# Airport Terminal Capacity Planning using Delay Time Approximations and Multistage Stochastic Programming

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### Abstract

An important part of the airport terminal design process is the determination of the optimal design capacities for different areas of the terminal under the uncertainty of future demand levels and expansion costs. Due to the difficulty of this task, most studies in this area either do not account for expandability or focus only on one particular area of the terminal. Furthermore, modeling passenger flow under the transient demand patterns is usually difficult due to the complex structure of an airport terminal. In this study, we aim to remedy these shortcomings first by developing functions to approximate maximum delays in passageways and processing stations during peak periods. We then use these delay functions to propose a multistage stochastic programming approach based on a multicommodity flow network representation of the whole airport terminal. Solution of the proposed model provides optimal capacity requirements for each area in an airport terminal during the initial building phase, as well as the optimal expansion policy under stochastic future demand.

*Keywords:* airport terminal; capacity planning; multistage stochastic programming

## 1. Introduction

Congestion is a significant problem for the hundreds of thousands of passengers flying in and out of major airports each day. This problem has been exacerbated over the last five years by the heightened levels of security. Hence, capacity planning during the airport terminal design process is more important than ever, suggesting a need for the development of more accurate analysis methods. However, the uncertainty associated with future passenger demand levels and the complexity of the airport terminals make this a difficult task.

Several studies have been conducted on capacity planning at airport terminals. [4] emphasizes the need for a solution to the problem of congestion caused by lack of capacity, arguing that if no remedial actions are taken, it could lead to an eventual functional breakdown of the airport system.

In practice, most such actions are realized in the form of costly expansion projects, because there are limited resources available during the initial construction, and great uncertainty as to future demand. However, it is crucial that the need for expansion and the costs associated with the initial design and future expansion projects are minimized. Significant, long-lasting increases in airport terminal capacity can only be achieved through the building of new terminals that are designed to be expandable from their very conception. Considering that upwards of twenty airports may need to be built worldwide in the next two decades, there is a distinct need for new terminal designs that are efficient and flexible enough to accommodate the wide range of demand scenarios that are possible, given the significant, historically observed uncertainty in the demand for air transportation.

Most studies on the optimum design of airport terminals consider the capacity problem using single period approaches based on short-term demand forecasts and the corresponding passenger flows within the terminal. Using this concept, [11] develops a resource utilization model in which the cost of oversizing or undersizing the terminal facilities is minimized, while [6] presents a capacity analysis model based on the study of movements within the terminal during discrete time intervals. In addition, [7] suggests a multi-channel queuing system approach to analyze passenger processing times under different capacity levels. Other queuing models have also been considered for passenger flow analysis at airport terminals. However, due to the high variability in the number of arrivals and departures during a typical day, a steady-state assumption is not valid for airport terminals. Hence, the well-known steady state results for queuing systems are mostly inapplicable. On the other hand, transient studies are generally intractable due to the complexity of flow in an airport terminal. Thus, most studies involve simulations to model this random and complex flow process. In these studies, simulation results are used to estimate the optimal capacity levels to make the operations more efficient. One such example is by [5], in which a simulation model is proposed to evaluate several terminal design alternatives.

Due to the difficulty of modeling passenger flow in a complex terminal structure with transient demand patterns, none of the existing models address the airport terminal capacity problem in a truly holistic fashion. Furthermore, expandability is never accounted for. To remedy these shortcomings, in this study we first develop time functions to approximate maximum delay in passageways and processing stations. It is assumed that the level of service at airport terminals is measured by the total time a passenger spends in the system. This is consistent with the criteria used in most design applications, where capacity is measured in terms of the processing times of passengers at different service stations[2]. Only those processes required for arrivals or departures are considered in total time calculations, which also include walking times. Using the developed time functions, optimal capacities corresponding to highest possible levels of service are calculated using a stochastic programming model based on a multicommodity flow network representation of the whole airport terminal. Optimal capacity levels at the processing stations and passageways of the terminal for multiple planning periods are the output of the model, as well as the optimal expansion decisions with recourse option under the uncertainty of demand. Section 2 describes the approximations for the time functions used in the optimization model, while the proposed multi-stage stochastic optimization model is discussed in Section 3. The results and the contributions of the study are presented in Section 4.

## 2. Approximation of Maximum Peak Period Delay

Capacity analysis of airport terminals aims at minimizing congestion related passenger delay in the terminals. Hence, approximation of walking times in passageways and delay times at processing stations as a function of the capacity and flow rates is an important part of any capacity planning model. Due to the stochastic and transient nature of demand, most such estimations are based on observational data or simulation models, which do not provide appropriate inputs for optimization models. In this study, we consider the walking and processing delays separately, and develop delay time approximations for the two areas. In addition, we analyze the validity of the developed time functions by comparing them with simulation results.

### 2.1 Maximum Delay in Passageways

Although there have been several studies on travel time functions for vehicular traffic, such studies are rare for pedestrians in transportation terminals. One such study is [17], in which pedestrian walking speeds are observed and analyzed in two major airport terminals. Results from this study suggest that free-flow walking speeds in airport terminals are normally distributed with a mean of 80.5 m (264 ft) per minute and a standard deviation of 15.9 m (52 ft) per minute. In addition, several analyses of general pedestrian traffic exist in the literature [3,9,12,13,14]. All such studies include estimations of the relationship between the speed of pedestrians and the congestion levels. Using the free-flow speeds from [17] to adjust the relationship suggested by [12], we determine the following linear function to represent the relation between the speed (m/s) and density (passengers/m$^2$) in airport terminal passageways:

$$s = -0.34\rho + 1.34 \tag{1}$$

where $s$ represents the speed and $\rho$ is the density. To approximate the maximum walking time in a passageway $l$ of length $L_l$, maximum density in the passageway can be estimated using the peak flow rate $f_l$ and width $w_l$ of the passageway. Assuming that the peak load is instantenous and that interarrival times $I_l = 1/f_l$ are exponentially distributed, the mean and variance of the number of passengers in the passageway, $N_l$, can be obtained using the following second order approximations based upon truncated Taylor series expansions[10]:

$$E[N_l] = E\big[\frac{t_0}{I_l}\big] \approx \frac{E[t_0]}{E[I_l]}\big(1 + \frac{Var[I_l]}{E^2[I_l]}\big) \tag{2}$$

$$Var[N_l] = Var\big[\frac{t_0}{I}\big] \approx \big(\frac{E[t_0]}{E[I_l]}\big)^2\big(\frac{Var[t_0]}{E^2[t_0]} + \frac{Var[I_l]}{E^2[I_l]} - \frac{Var^2[I_l]}{E^4[I_l]}\big) \tag{3}$$

where the random variable $t_0 = \frac{L_l}{s_0}$ is the walking time under free-flow conditions. $E[t_0]$ and $Var[t_0]$ can be estimated using similar approximations, i.e.

$$E[t_0] \approx \frac{L_l}{E[s_0]}\big(1 + \frac{Var[s_0]}{E^2[s_0]}\big) = \frac{L_l}{80.5}\big(1 + \frac{15.9^2}{80.5^2}\big) = 0.0130L_l \tag{4}$$

$$Var[t_0] \approx \big(\frac{L_l}{E[s_0]}\big)^2\big(\frac{Var[s_0]}{E^2[s_0]} - \frac{Var^2[s_0]}{E^4[s_0]}\big) = \big(\frac{L_l}{80.5}\big)^2\big(\frac{15.9^2}{80.5^2} - \frac{15.9^4}{80.5^4}\big) = 0.0024^2L_l^2 \tag{5}$$

It follows from (2) and (3) that

$$E[N_l] \approx \frac{0.0130L_l f_l}{60}(1+1) = 0.000433L_l f_l \tag{6}$$

$$Var[N_l] \approx \big(\frac{0.0130L_l f_l}{60}\big)^2\big(\frac{0.0024^2L_l^2}{0.0130^2L_l^2} + 1 - 1\big) = 0.00004^2L_l^2 f_l^2 \tag{7}$$

We assume that the distribution of $N_l$ is normal, and propose the following design density $\rho_l^d$ for passageway $l$:

$$\rho_l^d = \frac{E[N_l] + 3\sigma_{N_l}}{A_l} = \frac{0.000553f_l}{w_l} \tag{8}$$

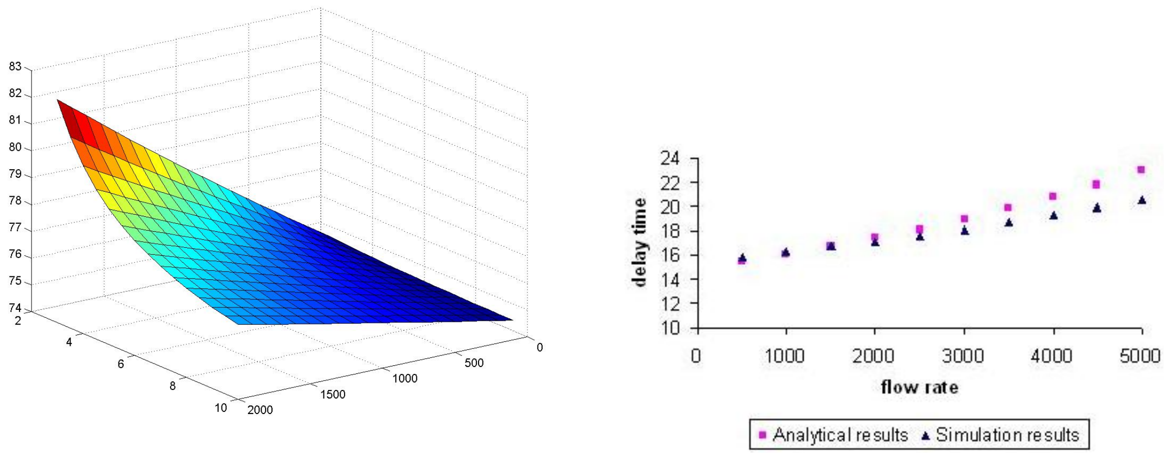
where $A_l = w_l L_l$ is the total effective area of the passageway. Hence, it follows from (1) that the maximum walking time in a passageway can be approximated by

$$t_l^w = \frac{L_l w_l}{-0.000188f_l + 1.34w_l} \tag{9}$$

where $L_l$ and $w_l$ are in meters, $t_l^w$ is in seconds, and $f_l$ is given in passengers per hour. Figure 1(a) is a surface plot of maximum walking times as a function of flow and passageway width. The approximation has been tested and validated on a simulation model based on the walking speed relation (1). Maximum walking times obtained through the simulation of different flow rates and the corresponding calculated values are displayed in Figure 1(b). The results suggest that as flow rate increases, the approximation in (9) starts to overestimate the delay. However, the estimation is accurate for typical peak passenger flow levels in airport terminals.

## 2.2 Maximum Delay in Processing Stations

Most congestion at airport terminals occur at processing stations such as security checkpoints and check-in counters. In this study, we develop relations to estimate the maximum delay at the processing stations in airport terminals as a function of flow and capacity. We first consider a deterministic approach with varying arrival rates and constant process rates, based on fluid approximations suggested by [8]. We then extend this approach to the stochastic case where the queue is assumed to evolve randomly.




(a) Walking times as a function of flow and passageway width (b) Analytical vs. simulated walking times

Figure 1: Maximum delay in passageways

### 2.2.1 Deterministic Approximation

Passenger arrival rates, estimated from flight schedules, can be plotted against time as shown in Figure 2(a). On this plot, the highest peak can be identified to be used in peak demand analysis for design purposes. A peak is defined as a period during which the arrival rate remains above the average arrival rate. We suggest three approximations that can be used to represent the shape of a peak, which lead to an estimation of the maximum queue length. [16] uses similar approximations in airport gate position estimation. Depending on the sharpness of the peak, either a triangular, parabolic or half-elliptical approximation may be appropriate to model the varying arrival rate. These shapes are shown in Figure 2(b). Other approximations or functions can also be used, if the peak can not be represented with these shapes.

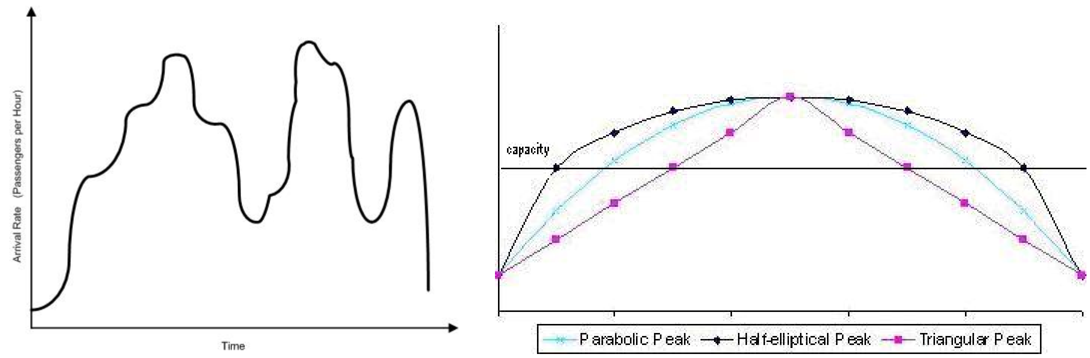
*Triangular Peak Approximation*

If $f_l(t)$ represents the flow rate into a processing station $l$ over time, then the triangular peak function can be expressed as

$$f_l(t) = \begin{cases} f_{l_{avg}} + a_T t & 0 \leq t \leq T_0/2 \\ f_{l_{avg}} + a_T(T_0 - t) & T_0/2 < t \leq T_0 \end{cases} \tag{10}$$

where $T_0$ is the time when the arrival rate drops below the average arrival rate, and $a_T = 2(f_l - f_{l_{avg}})/T_0$ with $f_l$ representing the maximum flow rate. Assuming that queue buildup occurs only after the arrival rate exceeds the capacity $u_l$ of the station, the maximum queue length can be estimated by calculating the area between the capacity line and the triangular arrival rate curve in Figure 2(b). This area is equal to $(t_2 - t_1)(f_l - u_l)/2$, where $t_2$ and $t_1$ can be obtained from the following relation.

$$u_l = f_{l_{avg}} + a_T t_1 = f_{l_{avg}} + a_T(T_0 - t_2) \tag{11}$$




(a) Arrival rates vs. time (b) Peak approximations

Figure 2: Arrival rates and the approximations for peak arrival periods

It follows that maximum queue length, $Q_{l_{max}}$, can be expressed as

$$Q_{l_{max}} = \frac{\big(T_0 - 2\big(\frac{u_l - f_{l_{avg}}}{a_T}\big)\big)\big(f_l - u_l\big)}{2} \tag{12}$$

Then, assuming deterministic service times, maximum delay at the process station for the triangular peak can be obtained from the following relation:

$$t_l^{T^d} = \frac{Q_{l_{max}} + 1}{u_l} = \frac{(u_l - f_l)^2 T_0}{2cf_l u_l} + \frac{1}{u_l} \approx \frac{(u_l - f_l)^2 T_0}{2cf_l u_l} \tag{13}$$

where $c = 1 - f_{l_{avg}}/f_l$ is a constant. The final result is simplified by dropping $\frac{1}{u_l}$, since service rates will typically be high during peak demand periods. Figure 3(a) shows the change in queuing delay as a function of the arrival and service rates. In addition, Figure 3(b) compares the deterministic approximation results with those obtained through a simulation study with Poisson arrivals and exponential process times. As shown in Table 1 deterministic approximations are very accurate at typical peak flow levels in airport terminals. In almost all instances, the differences from simulation results are within 10%.

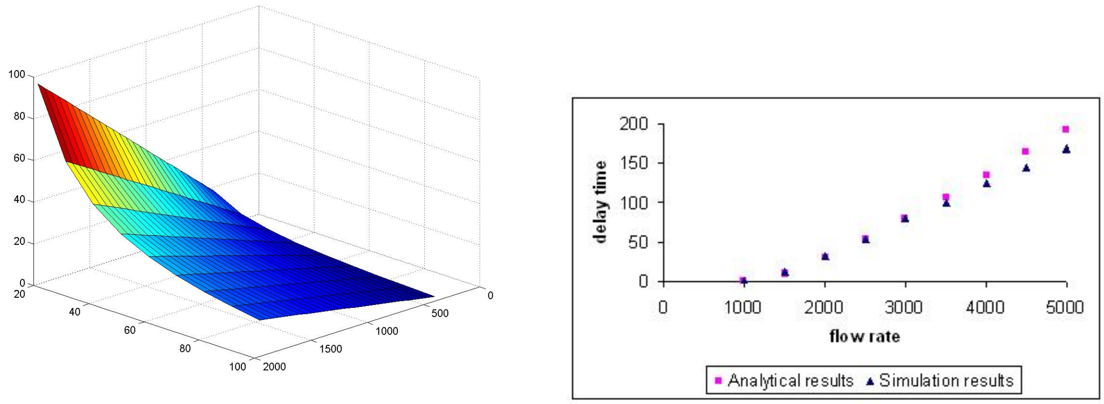
*Parabolic Peak Approximation*

For a parabolic approximation of the peak, the arrival rate curve is

$$f_l(t) = f_l - a_P\big(t - \frac{T_0}{2}\big)^2 \tag{14}$$

where $a_P = 4(f_l - f_{l_{avg}})/T_0^2$. Hence, $t_1$ and $t_2$ are the roots of the following polynomial function of the second degree:

$$f_l - a_P\big(t - \frac{T_0}{2}\big)^2 - u_l = t^2 - T_0 t - \big(\frac{4u_l - 4f_l + a_P T_0^2}{4a_P}\big) = 0 \tag{15}$$




(a) Delay times as a function of flow and capacity (b) Analytical vs. simulated delay times

Figure 3: Deterministic Triangular Approximation

It follows that

$$t_2 - t_1 = \frac{T_0 + \sqrt{T_0^2 - \frac{4u_l - 4f_l + a_P T_0^2}{a_P}}}{2} - \frac{T_0 - \sqrt{T_0^2 - \frac{4u_l - 4f_l + a_P T_0^2}{a_P}}}{2} = T_0 \sqrt{\frac{f_l - u_l}{c f_l}} \tag{16}$$

Similar to the triangular case, the area of the region between the capacity and parabolic arrival rate curves in Figure 2(b) represents the maximum queue length. Thus, maximum time spent for the deterministic parabolic approximation of the peak at process station $l$ can be expressed as follows.

$$t_l^{P^d} = \frac{\frac{2T_0}{3\sqrt{cf_l}}(f_l - u_l)^{\frac{3}{2}} + 1}{u_l} \approx \frac{2T_0 (f_l - u_l)^{3/2}}{3u_l \sqrt{cf_l}} \tag{17}$$

The surface plot of the above function is shown in Figure 4(a). Similarly, Figure 4(b) compares the estimates of the delay with simulation results, while Table 2 shows the percent differences from simulation results. The approximation appears to be very accurate with a maximum error around 5%.

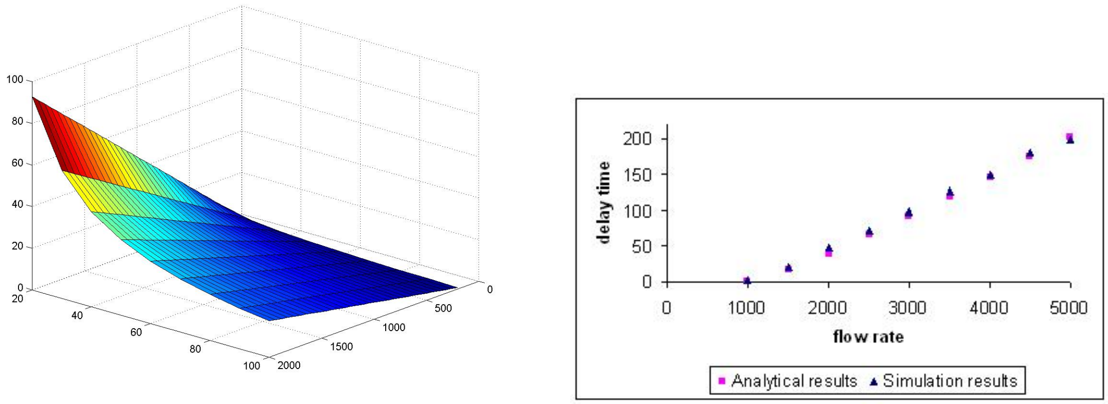
*Half-elliptical Peak Approximation*

Another approximation can be performed assuming that the peak has a half ellipsoid shape as shown in Figure 2(b). In this case, the arrival rate function can be expressed as

$$f_l(t) = f_{l_{avg}} + \sqrt{\left(f_l - f_{l_{avg}}\right)^2 \left(1 - \frac{(2t - T_0)^2}{T_0^2}\right)} \tag{18}$$

It follows that the queue builds up for a period of

$$t_2 - t_1 = \frac{T_0}{cf_l}\sqrt{(f_l - u_l)(u_l - f_l + 2cf_l)} \tag{19}$$




(a) Delay times as a function of flow and capacity

(b) Analytical vs. simulated delay times

Figure 4: Deterministic Parabolic Approximation

Thus, the maximum delay can be estimated as

$$t_l^{E^d} = \frac{T_0 \pi}{4 c u_l f_l} (f_l - u_l)^{3/2} \sqrt{u_l - f_l + 2 c f_l} \tag{20}$$

Although deterministic approximations of delay times provide a means to estimate the total passenger delay in an airport terminal, stochastic effects need to be taken into account for a more accurate representation of passenger flow. Moreover, these approximations must be numerically amenable.

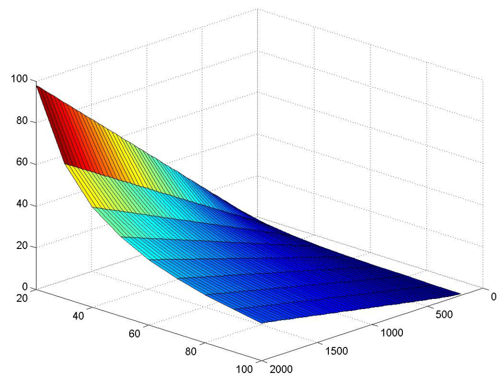
### 2.2.2 Stochastic Approximation

For the stochastic case, delay times can be approximated based on the estimation of the queue length distribution over time. An approach to this problem is suggested by [8], who studies the distribution of the queue length during peak periods using diffusion equations. More specifically, it is suggested that the distribution of $Q(t)$ during peak periods is normal with the following mean and variance, provided that the queue is unlikely to vanish in a short period of time:

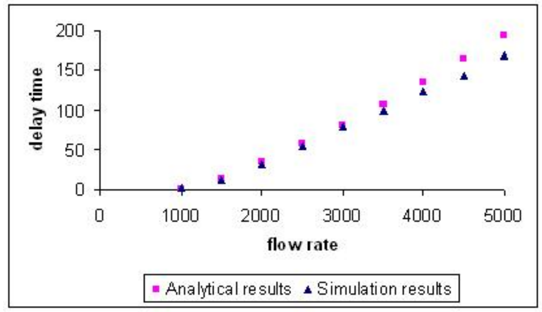
$$E[Q(t)] \approx 0.95 \sqrt[3]{u} + \int_0^t [f(t') - u] dt' \tag{21}$$

$$Var[Q(t)] \approx -0.3 (\sqrt[3]{u})^2 + \int_0^t [C_a f(t') + C_s u] dt' \tag{22}$$

where $C_a$ and $C_s$ are the squared coefficients of variation for the interarrival and service times, respectively. These results can be seen as stochastic corrections for the deterministic approximations discussed in Section 2.2.1. If $T_1$ represents the time period during which the arrival rate stays above the process rate, [8] also shows that the relation $T_1 \geq \frac{2}{(C_a + C_s) \sqrt[3]{u}}$ must hold for the approximations to be valid. For most practical applications, it is reasonable to assume that the arrivals occur according to a Poisson process, which implies $C_a = 1$. Similarly, service times can



(a) Delay times as a function of flow and capacity




(b) Analytical vs. simulated delay times

Figure 5: Stochastic Triangular Approximation

be represented by exponential distribution. In such situations $C_s = 1$, which further simplifies the calculations. Hence, for the validity of the approximations under these assumptions, it is required that $T_1 \geq u^{-1/3}$.

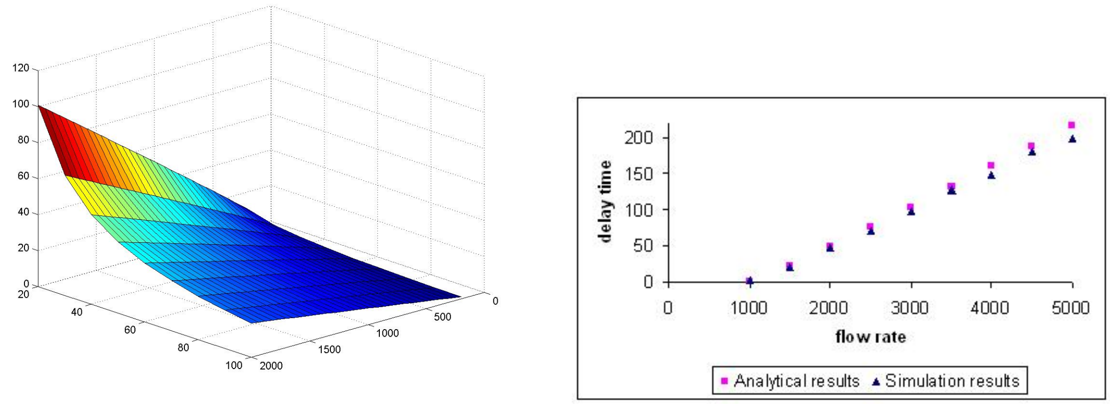
We assume that above conditions hold, and develop expressions to approximate the expected value of the maximum delay time for both the triangular and parabolic peak occurrences. However, the approach could be generalized to any shape that the peak can take. In situations where the peak does not resemble a geometric shape, piecewise calculations can be made to approximate the integrals in the expressions above. On the other hand, for an optimization model, it is necessary that a compact closed form expression is used to represent the delays in process stations. Assuming that the queue length is distributed normally over time, the design criteria for the maximum queue length can be chosen as $Q^d_{max} = E[Q(T_0)] + 3\sigma_{Q(T_0)}$. Using (21) and (22), we calculate the expected value and the variance of the queue length for a triangular peak as follows. For compactness, we leave out the subscript $l$ in $f_l$ and $u_l$.

$$\begin{aligned} E[Q(T_0)] &\approx 0.95\sqrt[3]{u} + \int_0^{T_0/2} [f_{avg} + a_T t - u]dt + \int_{T_0/2}^{T_0} [f_{avg} + a_T(T_0 - t) - u]dt \\ &\approx 0.95\sqrt[3]{u} + \frac{(u-f)^2 T_0}{2cf} \end{aligned} \tag{23}$$

$$\begin{aligned} Var[Q(T_0)] &\approx -0.3(\sqrt[3]{u})^2 + \int_0^{T_0/2} [f_{avg} + a_T t + u]dt + \int_{T_0/2}^{T_0} [f_{avg} + a_T(T_0 - t) + u]dt \\ &\approx -0.3(\sqrt[3]{u})^2 + 2T_0(f-u) - \frac{3T_0(f-u)^2}{2cf} \end{aligned} \tag{24}$$

It follows that,

$$t^{p_{T^s}} \approx \frac{Q^d_{max}}{u} = \frac{0.95\sqrt[3]{u} + \frac{(u-f)^2 T_0}{2cf} + 3\sqrt{-0.3(\sqrt[3]{u})^2 + 2T_0(f-u) - \frac{3T_0(f-u)^2}{2cf}}}{u} \tag{25}$$




(a) Delay times as a function of flow and capacity (b) Analytical vs. simulated delay times

Figure 6: Stochastic Parabolic Approximation

A similar analysis can also be performed for the parabolic peak case, which results with the following maximum delay function.

$$t^{p_{P^s}} \approx \frac{0.95\sqrt[3]{u} + \frac{2T_0}{3\sqrt{cf}}(f-u)^{\frac{3}{2}} + 3\sqrt{-0.3(\sqrt[3]{u})^2 + 2\sqrt{cf(f-u)} - \frac{4(f-u)^{3/2}}{3\sqrt{cf}}}}{u} \tag{26}$$

A similar function can also be obtained for the half-elliptical peak.

These approximations have been tested with simulation models, and Figures 5(b) and 6(b), as well as Tables 1 and 2 show the comparison of the results with the simulation studies on single processes. It is observed that stochastic approximations do not provide significant improvements when compared with the deterministic estimation. Especially for the parabolic case, stochastic approximation tends to overestimate the maximum delay, while deterministic approximation underestimates it. Although, in all cases the discrepancies are not large, the approximations also need to be considered in a network structure.

## 2.3 Approximations in Networks

The above approximations are valid when peak period analysis is performed on individual process stations. However, in a network structure the propagation of demand has to be considered, since flow into downstream processes will be affected by the capacity of preceding processes. Figure 7 displays the effect of the capacity on the departure process from a service station. As seen in this plot, the departure rate curve has a flat peak due to the capacity at the station. If the arrival rate curve has a parabolic peak, using the parabolic approximation for the departure rates may underestimate the actual arrival pattern at the next service station. A better approximation can be obtained through the half-ellipse approximation. Hence, given a parabolic arrival curve at a station, the arrival rates for all downstream processes should be approximated by using the time functions obtained for the half-elliptical peak.

Table 1: Comparison of Analytical and Simulation Results For Triangular Peak

| Max.flow rate | Max. Delay in Simulation | Deterministic Approx. | %Difference | Stochastic Approx. | % Difference |
|---|---|---|---|---|---|
| 1500 | 12.30 | 10.06 | -18.21 | 14.47 | 17.66 |
| 2000 | 31.40 | 30.06 | -4.27 | 34.47 | 9.78 |
| 2500 | 53.70 | 54.06 | 0.67 | 57.53 | 7.13 |
| 3000 | 78.50 | 80.06 | 1.99 | 80.57 | 2.64 |
| 3500 | 100.00 | 107.20 | 7.20 | 107.71 | 7.71 |
| 4000 | 123.00 | 135.06 | 9.80 | 135.57 | 10.22 |
| 4500 | 144.00 | 163.39 | 13.47 | 163.90 | 13.82 |
| 5000 | 168.00 | 192.06 | 14.32 | 192.57 | 14.63 |

On the other hand, if the arrivals at a station follow a triangular shape, then both the parabolic and elliptical approximations will significantly overestimate the delay. In these situations we suggest using a triangular approximation. If the original pattern is best represented by a half ellipse, then downstream arrivals can also be represented by this pattern. However, in all cases, it is possible to analyze the functions individually and determine the best approximating shape. In the following section, we use these general findings to develop an optimization model for capacity planning at airport terminals.

## 3. The Multistage Stochastic Optimization Model

For analysis purposes, we consider the terminal as a network, in which passengers with different origin-destination pairs move between nodes following corresponding demand patterns. Let $G(\mathcal{V}, \mathcal{A})$ denote this directed network, where $\mathcal{V} = \{v_1, v_2, ..., v_n\}$ is its set of nodes and $\mathcal{A} = \{a_1, a_2, ..., a_m\}$ is its set of arcs. Each node $v_i \in \mathcal{V}$ represents either a physical location in the terminal or the arrival and departure events at a service station, such as the ticket counters or the security checkpoints. Let $\mathcal{A} = \mathcal{A}_w \cup \mathcal{A}_p$ such that $\mathcal{A}_w$ is the set of arcs between location nodes and $\mathcal{A}_p$ is the set of arcs connecting the service arrival and departure nodes. For each passenger type $k = 1, 2, ..., K$, a pair of nodes $\{v_s^k, v_t^k\}$ represents the origin and destination, while $d^k$ denotes the peak demand rate for passenger type $k$. We also define three sets of nodes such that $\mathcal{V} = \mathcal{O} \cup \mathcal{D} \cup \mathcal{R}$. The sets $\mathcal{O}$ and $\mathcal{D}$ contain the origin and destination nodes respectively, and $\mathcal{R}$ is the set of process completion nodes in the network. Furthermore, $\mathcal{R}^k$ is the set of last process completion nodes that passenger type $k$ can visit before arriving at their destination. We also define a set $\mathcal{K}_d$ of passenger types such that $\mathcal{K}_d$ contains those that do not go through a process station to reach their destination. In addition,let $u_l$ denote the service capacity of a process represented by arc $l \in \mathcal{A}_p$. Similarly, let $w_l$ be the width of a passageway represented by arc $l \in \mathcal{A}_w$. In addition, $f_l$ represents peak flow rate on arc $l \in \mathcal{A}$, while $x_l^k$ is the flow of passenger type $k$ on arc $l$. For each arc $l \in \mathcal{A}$, let $t_l$ correspond to the maximum time spent by any passenger on that arc, which, due to congestion, varies with the amount of flow on the arc according to results obtained in Section 2. A simplified sample network representation of an airport terminal is shown in Figure 8.

The described network is similar to a multicommodity flow network, in which different types of passengers correspond to different commodities. Several objective functions can be considered for

Table 2: Comparison of Analytical and Simulation Results For Parabolic Peak

| Max.flow rate | Max. Delay in Simulation | Deterministic Approx. | %Difference | Stochastic Approx. | % Difference |
|---|---|---|---|---|---|
| 1500.00 | 20.40 | 16.33 | -19.95 | 23.12 | 13.34 |
| 2000.00 | 47.10 | 40.00 | -15.07 | 48.56 | 3.10 |
| 2500.00 | 71.10 | 65.73 | -7.55 | 75.66 | 6.42 |
| 3000.00 | 97.30 | 92.38 | -5.06 | 103.49 | 6.37 |
| 3500.00 | 127.00 | 119.53 | -5.88 | 131.69 | 3.70 |
| 4000.00 | 149.00 | 146.98 | -1.36 | 160.10 | 7.45 |
| 4500.00 | 180.00 | 174.62 | -2.99 | 188.63 | 4.80 |
| 5000.00 | 199.00 | 202.40 | 1.71 | 217.24 | 9.17 |

this flow model. An objective could be to find a routing for passengers through the network in such a manner that the maximum total time a passenger spends in the system for the worst case scenario is minimized for all routes. Another similar objective could be the minimization of maximum delay on each passageway and processing station. Consistent with the system equilibrium concept of [15], we assume that during peak demand periods, passenger flow is distributed optimally among alternate routes within the airport terminal. In the proposed model, minimization of maximum delay on each arc is chosen as the objective, and a weight factor corresponding to the arc flow rate $f_l$ is introduced in the objective function of the model for each arc travel time function to approximate this behavior. These delay functions depend on the capacity levels, which are maximized in the optimization model given a budget $B$.

Expandability and the decisions on when to expand play an important role in the determination of the optimal capacity levels for a terminal building. These expansions can be realized by building a separate terminal building or by expanding the existing one to accommodate increased demand. In any case, a fixed cost $\beta_l$ and a variable cost $\alpha_l$ will be incurred for each unit of added capacity $\epsilon_l$ on the component of the terminal represented by arc $l$. Assuming several planning epochs $i, i = 0, 1, 2, ..., T$ and deterministic demand forecasts, a multiperiod decision model based on the network structure described above can be formulated. However, such a deterministic model would not consider the variation in demand forecasts. On the other hand, the randomness associated with demand forecasts may play a significant role in the cost-effectiveness of an expansion policy. These factors can be accounted for by considering the described model in a stochastic setting.

Stochastic programming approaches to capacity expansion problems are common in the literature[1]. We assume that decisions on initial design capacities $u_l^0$ for servers and $w_l^0$ for passageways with associated unit costs $\alpha_l^0$ are made while the specific scenario to occur is unknown. Expansion decisions at future planning periods are made after the realization of demand, providing recourse options. Suppose that demand levels $d = \{d^k, k = 1, 2, ..., K\}$ between consecutive planning periods are assumed to occur at one of multiple levels, i.e. low, medium and high, for each scenario. Hence, a scenario tree $\mathcal{T}$, reflecting possible realizations of demand levels over the planning periods can be constructed. Each node $n$ of the tree corresponds to a state at some planning epoch $i, i = 0, 1, 2, ..., T$. The probability of being in state $n$ is given as $p_n$, and let the subscript $n$ refer to the values of all other parameters at state $n$. Furthermore, let $\mathcal{P}(n)$ represent a path from the root

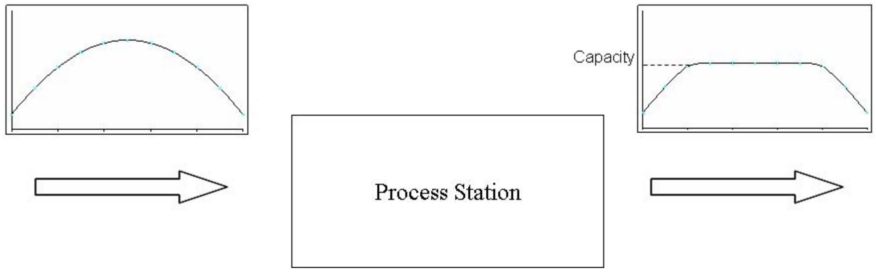



Figure 7: Effect of the capacity on the departure process from a service station

node in the scenario tree to node $n$, and let $\mathcal{N}$ denote the set of non-leaf nodes. If $v_{nl}$ is a boolean variable denoting whether an expansion on arc $l$ is realized at the planning epoch corresponding to node $n$, and $\epsilon_{nl}$ is the amount of expansion, then the following stochastic program can be used to obtain the optimal capacity expansion policy under stochastic demand:

*Airport Terminal Capacity Planning Problem(ATCPP):*

$$\text{minimize } z = \sum_{n\in\mathcal{T}}\sum_{k}\sum_{l\in\mathcal{A}_p} p_n f_{nl} t^p_{nl}(\cdot) + \sum_{n\in\mathcal{T}}\sum_{k}\sum_{l\in\mathcal{A}_w} p_n f_{nl} t^w_{nl}(\cdot) \tag{27}$$

s.t.

$$\sum_{j\in\mathcal{V}:(i,j)\in\mathcal{A}} x^k_{n,ij} - \sum_{j\in\mathcal{V}:(j,i)\in\mathcal{A}} x^k_{n,ji} = 0 \qquad \forall n,k,\forall i\notin\{\mathcal{O}\cup\mathcal{D}\cup\mathcal{R}\} \tag{28}$$

$$\sum_{j\in\mathcal{V}:(i,j)\in\mathcal{A}} x^k_{n,ij} - \sum_{j\in\mathcal{V}:(j,i)\in\mathcal{A}} x^k_{n,ji} = d^k_n \qquad \forall n,k,\forall i\in\mathcal{O} \tag{29}$$

$$\sum_{j\in\mathcal{V}:(i,j)\in\mathcal{A}} x^k_{n,ij} - \sum_{j\in\mathcal{V}:(j,i)\in\mathcal{A}} x^k_{n,ji} = -d^k_n \qquad \forall n,\forall k\in\mathcal{K}_d,\forall i\in\mathcal{D} \tag{30}$$

$$\sum_{j\in\mathcal{V}:(i,j)\in\mathcal{A}} x^k_{n,ij} - \sum_{j\in\mathcal{V}:(j,i)\in\mathcal{A}} x^k_{n,ji} = -\sum_{i'\in\mathcal{R}^k}\sum_{j'\in\mathcal{V}:(i',j')\in\mathcal{A}} x^k_{n,i'j'} \qquad \forall n,\forall k\notin\mathcal{K}_d,\forall i\in\mathcal{D} \tag{31}$$

$$\sum_{j\in\mathcal{V}:(i,j)\in\mathcal{A}} x^k_{n,ij} - u_{n,i'i}\frac{x^k_{n,i'i}}{f_{n,i'i}} = 0 \qquad \forall n,k,\forall(i',i)\in\mathcal{A}_p \tag{32}$$

$$u^0_{nl} + \sum_{m\in\mathcal{P}(n),m\neq n} \epsilon_{ml} = u_{nl} \qquad \forall n,\forall l\in\mathcal{A}_p \tag{33}$$

$$w^0_{nl} + \sum_{m\in\mathcal{P}(n),m\neq n} \epsilon_{ml} = w_{nl} \qquad \forall n,\forall l\in\mathcal{A}_w \tag{34}$$

$$\epsilon_{nl} - M_{nl}v_{nl} \leq 0 \qquad \forall n\in\mathcal{N},n\neq 0,\forall l \tag{35}$$

$$\sum_{l\in\mathcal{A}}(\alpha_{nl}\epsilon_{nl} + \beta_{nl}v_{nl}) \leq B_n \qquad \forall n\in\mathcal{N},n\neq 0 \tag{36}$$

$$\sum_{l\in\mathcal{A}_w}\alpha^0_l u^0_l + \sum_{l\in\mathcal{A}_p}\alpha^0_l w^0_l \leq B^0 \tag{37}$$

$$\sum_k x^k_{nl} = f_{nl} \qquad \forall n,l \tag{38}$$

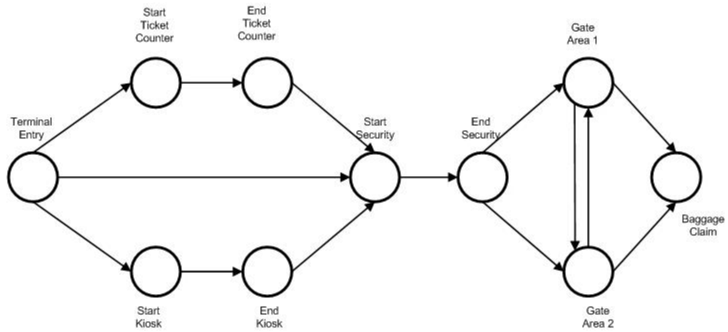



Figure 8: A simplified network representation of an airport terminal

$$u_{nl} - f_{nl} \leq 0 \qquad \forall n, \forall l \in \mathcal{A}_p \tag{39}$$

$$(1-c)f_{nl} - u_{nl} \leq 0 \qquad \forall n, \forall l \in \mathcal{A}_p \tag{40}$$

$$Q(x^k_{nl}, u_{nl}, f_{nl}, \epsilon_{nl}) \leq 0 \qquad \forall n, k, l \tag{41}$$

$$x^k_{nl}, u_{nl}, f_{nl}, \epsilon_{nl} \geq 0 \qquad \forall n, k, l \tag{42}$$

$$v_{nl} \in \{0, 1\} \qquad \forall n \in \mathcal{N}, n \neq 0, \forall l \tag{43}$$

where $t_{nl}(\cdot)$ in (27) represents the time function associated with each arc in the network. Constraints (28)-(32) are node balance constraints. In addition, (33) and (34) ensure that total available capacity is equal to the sum of expansions made up to the current planning epoch. Constraints (35) limit the amount of expansion to $M_{nl}$ and ensure that no expansion is made when $v_{nl} = 0$. Constraints (39) and (40) ensure that the capacity of a process station lies between the average and maximum flow rates into that station. The constant $c$ can be estimated as $c = 1 - (\sum_k d^k_{avg})/(\sum_k d^k)$, which follows from the assumption that $(\sum_k d^k_{avg})/(\sum_k d^k) = f_{l_{avg}}/f_l$.Finally, constraints (36) and (37) are the budget constraints, while (41) refers to a vector of additional constraints imposed on the flows and capacities. These additional constraints may include minimum flow and capacity requirements or those that require simultaneous expansions in different areas of the terminal. Integrality requirements for flows and capacities have been relaxed in the above model. Inputs to the model are the peak inflow rates $d^k$ for each level of demand realization, the cost terms $\alpha_l$ and $\beta_l$, and the arc time function expressions $t_l(\cdot)$, which are discussed in detail in Section 2. In the above formulation, to capture the transient behavior at process arcs during peak load periods, we assume that the departure rate form a process station is equal to the service rate of that station. Hence, $f_l$ in the time functions of downstream arcs is determined by a proportion of the departure rate from the preceding process arcs. This proportion is defined according to the ratio of flows into that preceding arc by constraints (32).

The above model can be implemented with any of the processing delay approximations discussed in Section 2. However, the highly nonlinear terms in the stochastic approximations make the problem more difficult to solve when these estimates are used. There are several other challenges that make the problem difficult, such as the increasing number of variables when multiple stages of the problem are considered. More importantly, constraint (32) is not convex, and thus any solution obtained in reasonable time through available solvers may not be the global optimum. All these factors make it necessary to develop approximation procedures and heuristics to solve this problem. We propose a solution heuristic which is based on the relaxation of the nonconvex constraints (32).

### 3.1 A Heuristic Procedure for ATCPP

The iterative procedure we suggest for ATCPP is based on the transformation of constraints (32) into a relaxed convex constraint. More specifically, we assume that the capacity at each process station can be considered to be composed of dedicated capacities for each passenger type. Hence, node balance constraints for each process completion node can be based on these individual flows and capacities. Then a relaxed version of the ATCPP can be obtained by replacing constraints (32) with (44) below, and also adding constraints (45) and (46):

*RATCPP:*

$$\sum_{j\in\mathcal{V}:(i,j)\in\mathcal{A}} x^k_{n,ij} - u^k_{n,i'i} = 0 \qquad \forall n, k, \forall(i', i) \in \mathcal{A}_p \tag{44}$$

$$x^k_{nl} - u^k_{nl} \geq 0 \qquad \forall n, k, \forall l \in \mathcal{A}_p \tag{45}$$

$$\sum_k u^k_{nl} = u_{nl} \qquad \forall n, l \tag{46}$$

Since this formulation does not enforce the required ratios of flow out of the process completion nodes, as ensured by (32), it forms a lower bound for the optimal objective function value of $ATCPP$. In the proposed heuristic below, we perform iterative steps to ensure that the required ratio restrictions are integrated into the formulation of $RATCPP$:

1. Let $i = 0$, and let $RATCPP_0 = RATCPP$. Solve $RATCPP_0$.
2. Let $\hat{x}^k_l$ and $\hat{f}_l$ be the flows on a processing station $l \in \mathcal{A}_p$ in the solution.
3. $i = i + 1$
4. Fix the flow on station $l$ by creating the following constraints to obtain $RATCPP_i$:

$$u^k_{nl} = u_{nl} * \frac{\hat{x}^k_l}{\hat{f}_l} \qquad \forall n, k \tag{47}$$

$$x^k_{nl} = \hat{x}^k_l \qquad \forall n, k \tag{48}$$

5. Solve $RATCPP_i$.
6. If flows on all processing stations have been fixed, stop. Otherwise, go to step 2.

The selection of the unfixed processing stations in the above algorithm should be such that upstream stations are selected first. Computational results suggest that for most problem instances the above procedure approximates the optimal value of $ATCPP$ with a great degree of accuracy. These computational studies are discussed in the following section. However, a formal analysis of the heuristic is also being studied.

## 4. Computational Results and Conclusions

A computational study was conducted using the simplified network representation of an airport terminal in Figure 8. In this test model, only unidirectional flow was assumed between arcs. However, bidirectional flow can be integrated into the model by approximating the delay times using the speed density relation (1) and assuming that density is based on flow in both directions. An arrival rate curve similar to Figure 2(a) was assumed to be available for each customer type, and lengths of passageways were assumed to be fixed constants. Two multistage models were studied, with three and four stages respectively. Solutions for all deterministic approximations were obtained in a few minutes using the SBB mixed integer nonlinear programming solver. However, additional analysis was necessary to determine the goodness of the solutions. The $ATCPP$ was solved using 100 random starting points, and almost all solutions converged at the same optimal value, suggesting a global optimum. This solution was used to assess the accuracy of the solutions obtained by the heuristic proposed in Section 3.1. In all instances, the heuristic solution was within 5% of the identified optimum. However, a formal analysis and additional computational studies on large problem instances are necessary for a valid assessment of the proposed heuristic.

Overall, results for the test problems under different scenarios suggest that the proposed model is an innovative and powerful tool that can be used in capacity planning at airport terminals. To the best of our knowledge, it is the first holistic model for airport terminals. Developed approximations of transient behavior provide a new and simpler approach to modeling peak periods in networks. Given the inefficiency and congestion associated with current airport designs, it is essential that accurate capacity planning is performed for new airport designs using concepts from this study. The results of the study are also applicable to existing airports. Current configurations of these airports can be modified according to obtained results to maximize efficiency. In addition, by minimizing the need for expansion and optimizing expansion schedules for airports, significant cost reductions can be achieved.

The model is currently being tested on very large networks representing the detailed structure of the terminals. An additional area currently being studied is the modification of the model to incorporate the uncertainties associated with expansions costs and budgets, as well as risks of disruptions in the normal operation of an airport terminal.

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