be able to do it using the fare differentiation, but capturing revenue management issues at scheduling level (network design, fleet assignment etc.) might made model extremely complex, if not intractable. Also, Passenger Mix Model (PMM) used in Lohatepanont & Barnhart (2004), assumes that airline will offer only one alternative itinerary to each passenger (If it can't provide the desired itinerary). PMM tries to incorporate different fare classes also as alternative. Even for given unconstrained itinerary demand, we could not find much work in literature combining user optimization for constrained demand, revenue management etc. However, if i a particular flight is part of more than one itineraries and the flight is getting full, then it might be upto airline's discretion to decide which itinerary availability to close at what time.

- **Recapture** represents the fraction of total demand spilled from airline's itinerary and captured back by airline's other itineraries. Recapture affects the itineraries demand. Demand is one of the most important factors in airline planning models and hence can influence the optimal solution of these models. Here we are deciding the flights to offer and corresponding capacity in each market based on demand and so, it becomes important to calculate recapture as accurately as possible. It would finally help in a better optimization model.

We assume the total recapture in each market to be 100%. It means that as long as capacity for air travel exceed demand in a market, the spill demand wont be lost. It would just get redirected to other itineraries in the market. The model can incorporate recapture other than 100%. Other case can be that this percentages varies from itinerary to itinerary.

We are using logit-based models for demand estimation. But most of the literature, capturing demand-supply interaction on these stages, uses QSI-based demand calculation for optimization models. As, we could not find much work on these optimization problems using logit-based models, there was no reference on use of recapture metric in these models. Though logit-based models have been used in other optimization problems ((Pels et al., 2000)), but these models don't deal with recapture directly.

In this paper, we use recapture metric provided by ((Ratliff, 2006)) to calculate recapture. The concept behind the metric is that, if an itinerary becomes unavailable, then the expected demand of that itinerary gets distributed, among remaining itineraries in the market, proportional to the original share of those itineraries. We explain the recapture rate calculation with the help of following example.

Suppose, a market has four itineraries 1, 2, 3, 4, where the itineraries 1, 2, 3 are airline's itineraries and 4 is competitors itinerary. The market shares for itineraries 1, 2, 3, 4 are - 0.3, 0.2, 0.2, 0.3 respectively. If itinerary 1 gets deleted from the market, its demand gets redistributed to itineraries (100% - We assume that capacity is available on itineraries 2, 3, 4). So, the recapture rate from 1 to 2 would be

$$recap_{1 \rightarrow 2} = share_2 / (1 - share_1)$$

In our example,

$$recap_{1 \rightarrow 2} = 0.2 / (1 - 0.3) = 0.2 / 0.7$$

Similarly it can be calculated for other itineraries (3, & 4).

Though we have already discussed the utility of logit-based models for demand estimation performance in previous sections. (Ratliff, 2006) explains another another advantages of using these models as - By describing flight services demand in terms of share, it's easy to estimate shifts in demand caused by flight service and/or fare change.

Next issue is of the calculation of itineraries original market share. As we are not considering unconstrained demand shares for itineraries as an input parameter. Instead these are variables for the model. We assume all the itineraries to be operating and use the corresponding market shares for recapture rates calculation.

Our approach is difficult from the one in literature due to following reasons:

Literature uses $Spill_i^j * (RecaptureRate_i^j)$ for recapture modeling. Here $Spill_i^j$ denotes spill from itinerary i to itinerary j and $(RecaptureRate_i^j)$ denotes recapture rate of itinerary j when the spill is from itinerary i . But in this method, it would be difficult to ensure that recapture happens only if the itinerary to recapture is being operated (As there is no variable for recapture). Though the capacity constraint discussed in Spill issue above will have Capacity zero if itinerary is not being operated. But still Rcp and Spill combination might try to make left hand side positive, if it is profitable. We use Rcp variable along with the constraint that Rcp_j (i.e. Recapture by

itinerary j occurs only if it is being operated). And Rcp_j gets calculated with the help of following constraint

$$Rcp_j \leq \sum_i RecaptureRate_i^j * Spill_i$$

And as in maximization objective Rcp_j is a positive term, it will try to be maximum on left hand side and hence would try to be equal to right hand side.

- **Departure-Time** is assumed to be fixed for each flight. Though the concept of time-window has been proved to be useful, we don't consider it in this model. If required, it can be incorporated in our model (Though, it would increase complexity of the model).
- **Code-sharing** represents sharing of seats by an airline with another airline in terms of using each other's capacity under mutually agreed set of conditions. For our model, we assume that code-sharing is not present.
- **Cost** in our model consist of three components - Cost of assigning a particular fleet type to a flight, Allocation cost an of operating a flight from a particular airport and Per passenger cost. We assume that the first two costs can be combined and used in model along with binary variable, indicating whether a flight is being operated or not. Per passenger cost is assumed to be adjusted in fare component.

Allocation is the cost of obtaining slots at airports to use different ground facilities. It might be concern in selection of itineraries. Though it might one of the important considerations, but we assume that the potential itineraries are considered, taking care of the aspect whether a slot would be available or not. And the corresponding cost can be integrated in the model. Though if the departure time is a variable, then it might be a concern. Although, the time-window considered is normally quite small, still one might need to check the feasibility in terms of procuring slots at airport.

- We assume competition to be present and static. It means that for market share (and demand) calculation etc., all the itineraries (Airline's and competitors) in the market have been considered. But there is no variable for competitors's itineraries, i.e. the itineraries offered by competitors and their corresponding characteristics are known. We distinguish between airline's and competitors itineraries by considering the set of airline's itineraries as a subset of set of all itineraries in the market. In model, for profit calculation expressions, we use the this subset having only the airline's itineraries.
- **NULL Itinerary** - We consider that Null itinerary exists and captures demand only when capacity in market is less than total demand. We assume that passenger spilled from an itinerary, would not get lost from air travel market, if market has capacity available in any other itinerary. But if total market capacity is less than total market demand, only then the demand would go to NULL itinerary.
- **Number of fleets** - are assumed to be fixed and known.
- We assume only one travel class and don't distinguish between business and economy classes (or different classes). As discussed above also, we do not consider different fare levels and use average fare for each itinerary.

The reason is that we want to focus on the modeling of capturing demand-supply interactions. To do so and keep the model simpler, we formulate these assumptions. These assumptions are not restrictive, as these extensions can be easily incorporated in the model.

- **Time Horizon** - The model can be used for any planning horizon length (1 or more days). We assume the planning horizon to be independent of adjacent periods. For example, we assume that demand for a day has to be satisfied by same day's itineraries. It does not interact with previous or next day's demand. But in real cases, passengers do shift across days.

The above list provides the set of assumptions and defines the scope of our model. We model and describe the schedule design and fleet assignment problem in the following section.

3 Notations and Definitions

List of notations, used in our models, is as following:

Sets

J - Set of all Itineraries, i.e. consists of airline's and competitor's itineraries

I - Set of airline's itineraries

W - Set of fleet types

P - Set of flights

A - Set of airports

LS_i - Set of flight in itinerary i

Indices

i - Index for itineraries in set I

j, k - Index for itineraries in set J

w - Index for fleet type in set W

p - Index for flight in set P

a - Index for airport in set A

Parameters

F_w - Number of fleet type w available

$Fare_i$ - Average fare for itinerary i

Avl_w - Number of seats available on fleet type w

$C_{p,w}$ - Cost of assigning fleet type w to flight p

$Nleg_i$ - Number of legs in itinerary i

t_i - Departure time of itinerary i

α_p^i - 0/1 indicator, tell if flight p is a part of itinerary i or not

τ_h - Desired departure time points

B_j - Utility value of itinerary i

$m(i)$ - Market for itinerary i

V_i - Spill penalty for itineary i , calculated as sum of $Fare_i$ and a big penalty number

r_j^i - Recapture rate for potential customers spilled from itinerary j and recaptured on itinerary i

Variables

$U_{h,m}$ - Demand at departure time h in market m

Z_i - 0/1 variable;

equals 1 if itinerary i is being operated;

equals 0 otherwise

$f_{p,w}$ - 0/1 variable;

equals 1 if flight p is assigned fleet type w ;

equals 0 otherwise

$y_{w,a,t}$ - Indicates the number of fleet type w available at airport a at time t

$y_{w,a,t}^-$ - Number of fleet type w available at airport a just before time t

$y_{w,a,t}^+$ - Number of fleet type w available at airport a just after time t

$Spill_i$ - Number of potential customers spilled (Denied service) from itinerary i

Rcp_i - Number of customers Recaptured by itinerary i

Function

G_h - Utility function for schedule delay at departure time h ; For other attributes it has been assumed to be linear.

4 Model

We explain the objective function and constraints of our model in this section. While we provide our model, some of the constraints like flow balance etc. represent the standard constraint structure of literature ((Hane et al., 1995), (Kniker, 1998)) We describe the model as following:

(1) Objective Function

Objective of the model is to maximize profit. It constitutes revenue obtained from unconstrained and recaptured demand. The cost consists of cost of assigning a particular fleet type to each flight being operated (operating cost of fleet and cost of slot at airport). We also subtract the cost of spill to airline to minimize spill.

Objective function is formulated as:

$$\sum_{i \in I} \left(\sum_h U_h \frac{e^{B_i} \cdot Z_i}{\sum_{j \in J; m(i)=m(j)} e^{B_j} \cdot Z_j} + Rcp_i \right) \cdot Fare_i - Spill_i^- \cdot V_i - Spill_i^+ \cdot V_i - \sum_{p \in P} \sum_{w \in W} C_{p,w} f_{p,w} \quad (6)$$

We provide the explanation of each term as following. $\sum_{i \in I} \left(\sum_h U_h \frac{e^{B_i} \cdot Z_i}{\sum_{j \in J; m(i)=m(j)} e^{B_j} \cdot Z_j} \right) \cdot$

$Fare_i \cdot Z_i$ represents Unconstrained Demand Revenue. $Spill_i^- \cdot Fare_i \cdot Z_i$ denotes the revenue lost due to spill. $Rcp_i \cdot Z_j \cdot Z_i$ denotes the revenue due to recapture demand. $\sum_{p \in P} \sum_{w \in W} C_{p,w} f_{p,w}$ expresses cost of fleet assignment to flights.

(2) Coverage Constraint

It states that each flight should be assigned maximum of one fleet. The constraint is formulated as:

$$\sum_{w \in W} f_{p,w} \leq 1 \forall p \in P \quad (7)$$

It tells that if the flight is being operated (i.e., being assigned a fleet), exactly one of $f_{p,w}$ would equal to one. Otherwise, the variable $f_{p,w}$ would be zero for all w .

(3) **Capacity Constraint**

The constraint explains that the demand carried by a flight should be less than equal to its capacity. But we formulate it as an equality constraint because of the reason explained in previous section with regard to spill.

$$\sum_{j \in J} \left(\sum_h U_h \frac{e^{B_j}}{\sum_{k \in J; m(j)=m(k)} e^{B_k} \cdot Z_k} \cdot \alpha_p^j \cdot Z_j - Spill_j^- \cdot \alpha_p^j + Spill_j^+ \cdot \alpha_p^j + \alpha_p^j \cdot Rcp_j \right) = \sum_w Avl_w f_{p,w} \forall p \in P \quad (8)$$

Here the first term denotes unconstrained demand. Spill has been divided into positive and negative components to take care of the fact that each one becomes positive depending on whether the capacity is less than demand or not. One term of the pair is always zero. This is done to take care of the fact that Spill should take positive values, only if capacity is less than demand. The next term denotes the recapture of each itinerary. The left hand side explains that the net demand carried by fleet should be less than the number of seats available on fleet.

(4) **Spill Constraint**

The constraints tell that spill from an itinerary is possible only if the itinerary is being operated. It can be stated as following:

$$\begin{aligned} Spill_i^- &\leq M \cdot Z_i \\ Spill_i^+ &\leq M \cdot Z_i \end{aligned}$$

Here M is a big number.

(5) **Recapture Constraint**

Recapture is calculated with the help of following constraint

$$Rcp_i \leq \sum_{j \in J; m(i)=m(j)} r_j^i \cdot Spill_i^-$$

Recapture from different itineraries adds to the value of maximization objective function. So, we try to be equal to right hand side. Another constraint related to recapture is:

$$Rcp_i \leq M \cdot Z_i$$

It ensures that recapture happens only if the itinerary is present.

Modeling of recapture (and Spill) is different in our approach as compared to ones existing in literature ((Kniker, 1998), (Lohatepanont & Barnhart, 2004)).

(6) **Flow Balance Constraint**

The following constraint tells that the number of fleet of each type would get affected at each airport whenever some operation (departure or arrival) happens. It states that the difference between number of fleet at the airport would equal to net affect of departure and arrival such that each departure would reduce the fleet on airport and arrival would increase it. The constraint is formulated as:

$$y_{w,a,t^-} + \sum_{p \in I(w,a,t)} f_{p,w} - y_{w,a,t^+} - \sum_{p \in O(w,a,t)} f_{p,w} = 0 \forall \{w, a, t\} \in N \quad (9)$$

(7) **Fleet Availability Constraint**

The constraint tells that the number of fleet on ground and air at any point of time should be less than or equal to the fleet available.

$$\sum_{a \in A} y_{w,a,t_n} + \sum_{p \in CL(w)} f_{p,w} \leq Num_w \quad (10)$$

(8) **Flight-Itinerary 0/1 Constraint**

The following two constraints take care of the fact that an itinerary variable is 1 only if all the flights in the itinerary are operating.

$$Z_i - \sum_{w \in W} f_{p,w} \leq 0 \forall p \in LS(i) \quad (11)$$

$$Z_i - \sum_{p \in L(i)} \sum_{w \in W} f_{p,w} \geq 1 - Nleg_i \forall i \quad (12)$$

(9) **Range Constraint**

This set of constraint explains which variables are 0-1; non-negative etc. These can be shown as following:

$$f_{p,w} \in \{0, 1\} \quad (13)$$

$$Z_i \in \{0, 1\} \quad (14)$$

$$y_{w,a,t} \geq 0 \forall w, a, t \in L \quad (15)$$

$$spill^+, spill^- \geq 0 \quad (16)$$

Model

We present the full model as following:

Maximize

$$\sum_{i \in I} \left(\sum_h U_h \frac{e^{B_i} \cdot Z_i}{\sum_{j \in J; m(i)=m(j)} e^{B_j} \cdot Z_j} + Rcp_i \right) \cdot Fare_i - Spill_i^- \cdot V_i - Spill_i^+ \cdot V_i - \sum_{p \in P} \sum_{w \in W} C_{p,w} f_{p,w} \quad (17)$$

such that

$$\sum_{w \in W} f_{p,w} \leq 1 \forall p \in P \quad (18)$$

$$\sum_{j \in J} \left(\sum_h U_h \frac{e^{B_j}}{\sum_{k \in J; m(j)=m(k)} e^{B_k}} \cdot \alpha_p^j \cdot Z_j - Spill_j^- \cdot \alpha_p^j + Spill_j^+ \cdot \alpha_p^j + \alpha_p^j \cdot Rcp_j \right) = \sum_w Avl_w f_{p,w} \forall p \in P \quad (19)$$

$$Spill_i^- \leq M \cdot Z_i \quad (20)$$

$$Spill_i^+ \leq M \cdot Z_i \quad (21)$$

$$Rcp_i \leq \sum_{j \in J; m(i)=m(j)} r_j^i \cdot Spill_i^- \quad (22)$$

$$Rcp_i \leq M \cdot Z_i \quad (23)$$

$$y_{w,a,t^-} + \sum_{p \in I(w,a,t)} f_{p,w} - y_{w,a,t^+} - \sum_{p \in O(w,a,t)} f_{p,w} = 0 \forall \{w, a, t\} \in N \quad (24)$$

$$\sum_{a \in A} y_{w,a,t_n} + \sum_{p \in CL(w)} f_{p,w} \leq Num_w \quad (25)$$

$$Z_i - \sum_{w \in W} f_{p,w} \leq 0 \forall p \in LS(i) \quad (26)$$

$$Z_i - \sum_{p \in L(i)} \sum_{w \in W} f_{p,w} \geq 1 - Nleg_i \forall i \quad (27)$$

$$f_{p,w} \in \{0, 1\} \quad (28)$$

$$Z_i \in \{0, 1\} \quad (29)$$

$$y_{w,a,t} \geq 0 \forall w, a, t \in L \quad (30)$$

$$spill^+, spill^- \geq 0 \quad (31)$$

It is a non-linear mixed integer model. We discuss the transformation to handle the non-linearity in the following section.

5 Non-linearity in the Model

Non-linearity exists due to terms $D_i \cdot Z_i$, where

$$D_i = \sum_h U_h \cdot \frac{e^{B_i}}{\sum_j e^{B_j} \cdot Z_j}$$

This structure can be mapped to Multiple-Ratio 0-1 Fractional Programming (Hyperbolic Programming) - Maximization of sum of the ratios of linear 0-1 functions. We use the approach given by (Wu, 1997) to solve our problem. It can be explained as below:

Say the expression is $\frac{a_i \cdot Z_i}{\sum_j a_j \cdot Z_j}$

(For our problem - a_i is the exponential of utility in the demand expression)

put $c = \frac{a_i \cdot Z_i}{\sum_j a_j \cdot Z_j}$

The term $D_i \cdot Z_i$ becomes $c \cdot Z_i$. Transformation for such non-linearity has been explained as below.

Polynomial mixed 0-1 term $y_i = c \cdot Z_i$ where c is a continuous variable and Z_i is a binary variable, can be represented by following inequalities (K - big number)

$$c - y_i \leq K - K \cdot Z_i$$

$$y_i \leq c$$

$$y_i \leq K \cdot Z_i$$

$$y_i \geq 0$$

So, we use the above type of transformation for our model.

The resultant model is a mixed integer linear programming model and we solve the above model using ILOG CPLEX.

6 Computational Experiments

We illustrate the functioning of our model with the help of following two examples.

(1) **Example 1.**

In this small example, while we solve our model; we also try to solve the problem, considering unconstrained demand as an input parameter.

The characteristics of the problem are as following:

- 2 Airports - A and B
- Number of fleet available - 2
- Fleet type - 1
- Number of flights = Number of itineraries =7
(Market 1 (A to B) has 4; Market 2 (B to A) has 3)
- Competition present - yes
- Competitor's itineraries (same as competitor's flights): (Market 1 has 2); (Market 2 has 3)

Itinerary	Market	Airline	Dep time	Arrtime	Fare
1	1	1	7	9	2500
2	1	1	8	10	2500
3	1	1	12	14	2500
4	1	1	18	20	2800
5	2	1	11	13	3000
6	2	1	14.5	16.5	3000
7	2	1	17	19	2500
8	1	2	9	11	2700
9	1	2	14	16	2500
10	2	2	16	18	3000

Table 2 **Example 1**

The demand in two markets as a function of time is as following. In market 1 (A to B), demand has been considered at 4 desired departure time: 6:30, 9, 12 and 17. The corresponding demand is 180, 160, 40 and 170. Market 2 considers demand at 3 desired departure time: 8, 13 and 17. Demand at this points is 170, 110 and 150 respectively.

We solved the above problem using two approaches: First one is to use model presented in this paper. The second one, solves the problem without taking demand-supply interactions into account. For the second model's input demand calculation, all itineraries are supposed to be op-

erating and hence are used for demand share calculation. For illustration purpose, only departure time has been considered in utility function. Though the model considered in the paper is a general model, capable of taking different attributes.

The comparison of two approaches can be discussed as following:

The itineraries selected in Market 2 (i.e. B to A) are same in both the approaches. But the itineraries selected in market 1 differ.

The following table discusses the comparison between two approaches (for market 1):

Criteria	Our Model	Without Demand-Supply Interactions
Itineraries Selected	2, 3, 4	1, 2, 4
Expected Revenue (units)	857723	855931

Table 3 **Comparison**

The above table shows the performance of two models. For second model (i.e. not the one presented in this paper), we calculated expected demand and hence expected revenue by using the demand calculation of itineraries such only the set of itineraries operated are considered.

Next, we checked the performance of two solutions in market 1 with the help of simulation. We simulated the demand for 32 days. We assumed the normal distribution, with expected value of demand as calculated by above results and a common standard deviation of 0.3. We compared the results for solution obtained by our model with the one having no demand-supply interactions. For the same itinerary, Mean demand values were different in our approach from the other approach. We explain the reason as following. For example, itinerary 4 is present in both the solutions. But its average demand is different in both the cases, as the set of other itineraries is different. We did the simulation three times. It means each time we generated demand for 6 itineraries (3 in our solution and 3 in other one) for 32 days with their respective mean and standard deviation. The means for itineraries in our solution are - 115, 107 and 108 (itinerary 2, 3 and 4 respectively). The mean values for other solution itineraries are - 103, 112 and 112. The solution obtained are as following: Our approach total revenue over 32 days is (28136428, 27398424 and 28728404) for set 1,2 and 3. The revenue for obtained by second approach is (27619173, 26841885, 273392830) for set 1, 2 and 3 respectively.

We illustrated the functioning of our model vis. a vis. with a model that does not capture demand-supply interactions. Even in this small example we show that there is a possibility of the solution deviating from the optimal solution. So, in big problems where the network effects can increase a lot, measuring total affect of such errors might be much more difficult.

(2) **Example 2**

The problem can be explained as following:

Itinerary	Market	Airline	Dep time	Arvertime	Fare
1	1	1	7	9	250
2	1	1	8	10	250
3	1	1	10	12	250
4	1	1	12	14	250
5	1	1	16.5	18.5	265
6	1	1	18	20	280
7	1	1	16.5	18.5	265
8	1	1	18.75	20.75	280
9	2	1	6	8	310
10	2	1	10.5	12.5	290
11	2	1	11.5	13.5	300
12	2	1	14.5	16.5	300
13	2	1	17	19	280
14	1	2	9	11	270
15	1	2	14	16	250
16	2	1	16	18	300

Table 4 **Example 2**

Market demand as a function of time of day is as following:

Market	Demand at 6.5	9	12	17
1	180	160	40	190
2	150	85	50	170

Table 5 **Demand - Example 2**

Fleet of airline consists of 2 aircrafts of same type. Capacity of the aircrafts is 180.

The above problem was solved using our model. The solution obtained is as following. The final set of itineraries operating consists of - 2, 3, 5, 6, 8, 10, 11. There was no spill and hence no recapture in the model.

Other than these two examples, we checked the our model on another problem with 4 airports, 12 markets, 3 fleet type (having total 7 aircraft - type one has 2 aircrafts, type two has 3 aircrafts and type three has

2 aircrafts) and 70 flights. However, we do not provide the details like departure time of each flight here because of space constraint.

7 Contribution from the Paper

We developed a model that provides the framework and model for incorporating unconstrained and constrained demand as a function of supply. The issue of capturing demand in a methodical and less subjective way has been discussed in detail. We provide a methodical approach for unconstrained demand calculation, which has less subjective components and hence makes our model more scalable than existing models. On constraint demand side also, we contribute in terms of providing a new approach, that helps in overcoming the gaps in existing approaches. We consider competition in demand share models. The demand share models used are logit-based models. According to our knowledge, there is no other work using logit-based model for these planning stages. We discuss the appropriate transformations to remove non-linearity of our model and validate the resulting model with the help of examples. While our paper solves the defined problem, it also provides framework to solve similar problems. For example, it can be extended to solve code-sharing problems.

Appendix

(1) Important Definitions

In this section, we provide brief introduction to some of the demand basics. ((Lohatepanont & Barnhart, 2004))

- **Unconstrained Demand**, is defined as the number of people who would like to travel by an itinerary when the capacity constraints are not taken into account. Unconstrained demand can be at market level and is generally calculated using socio-economic, presence of air travel facilities and distance etc. Itinerary level unconstrained demand can be calculated from market level demand using different share calculation models. Examples of such models are Quality Service Index (QSI) and Logit based models. These models take into account the availability of itineraries (in the market) and their corresponding characteristics (e.g. departure-time, direct, through, time-of-day demand etc.) to calculate the share of demand an itinerary would obtain. For example, in a given market $m1$, the unconstrained demand for an itinerary i is defined as the number of people who would like to travel by itinerary i irrespective of the capacity available on the itinerary i .
- **Constrained Demand**, is the demand after the capacity restriction have been taken into account. One of the simple examples can be provided as - say, for itinerary i the unconstrained demand is 100 and the capacity of the fleet is 80. Hence, airline can't satisfy the full de-

mand. So, the constrained demand can take the maximum value of 80 (Though, in practice, it might be less than that because of the last minute cancellations etc.). The next question, that is which passengers, airline prefers to carry above others and how does it decide, constitutes a set of problems to be solved. Examples of some ways, that airlines use to balance demand over different itineraries are - using appropriate fleet assignment (at the planning level), using fare differentiation (planning level and later revenue management), while taking care of the airline's objectives. For example, assume that the fare for the itineraries i and j was same, and the unconstrained demand for the them were 100 and 60 respectively. Assume that the capacity of fleet assigned to i is 80. So, if fares of i and/or j are changed, some demand can be shifted from i to j . So, the demand for an itinerary can be changed, for the airline's benefit, to some extent.

- **Spill - (Belobaba, 2006)** is total number of potential passengers who cannot obtain a reservation and travel on a given flight due to insufficient capacity.
 - Spill is by definition equal to total demand minus the total load of the flight
 - When demand is less than capacity, spill is zero
 - When demand exceeds capacity, spill is equal to demand minus capacity
- **Recapture, (Lohatepanont & Barnhart, 2004)** occurs when passengers spilled from their desired itineraries are accommodated on alternative itineraries of same airline.

(2) **Calculation/Forecasting of Demand**

Demand forecasting is very crucial for an airline, as it affects various scheduling and planning decisions. Demand calculation for an itinerary is done in two steps:

- (a) Market Demand Forecast - for a given market at a given time
- (b) Itinerary/flight share models tell share of an itinerary based on the total itinerary offering in the market and the corresponding attributes of the itineraries.

Demand forecast for a market is based on various demographic and economic factors. The total demand can also be affected by the total frequency of the flights available, fares and the other factors. One of the other important factors about the total demand is time-of-day. Total demand for a market is generally represented as a function of time-of-day (time-of-day demand curve). The curve has been shown to follow an M-pattern in literature, which means that in the first half of day, demand first increases and then decreases and similar is the case for the second half. Though, normally the two peaks are at different levels.

The paper mainly assumes that the demand forecast for the markets is known. We are interested in finding out the share of this forecast, a particular itinerary will get.

The next step is to find out the share of each itinerary in a market. This is done with the help of the share models. Going into the details of the share models in airline literature, two types of models are used - QSI (Quality Service Index) based and logit based (Discrete choice models/Random utility models). However other classes of models do exist in literature like EBA and tree-based. These models have been used in the field of psychology and marketing. But they have not been used much in the airlines literature. EBA model might be useful in providing the information about the relation between different pairs of itineraries and hence in the analysis of the system. But the technique requires to estimate a large number of parameters. And in the domain of airline scheduling, it might grow exponentially (as the number of itineraries is very large).

In the recent times, there are many developments in the study of these discrete choice models and also in their applications to the airline industry ((Coldren, 2005)). But what are the marginal effects of change in the value of the attributes of the itineraries, is also an important field of study, as it provides insights about the attributes of the itineraries and their relative affects. The work in the elasticity study part is comparatively less work . The objective, in these studies, might be to find attribute elasticity and availability cross-effects.

In this paper, we use the demand share model given by (Parker & Walker, 2006). But the additional assumption made is that the day has been divided into various discrete time points (Denoting desired travel time) and the demand is represented corresponding to each desired departure time.

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