Rethinking the Airline Crew Scheduling Process

Chunhua Gao (cgao@isye.gatech.edu)
Ellis Johnson (ejohnson@isye.gatech.edu)
School of Industrial and Systems Engineering
Georgia Institute of Technology
Atlanta, GA 30332

Abstract

This research on the airline crew scheduling problem focuses on more efficient solution approaches as well as integrating the crew problem with other airline scheduling problems. It is known that for some schedules there are very good pairing solutions with low pay and credit, while for other schedules the pairing solutions are poor and may be impossible to improve by more computational effort. We explore the reasons intrinsic to schedules that prevent getting good pairing solutions. In this paper, two schedule analysis methods are proposed to evaluate the crew friendliness of a given schedule, and guidelines are given for making small modifications of the schedule when analysis indicates problems. Based on the result and methodology of schedule analysis, application areas such as a new duty partition formulation for crew pairing problem, integrated planning, and integrated recovery are discussed.

1 Introduction

The airline schedule planning process includes a sequence of decision-making phases. These phases are schedule development, fleet assignment, aircraft maintenance routing and crew scheduling. The objective of the crew scheduling problem is to find a minimum cost assignment of flight crews to a given flight schedule. The flight schedule considered includes all flight legs that have been assigned to a single aircraft type, which is the output from the upstream decisions: schedule development and fleet assignment. The crew scheduling problem is typically broken into two sequentially solved subproblems, the crew pairing problem and the crew rostering problem. The crew pairing problem generates minimum cost pairings that cover the collection of flight legs in the schedule, while the crew rostering problem combines the pairings into month-long crew schedules and assigns them to individual crew members. Usually the overall quality of a crew scheduling solution is evaluated by the pay and credit of the crew pairing solution, which is the excess cost of crew beyond the required flying hours. Much of the current research on the airline crew scheduling problem focuses on seeking more efficient solution approaches as well as integrating the crew problem with other airline scheduling problems. Our work is in those directions.

In this paper, schedule analysis methods are presented to evaluate a given schedule for "friendliness" to the crew pairing problem. The analysis is used to find the reasons or structure in the schedule that may cause high pay and credit in the crew pairing solution and to explore guidelines for tweaking the schedule in a small range in order to improve the crew pairing solution. Two types of analysis methods, called *global analysis* and *pattern analysis* respectively, are proposed. In global analysis, the minimum number of duties is one of the important indices. First we attempt to get the number of aircraft and number of crews that overnight at each station. Then we estimate the number of duties needed in the pairing solution, which can finally provide us the information about average duty time. By comparing this average duty time with the parameter of average duty guarantee in the pairing cost function, we are able to know how good the performance of the pairing solution is. In pattern analysis, by further analyzing the two different types of crew overnights, legal short rests and midday breakouts, we can get the patterns of duties and pairings, as well as the number of pairings needed in the pairing solution. Overall, the pattern analysis can provide us clues to adjust the schedule in a reasonable range for improving the crew friendliness.

Most of this paper is concerned with explaining and making concrete the analysis of schedules and how to make small schedule changes to improve subsequent crew solutions. However, we also look at using schedule analysis in other ways. Based on the results and methodologies of schedule analysis, a new duty partition formulation is presented for the crew pairing problem. In addition, both the duty partition model and schedule analysis may be used in an integrated fleet assignment (FAM) and crew pairing model. Finally, integrated recovery is discussed. The method we propose for integrated recovery incorporates schedule adjustment into a dated FAM plus crew model. Then a Benders approach with feasibility and optimality cuts for maintenance routing and passenger service is proposed.

This paper is organized as follows. In section 2 we describe the crew scheduling problem and present an overview of prior work on crew scheduling. In section 3 we present two methods of schedule analysis. Case studies are conducted. In section 4 we discuss the applications of schedule analysis in developing the duty partition model for crew pairing problem. Section 5 presents prior work on integrating FAM and crew. Our proposed model is presented. Section 6 looks at the potential use of these methods in integrated recovery. Section 7 summarizes our work.

2 Problem Description

In our study, we consider the domestic daily crew pairing problem, that is, to construct a set of pairings that cover the collection of domestic flights flown by a given aircraft type in one day at minimum cost. A crew pairing is a sequence of flights that starts at a crew base and ends at that same crew base. It can span several days in which crew members will rest, usually overnight, at some location other than where they reside. The periods between rests in a pairing are called duties. Duties can be thought of as a day's assignment. Note however that a duty may last only a few hours and a rest can occur in the middle of the day.

2.1 Restrictions and Cost Structure

The Federal Aviation Administration (FAA) and contractual restrictions define the structure and cost of legal duties and pairings. A duty begins with a briefing, typically 45 minutes, and ends with a debriefing, typically 15 minutes. These times are part of the duty and must be counted in the elapsed time of the duty. Each duty period must satisfy work rules limiting the maximum number of flights in a duty period, the maximum flying time per duty period, the maximum elapsed time of a duty period, and the minimum and maximum sit time between consecutive flights in a duty period. The cost of a duty, expressed in hours is the maximum of three quantities: the actual flying time in the duty period; a fraction, coeff1, times the elapsed time of the duty; and a guaranteed minimum number of hours, called minimum duty guarantee. The cost can be expressed as:

$$\text{duty cost} = \max\{\sum blocktime, \text{coeff1*elapsed time, min duty guarantee}\}. \quad (1)$$

Legal pairings may be composed of up to a maximum number of duties. A pairing must allow a minimum number of hours of rest between duties. A complicated FAA rule for pairings is the so called "8-in-24". For instance, if the flying time in 24 consecutive hours is larger than 8 hours, while the consecutive hours of rest in that 24 hours is less than 9 hours, than a compensatory rest of at least 10 hours is needed for the next rest. Modeling the problems on a daily basis asks no leg repetition in a pairing, since each flight in each pairing must be covered exactly once each day. The cost of a pairing in hours is the maximum of three quantities: the sum of the costs of the individual duties that make up the pairing; a fraction, coeff2, times the total elapsed time of the pairing, TAFB; and an average duty guarantee times the number of duties in the pairing. The cost can be expressed as:

$$\text{pairing cost} = \max\{\sum duty\cos t, \text{coeff2*TAFB, \#duties*average duty guarantee}\}. \quad (2)$$

The quality of a crew pairing solution is evaluated by pay and credit, defined as:

$$\text{pay\&credit} = \frac{\sum pairing\cos t - total\ blocktime}{total\ blocktime}. \quad (3)$$

Because of the cost structure for duties and pairings, a lower bound on the cost of a given schedule is the total number of hours of flying in the schedule. Pay and credit tells how many percentage of excess cost for the pairings. (Anbil et al 1992) mentions three main causes of pay and credit: pairings that include (1) long or frequent sits within a duty, (2) long overnight rests between duties, and (3) "deadheading". In this paper, reasons intrinsic to schedules for causing pay and credit will be explored in detail in section 3.

2.2 Previous work on crew scheduling

A survey of older work on crew scheduling can be found in Arabeyre et al. (1969), Etschmaier and Mathaisel (1985). More recent survey papers on overall airline scheduling process can be found in Barnhart et al (2003), Clarke and Smith (2004), Barnhart and Cohn (2004).

Because of the complicated restrictions and non-linear cost structures defined on legal duties and pairings, the set partitioning model is powerful to model crew pairing problem. There is one binary decision variable for each possible pairing, and the objective is to minimize the cost of the selected pairings such that for each flight, exactly one pairing containing that flight is chosen. By defining pairings as variables, explicit formulation of complicated working rules can be avoided, also non-linear pairing cost can be computed up in front so that non-linearity can be excluded from the model. The set partitioning model for the crew pairing problem is formulated as:

$$\min \sum_{p \in P} c_p X_p,$$
$$s.t. \quad \sum_{p: l \in p} X_p = 1, \quad \forall l \in L,$$
$$X_p \in \{0,1\}, \quad \forall p \in P$$

Where $X_p = 1$ if pairing p is in the solution, and 0 otherwise. L is the set of flight legs and P is the set of pairings. A column p has a 1 in row l if flight l is flown by pairing p, c_p is the cost of pairing p.

The drawback to this modeling approach is that there are potentially many billions of possible crew pairings, especially in hub-and-spoke networks. Pairings must be either enumerated or generated dynamically by column generation. Enumerating pairings can be accomplished by first enumerating all the possible duties for the schedule and then enumerating all the possible ways to form pairings from the duties. Both duty and pairing enumeration can be accomplished by a depth-first-search approach. A local optimization approach is adopted in Anbil et al. (1991) and Gershkoff (1989). Anbil et al (1992) found an optimal solution over a large subset of the possible pairings to the LP relaxation. Five and a half million feasible pairings were enumerated. Hu and Johnson (1999) present compact storage scheme and primal–dual subproblem simplex method to speed up linear programming solution times for the crew pairing problem. The other approach uses constrained shortest path methods on specially structured networks to price out attractive pairings, as in Lavoie et al. (1988) and Barnhart et al. (1994). In Barnhart et al. (1994), two alternative network representations, the time-space and the time-line network were investigated. In both networks, for each duty there is one arc corresponding to it. The tail node of a duty arc represents the first flight in the duty and the head node of a duty arc represents the last flight in the duty period. Shaw (2003) proposes hybrid column generation method which combines delayed column generation and compact storage for enumerated extended duties. To get the integer solution, a branch-on-follow-ons branching rule is typically used, Ryan and Foster (1981), Chu et al. (1997).

Vance et al. (1997) presented a duty-period-based formulation and proved that its LP relaxation provides a stronger bound than the traditional set partitioning model. In their formulation, the decision process is broken into two stages. First, a set of duty periods is selected that partitions the flight segments. Second, a set of pairings is selected that partitions these duty periods.

The most recent research on crew scheduling includes integrating the crew problem with other airline scheduling problems. Cohn and Barnhart (2003), Cordeau et al. (2001, 2005) present models integrating aircraft maintenance routing and crew scheduling. Clarke et al. (1996) was the first attempt to take into account the crew factors in the fleet assignment model. Sandhu and Klabjan (2004) propose a model that considers fleeting, aircraft rotation and crew pairing simultaneously. Pairings are generated by delayed column generation and the rotation problem is captured by the plane count constraints. Integrated models are considered in sections 5 and 6.

3 Schedule Analysis

It is known that for some schedules there are very good pairing solutions with low pay and credit, while for other schedules the pairing solutions are poor and may be impossible to improve by more computational effort. Therefore, it is necessary to explore reasons or structure in the schedule that prevent us from getting good pairing solutions. Two types of analyzing methods, called global analysis and pattern analysis respectively, are developed. From the analysis, two main types of schedule are found. Although both types can have very low average duty time, they have different duty patterns. Methodology to make schedule adjustments on both types of schedule is discussed. Case studies are investigated.

3.1 Global Analysis

Due to the cost structure for pairings, in a poor pairing solution, frequently we found that the average duty guarantee dominates among the three items. Thus, it becomes crucial to know the average duty time that the schedule actually has. For getting this, we need to estimate the number of duties.

There are two ways to estimate the number of duties.

First, from the schedule, we can calculate the minimum number of planes needed to fly these flights. Dividing total blocktime by the minimum number of planes gives the plane average flying time. If the duration of a plane rotation is long, e.g. greater than eight hours (the maximum blocktime for a duty period), at least two duties are needed for one plane rotation, even crew does not stay with plane. Generally large plane average flying time means long plane rotations. In such case, 2*{the minimum number of planes} gives a good estimation of the number of duties. If the plane average flying time is small, it is possible that the flying time in some plane rotation is less than eight hours, and can be covered by a single duty. In this case, we can try to get the plane rotation by First-In-First-Out heuristic, and obtain the exact number of short plane rotations. Note that it is not dependent on exact plane rotations. By taking this approach, a more precise estimation of the number of duties can be obtained. We denote this first estimation of the number of duties as NumDuty(Aircraft).

Second, from the schedule and legality rules about the overnight rest time, we can find the minimum number of crew overnight rests needed at each station. Then, the total number of crew overnight rests can be calculated. Knowing the maximum length of a pairing, we can obtain the number of duties from the number of crew overnights. For instance, the maximum length of pairing is three means that a pairing consists of at most three duties, as well as two overnight rests need at least three duties. So, {the total number of crew overnights}*3/2 is the second way to estimate the minimum number of duties needed. We denote this second estimation of the number of duties as NumDuty(Crew).

To sum up, the number of duties needed should take the maximum of these two estimations. That is, NumDuty = max { NumDuty(Aircraft), NumDuty(Crew) }. Dividing the total blocktime by NumDuty will give us the average duty time. If this average duty time is much smaller than the parameter of average duty guarantee in the pairing cost function, a poor pairing solution will be anticipated.

Moreover, the difference between these two estimations generates two different types of schedule. For the first type, the number of duties required by crew overnights is much greater than the number of duties required by plane rotation, i.e., NumDuty(Crew) >> NumDuty(Aircraft). This is called **Type I schedule**. For the second type, the number of duties required by crew overnights is much smaller than the number of duties required by plane rotation, i.e., NumDuty(Crew) << NumDuty(Aircraft). This is called **Type II schedule**. Both types could cause poor performance of the pairing solution. A balanced version would be preferable in the crew friendliness point of view. Examples for both types will be illustrated in case studies in section 3.4.

3.2 Pattern Analysis

Based on the information of crew overnight rests and the number of planes overnighting at the crew bases, we can analyze the patterns of duties and pairings. First we classify the starting and ending patterns of duties. From the duty patterns, we can obtain the pairing patterns.

Crew overnight rests consist of legal short rests and midday breakouts. Legal short rest means that an evening arrival flight A legally connects to a morning departure flight B. On the other hand, midday breakouts mean that there is not enough rest time between A and B, instead of a double overnight, it would be better for the pairing solution that a pair of midday flights (arrival flight C and departure flight D) is used to connect with A and B. In other words, one crew arrives in the middle of the day via C and leaves the next morning via B, the other crew arrives in the evening via A and leaves in the middle of the next day via D, as shown in Figure 1. Figure 1 shows that a pair of midday breakout gives a pair of rests of style AM-AM and PM-PM simultaneously. In contrast, a legal short rest is in style of PM-AM.

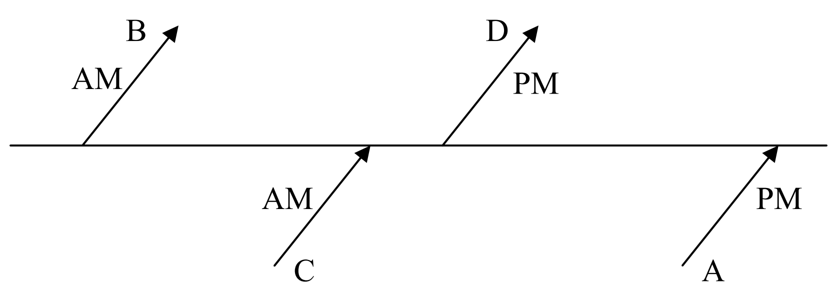


Figure 1 Midday breakouts

Table 1 gives the notation related to defining duty patterns.

Table 1 Notation related to defining duty patterns

Notation	Meaning
CB(AM)-start	Duty starts from a crew base using the overnight plane
CB(PM)-start	Duty starts from a crew base without using the overnight plane
Rest(AM)-start	Duty starts from non-crewbase stations using the morning flights
Rest(PM)-start	Duty starts from non-crewbase stations using the afternoon flights
CB(AM)-end	Duty ends at a crew base without using the overnight plane
CB(PM)-end	Duty ends at a crew base using the overnight plane
Rest (AM)-end	Duty ends at non-crewbase stations using the morning flights
Rest (PM)-end	Duty ends at non-crewbase stations using the afternoon flights.

In Table 1, the AM or PM starting and ending of a duty is not the absolute morning flights or evening flights. For crew-base stations, AM or PM is dependent on whether it is flown by the overnight plane. For non-crewbase stations, AM or PM is determined by whether it is flown by the overnight plane and also by how the midday breakout pair of flights is chosen. We assume that for crew, there is no overnight rest allowed at other crew bases. Then a duty can only have Rest(AM)-start after a legal short rest PM-AM, or the AM-AM rest in a midday breakout. So, the number of duties that have Rest(AM)-start is:

#Rest(AM)-start = # midday breakouts + # legal short rests (4)

Similarly, we can have the following:

#Rest(PM)-start = # midday breakouts (5)

#Rest(PM)-end = # midday breakouts + # legal short rests (6)

#Rest(AM)- end = # midday breakouts (7)

For crew-base stations, since there is no overnighting crew, a duty starts at a crew base can either use the overnight plane or wait for an inbound plane. Due to the convention we defined for AM/PM at crew bases, we have,

#CB(AM)-start = # planes overnight at the crewbase (8)

#CB(PM)- end = # planes overnight at the crewbase (9)

The total number of pairings is #CB(AM)-start + #CB(PM)-start. The total number of duties is SUM{#Rest(AM)-start, #Rest(PM)-start, #CB(AM)-start, #CB(PM)-start}.

3.2.1 Duty pattern for Type I schedule

A typical duty pattern for Type I schedule is shown in Figure 2. In Type I schedule, the number of duties is greater than twice of the number of planes. Thus, besides the "half-day" duties, we may expect some very short duties, illustrated as dash-dot lines in Figure 2.

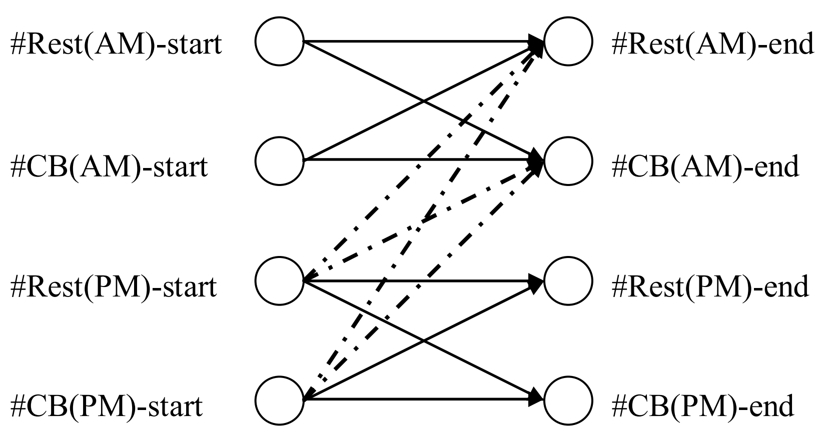


Figure 2 Typical duty pattern for Type I schedule

3.2.2 Duty pattern for Type II schedule

A typical duty pattern for Type II schedule is shown in Figure 3. In Type II schedule, due to short plane rotations that can be covered by a single duty, there will be duties having (AM)-start and (PM)-end, as illustrated by dash-dot lines in Figure 3.

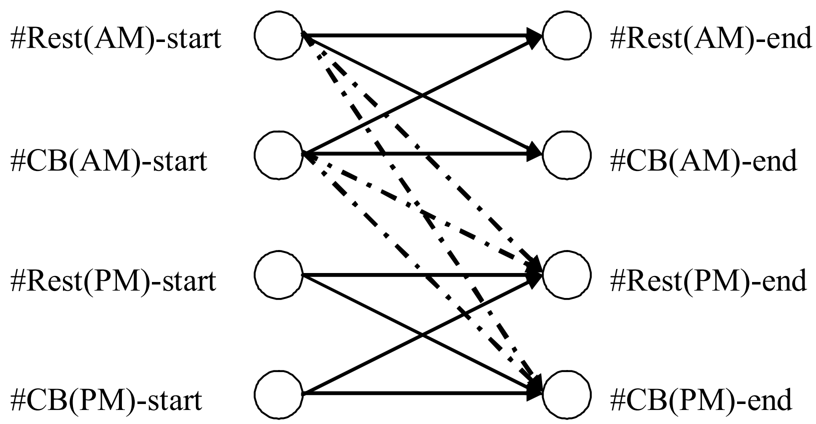


Figure 3 Typical duty pattern for Type II schedule

3.2.3 Pairing pattern

Extend the duty pattern into layers, pairing pattern can be obtained. Figure 4 shows an example of pairing patterns. The maximum length of a pairing is 3 duties. The total number of legal short rests and midday breakouts, as well as the number of different types of duties should be consistent with those in the duty pattern.

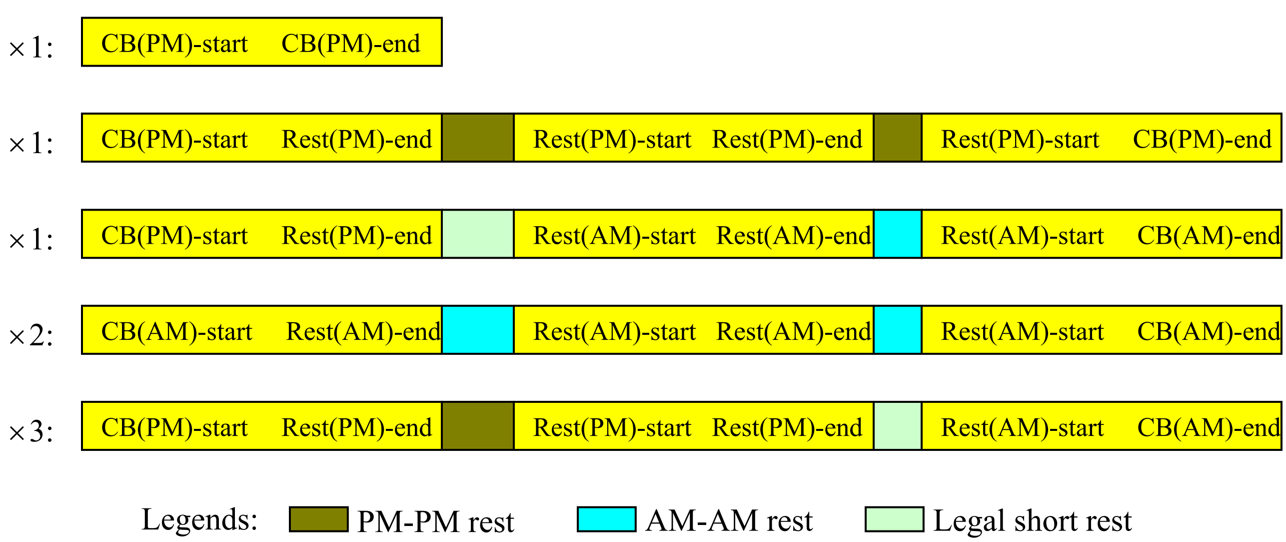


Figure 4 An example of pairing patterns

3.3 Schedule Adjustment

When the pairing solution is poor, then probably the schedule has very low average duty time. In order to improve, it is necessary to reduce the total number of duties. It is noted that concerning sequential decision-making in airline planning, a reduced number of duties would also benefit the crew rostering process by making the days off requirements for crew easier to satisfy. For the two different types of schedule, we have different methods to adjust.

3.3.1 Type I schedule

From the duty pattern of Type I schedule, it is observed that there are very short duties with (PM)-start and (AM)-end. To reduce the number of duties, we want to eliminate such short duties. Figure 5 shows the ideal duty pattern for Type I schedule.

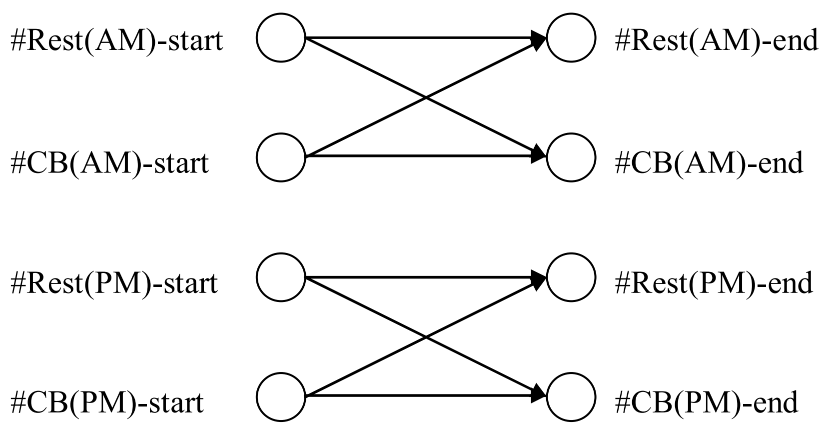


Figure 5 Ideal duty pattern for Type I schedule

From Figure 5, we can get the following formula,

pairings = # legal short rests + 2 * # planes overnight at CB (10)

Usually it is needed to increase items at the right hand side: {# legal short rests} and {# planes overnight at CB}. We anticipate improving the crew friendliness of the schedule by retiming the schedule in a small range, as well as by adding or dropping a small number of legs.

At the station where we want to change a midday breakout to legal short rest, we can try to make the early evening arrival flight arrive earlier, and/or make the late morning departure flight depart later. However, we try not to change the number of planes overnight at this station.

If there are long sits or long rests, we may want to add a pair of flights in between. An extreme example is the lonely double overnight. At the station where we want to move a plane overnight to some crew base, we can add 2 legs. Let one leg depart after the early evening arrival flight, at the same time, let the other leg arrive before the late morning departure flight, as shown in Figure 6. This will not save the total number of crew overnights, but it will not add crew overnights either. Since adding new legs may violate crew base balance constraint, it might be necessary to drop some legs.

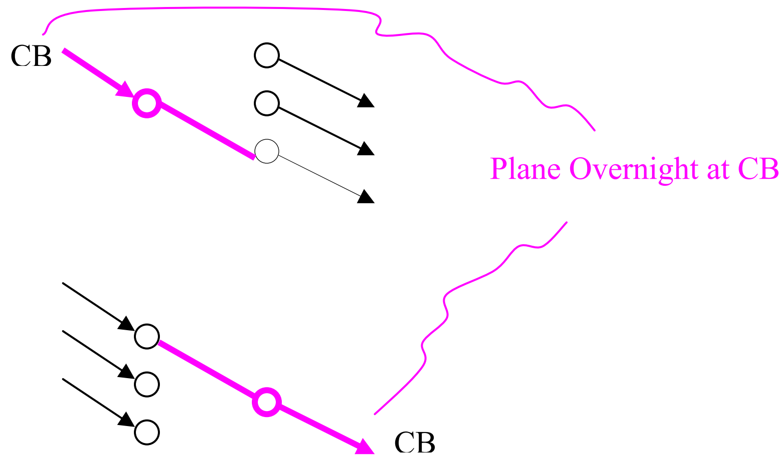


Figure 6 Move plane overnight to a crew base by adding new legs

3.3.2 Type II schedule

For Type II schedule, in order to reduce the number of duties, we want to increase the number of short plane rotations so that a plane rotation that needs at least two duties earlier can now be covered by one duty. So we want to have more duties with (AM)-start and (PM)-end. The ideal duty pattern for Type II schedule looks same as the typical pattern for Type II schedule as shown in Figure 3. But the total number of duties and the number of duties with (AM)-start and (PM)-end will be different. We can do this by retiming some flights. We can try to move the first flight in the plane rotation later, also move the last flight in the plane rotation as early as possible so as to shorten the plane rotation.

3.4 Case Study

A schedule of 164 legs for two fleet types is analyzed, in which Fleet 1 schedule has 102 legs, Fleet 2 schedule has 62 legs. Using our schedule analysis, these two sets of schedule are categorized as Type I schedule and Type II schedule respectively. Schedule adjustment methods are investigated for both schedules.

3.4.1 Fleet I schedule

For Fleet I example, we can see from Table 2 that NumDuty(Crew)=26, NumDuty(Aircraft)=22, and NumDuty(Crew) > NumDuty(Aircraft). This is a Type I schedule. The number of duties needed for the original schedule is 26, which determines the average duty time to be 238 minutes, much smaller than the average duty guarantee, 270 minutes. This explains the high pay and credit of the optimal pairing solution for the original schedule. Now look at it in the other way, 26 duties implies at least 9 pairings in the pairing solution provided that the maximum length of pairing is three. While

legal short rests + 2 * # planes overnight at CB = 3+2*1 = 5,

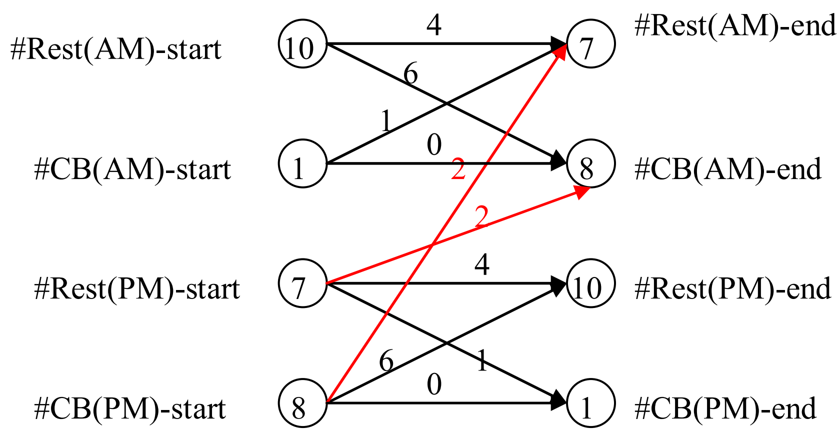
the original Fleet I schedule cannot produce the ideal duty pattern shown in Figure 5.

The original schedule has a small number of planes overnighting at the crew base, and many crew midday breakouts. In order to improve, we need to decrease the number of duties, increase the number of planes overnight at crew bases, as well as increase the number of legal short rests. By changing the midday breakouts to legal short rests, we can increase the number of legal short rests, reduce the total number of crew overnight rests, so as to reduce the number of duties. Also the method of adding legs as shown in Figure 6 can be adopted in order to move one plane overnight from other station to the crew base. For improving the original Fleet I schedule, two legs are added (creating one more crew overnighting at the CB and one less midday breakout), and one leg is retimed (removing another midday breakout). Solving the crew pairing problem for the new schedule, zero pay-and-credit can be achieved. Obviously, the adjustments made a tremendous improvement, i.e., even with the cost for the two newly added legs, the pairing cost for the new schedule is still significantly smaller than the original cost. In addition, this adjustment wouldn't violate the crew base balance constraints.

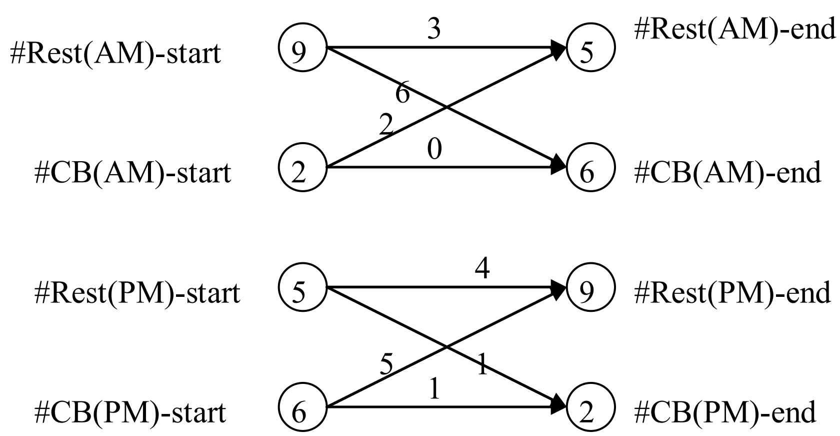
We compared the new schedule with the original schedule in terms of duty and pairing patterns. The comparison of the duty patterns is shown in figure 7. The new schedule achieved the ideal duty pattern for Type I schedule. Therefore, it can be concluded that the duty pattern illustrated in Figure 5 is appealing indeed.

Table 2 Fleet I Schedule Analysis
(Original schedule / New schedule)

Stations	Crewbase?	# planes overnight		# crews overnight		# midday breakouts		# legal short rests	
FAT	---	2	2	3	3	1	1	1	1
LAX	Yes	1	2	---	---	---	---	---	---
MRY	---	2	2	3	3	1	1	1	1
PSP	---	0	0	0	0	0	0	0	0
SAN	---	2	1	4	2	2	1	0	0
SBA	---	2	2	4	3	2	1	0	1
SBP	---	2	2	3	3	1	1	1	1
Total		11	11	17	14	7	5	3	4
Plane average flying time = Total blocktime / # planes = 9.382 (hrs)									
NumDuty = max{ NumDuty(Aircraft), NumDuty(Crew) } = max{22,26} = 26 NumDuty = max{ NumDuty(Aircraft), NumDuty(Crew) } = max{22,21} = 22									
Average duty time = Total blocktime/NumDuty = 238 min Average duty time = Total blocktime/NumDuty = 281 min				Average duty guarantee = 270 min					
Midday breakouts percentage:				41.2%			35.7%		
Pairing solution cost in Pay & Credit:				13.37%			0.00%		
Pairing solution cost in minutes:				7020			6302		



(7.a) duty pattern for the original schedule



(7.b) duty pattern for the new schedule

Figure 7 Case study: Fleet I Schedule

3.4.2 Fleet II schedule

In this example, as shown in Table 3, NumDuty(Crew) < NumDuty(Aircraft). This is a Type II schedule. The average plane flying time is not large. There might be some long duties cover the whole plane rotations. So, NumDuty(Aircraft) is 17, instead of 2*#planes=18. The average duty time is comparable to the average duty guarantee. Correspondingly, the pay and credit of the original pairing solution is not very poor.

To improve, we make early evening arrival flights arrive earlier, and late morning departure flights depart later, resulting in more planes with shorter plane rotations that can be covered by one single duty. As a result, the number of duties can be reduced. Besides, a few small changes on departure times are made in order to create more day connections. The pay-and-credit is reduced from 2.19% to 1.65%. In addition, TAFB

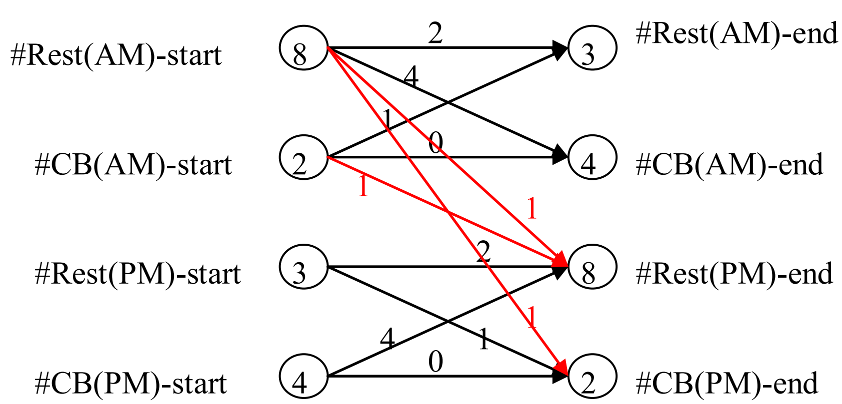
(Time Away From Base) in the pairing solution is reduced apparently after the schedule adjustment.

Duty patterns and pairing patterns of the original and modified schedule for Fleet II are compared. The comparison of duty patterns is shown in Figure 8. It can be seen that the number of duties with (AM)-start and (PM)-end is increased from 3 to 4. The total number of duties is decreased from 17 to 16 with the schedule adjustment.

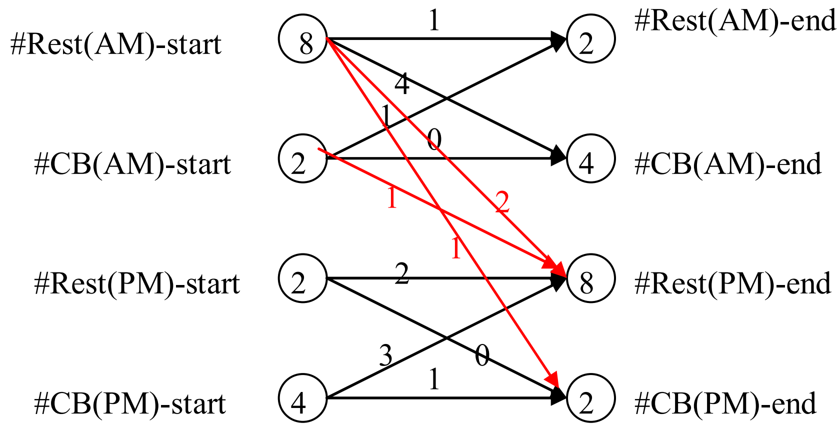
Table 3 Fleet II Schedule Analysis

(Original schedule / New schedule)

	Crewbase?	# planes overnight		# crews overnight		# midday breakouts		# legal short rests	
LAX	Yes	2	2	---	---	---	---	---	---
SAN	---	1	1	1	1	0	0	1	1
SBP	---	1	1	1	1	0	0	1	1
SFO	---	1	1	1	1	0	0	1	1
SJC	---	3	3	5	5	2	2	1	1
SNA	---	1	1	1	1	0	0	1	1
Total		9	9	9	9	2	2	5	5
Plane average flying time = Total blocktime / # planes = 8.767 (hrs)									
NumDuty = max{ NumDuty(Aircraft), NumDuty(Crew) } = max{17,15} = 17 NumDuty = max{ NumDuty(Aircraft), NumDuty(Crew) } = max{16,15} = 16									
Average duty time = 278 Average duty time = 296				Average duty guarantee = 270 min					
Pairing solution cost in Pay & Credit:				2.19%			1.65%		
Pairing solution cost in minutes:				4837			4812		
Total TAFB of the pairing solution:				17055			15624		



(8.a) duty pattern for the original schedule



(8.b) duty pattern for the new schedule

Figure 8 Case study: Fleet II Schedule

4 Duty Partition model for the crew pairing problem

Instead of considering crew scheduling as choosing pairings to partition the scheduled flights, we choose duties to partition the flights based on observations of duty features to constitute good pairing solution from schedule analysis. In other words, the duty partition model gives a set of duties as solution that partitions the flight segments. The main objective of this model is to reduce the number of duties. In the section on schedule analysis, we have explained how the number of duties affects the quality of crew pairing solutions. In addition, the objective function will penalize poor duties, in particular those that are short or have long sit-times.

Two different models are investigated. The first one is Duty Partition + Duty Pattern model. We apply the duty pattern result obtained from schedule analysis into the duty partition model by exactly limiting the number of various duty types, for example, duty type like (Rest(AM)-start, Rest(AM)-end). The purpose is to study whether the duty pattern can help find a set of good duties that will eventually constitute a legal and good pairing solution. Computational results show that this model can give the right number of duties, but it is not guaranteed that the duty solution can be grouped into legal pairings. So, we can conclude that the ideal duty pattern is a necessary condition for the existence of a good pairing solution. However, it is not sufficient to find a good pairing solution.

The second model is Duty Partition + Pairing Pattern model. The selected duties are guaranteed to constitute a legal pairing solution by means of a duty connection network. Pairing Legalities such as 8-in-24 rule are considered when constructing the duty connection network. The second model is more precise in the sense of finding pairing solutions since it conforms to pairing pattern. This model is not designed to find the optimal crew pairing solution, but it can quickly find a very good legal pairing solution, which makes it promising in an integrated planning or real-time recovery framework.

4.1 Create Smart Duties

There are plenty of ways to partition duties. In the duty partition model, the objective is to reduce the total number of duties in the pairing solution. The solution will naturally choose duties that start from the morning flights by overnighting aircraft and end at the evening flights which lead to aircraft overnights, if the elapsed time and flying time of these duties are legal. If not, it is preferred that the plane rotation be split into two duties, so that one of them includes the first flight in the plane rotation, while the other includes the last flight in the plane rotation. So, most of the duties in a good pairing solution either start from the morning flights by overnighting aircraft or end at the evening flights which lead to aircraft overnights, or both. To look for the corresponding morning flights and evening flights, we only need to look at the "overnight island". Hane et al (1995) introduces the idea of "islands" in the fleet assignment network. We try to identify the morning beginning leg and evening ending leg from the island that includes plane overnight. In our case, this feature is applicable to both hubs and spokes.

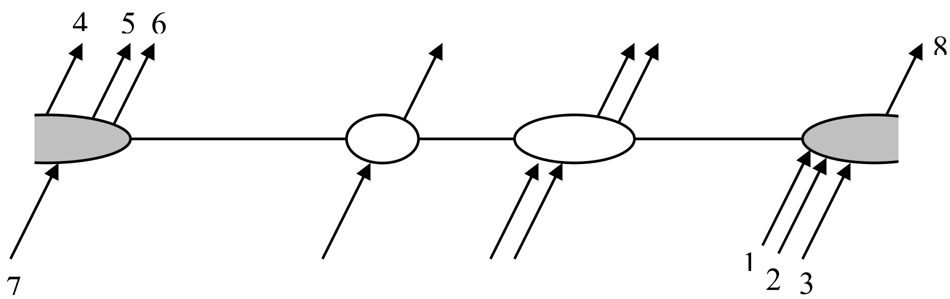


Figure 9 How to identify beginning and ending leg set

Figure 9 shows an example. In Figure 9, the oval with shade identifies the overnight island. There are two airplanes overnight at this station. So there will be two actual beginning legs and two actual ending legs. It can be seen that leg 4 must be a beginning leg, while only one of leg 5 and leg 6 can be beginning leg, since one of them has connection with the incoming flight 7. As to the ending leg, two of the legs among 1,2,3 will be ending legs, since one of them need to connect to leg 8.

4.2 Duty Connection Network

The duty connection network is constructed to ensure that the chosen duties can be grouped into legal pairings so that although the solution is about duties, there is a legal pairing solution embedded in the network. Three duty pairings are taken into account in this context. Pairings with more duties, say 4, can be constructed in a similar way.

The duty connection network depicted in Figure 10 contains three types of arcs: duty arcs, rest arcs and dummy arcs. There are 6 types of nodes: source node s, sink node t, crew base begin leg nodes (CBiBEG), crew base end leg nodes (CBiEND), rest of stations begin leg nodes (RestBEG), and rest of stations end leg node (RestEND). Rather than using duties as node set, using flights that start or end a duty as node set can greatly reduce the size of the network. From the enumerated duty set D, the node sets of CBiBEG, CBiEND, RestBEG, and RestEND can be easily obtained. The nodes of RestBEG and RestEND are shared by multiple commodities. So this is a multi-commodity network. The nodes are connected correspondingly by duty arcs, rest arcs and dummy arcs, where duty arcs connect CBiBEG with CBiEND, CBiBEG with RestEND, RestBEG with CBiEND, RestBEG with RestEND; Rest arcs connect RestEND with RestBEG; Dummy arcs connect source/sink node to CBiBEG/CBiEND respectively.

It is noted that there is another kind of dummy arc lying between the leg node pairs in RestEND. This is designed to guarantee no 8-in-24 violations in the embedded pairing solution. Each leg in RestEND corresponds to two nodes. The first node connects to long overnight rest arcs, like midday breakouts as AM-AM rests or PM-PM rests. The second node connects to short overnight rest arcs, like PM-AM legal short rests. If a duty that ends at this leg is long, then the corresponding duty arc should be connected to the first node, as shown in Figure 11. This mechanism can ensure that a short rest will never connect two long duties, so that 8-in-24 rule should not be violated. The way to split single node into two due to short vs. long overnight rest is similar to the treatment in Barnhart, Johnson, et al, (1994). But they have different motivations. In this context the main concern is to avoid getting into 8-in-24 violations. Another legality rule on daily pairing problem is no leg repetition. This can be assured by the duty partition constraints.

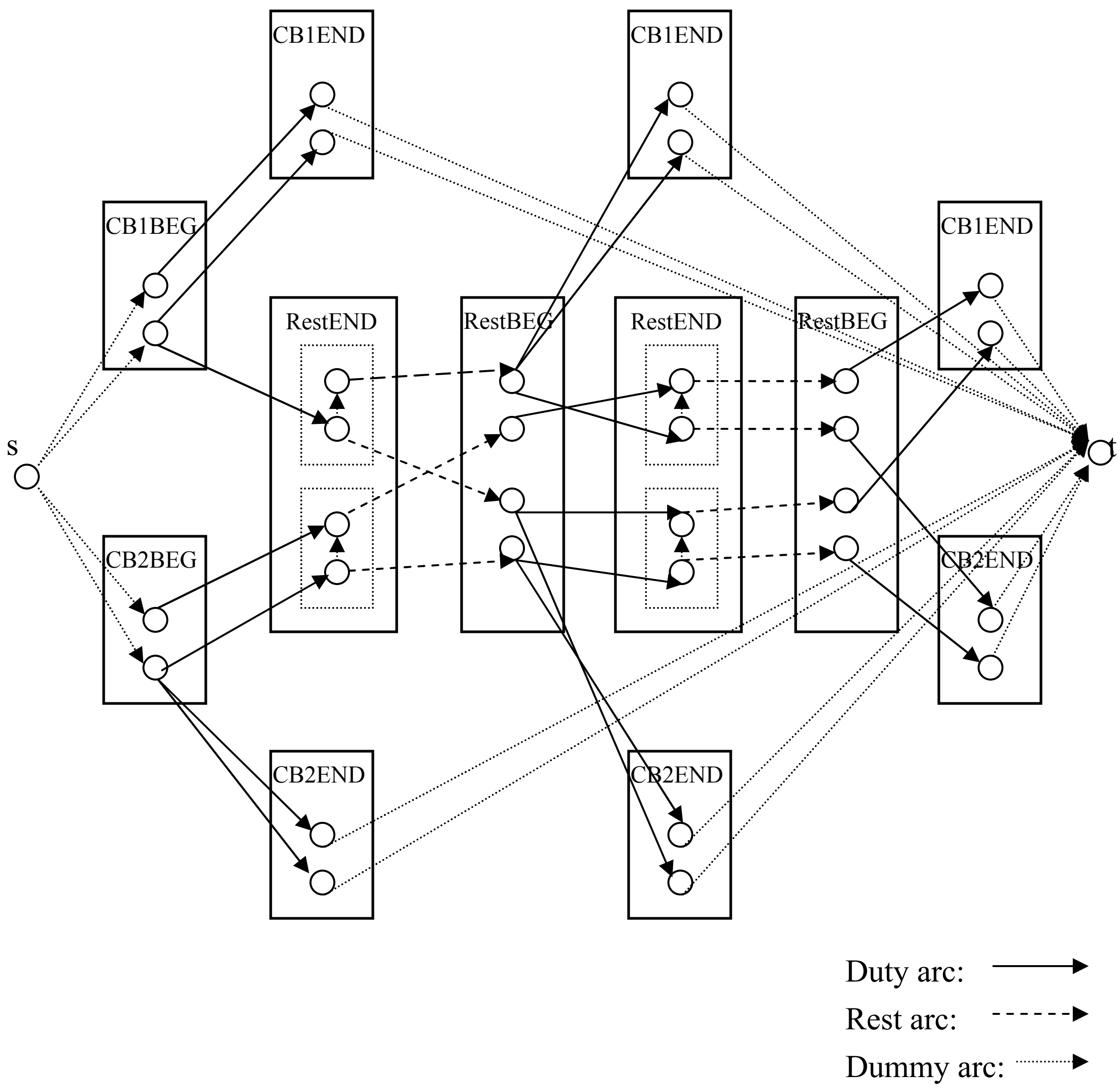


Figure 10 Duty connection network

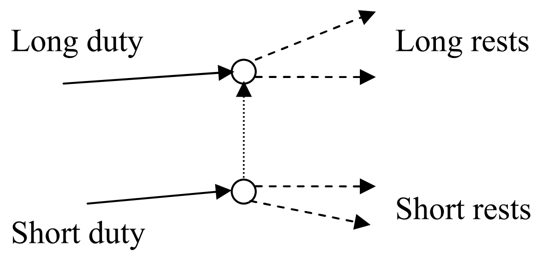


Figure 11 How to ensure no violation on 8-in-24 rule

4.3 The Formulation

The formulation of the duty partition model with pairing patterns is given by:

$$\min \sum_{d \in D} (1 + P_d) X_d \quad (11)$$

s.t.

$$\sum_{d: l \in d} X_d = 1, \qquad \forall l \in L, \quad (12)$$

$$\sum_{a \in O(n)} Z_{a,cb} = \sum_{a \in I(n)} Z_{a,cb}, \qquad \forall n \in N, \forall cb \in CB, \quad (13)$$

$$\sum_{cb} \sum_{a: a=d} Z_{a,cb} = X_d, \qquad \forall d \in D, \quad (14)$$

$$X_d \in \{0,1\}, \qquad \forall d \in D, \quad (15)$$

$$Z_{a,cb} \in \{0,1\}, \qquad \forall a \in A, \forall cb \in CB, \quad (16)$$

Where $X_d = 1$ if duty d is chosen, and 0 otherwise; $Z_{a,cb} = 1$ if arc a in the duty connection network is chosen and assigned commodity of crew base cb, 0 otherwise. L is the set of flight segments. D is the set of enumerated duties. N is the node set of the duty connection network. A is the arc set of the duty connection network. CB is the set of crew bases. The duty connection network is a multi-commodity flow network, where crew bases are the commodities. s is the source node of the network, and t is the sink node if the network. O(n) and I(n) represent the sets of out-going arcs and incoming arcs of node n, respectively.

The main objective is to reduce the total number of duties. So each duty variable has cost coefficient 1. P_d is a penalty function defined to promote good duties in the solution. If duty cost is larger than duty block time (sum of block-times on flight segments in the duty), $P_d = \overline{P}_d + 0.01$; Otherwise $P_d = \overline{P}_d$. $\overline{P}_d$ is defined as shown in Figure 12. In this way, short duties and duties with long sit-times will be penalized.

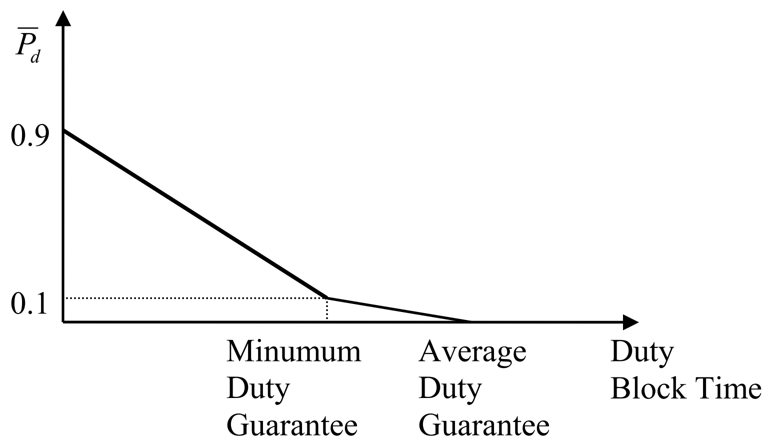


Figure 12 Definition of $\overline{P}_d$

Expressions (12)-(14) give the constraints of this model. Expression (12) enforces that each flight is covered by exactly one duty. Expression (13) gives constraints related to the duty connection network. For each node in the network, flow conservation constraints are required on separate commodities. It is noted that we can fix a lot of variables to 0, that is, set $Z_{a,cb} = 0,$ if a is the input or output arc of a node in CBiBEG or CBiEND, while cb is not correspondingly the ith crew base. Expression (14) ensures that each duty chosen can only appear once in the pairing solution embedded in the network. It is the constraint linking duty partition and duty connection network.

Note that crew availability balance constraints can be easily included in this model by simply adding up the flying hours of duty arcs assigned to each crew base.

4.4 Preliminary Computational Results

Computational experiments were performed using flight schedule data for a single aircraft fleet provided by a major domestic carrier. In particular, Problem 1 is the modified schedule for fleet I discussed in section 3.4.1. CPLEX 9.0 is applied to solve the IP problem. As shown in Table 4, for Problem 1, the proposed model can be solved in 6 seconds and give a very good pairing solution. For Problem 2, it takes about half an hour to get the IP solution, which embeds a reasonably good pairing solution.

Table 4 Computational Results of duty partition model

	Problem 1	Problem 2
Flights	104	350
Duties	3332	16428
Crew Bases	1	3
Nodes	507	1579
Arcs	4287	20824
Solving time (in Seconds)	6.031	1970
Pay & Credit	0.64%	2.36%
Optimal Pay & Credit	0.00%	0.44%

5 Integrated Planning

Schedule analysis highlights the advantages and importance of integrated planning in two ways. First, it suggests small schedule adjustments which can make a big difference on crew scheduling. Second, by doing schedule analysis on combined schedule of different fleet types, it shows the advantage of integrating fleet assignment and crew planning. It is good to adopt the proposed duty partition model in the integrated fleet assignment and crew pairing planning framework because of its computational efficiency. In this section, first we continue with the case studies in section 3.4. Schedule analysis is applied to the combined schedule of two fleet types. The analyses show the importance to integrate FAM and crew. Then we propose an integrated planning model which integrates FAM, crew and schedule adjustment.

5.1 Advantages of Integrating FAM and Crew

Note that the characteristics of Type I and Type II schedules are opposite to each other. If we integrate fleet assignment and crew planning, balanced resource utilization can be expected. We apply the schedule analysis to the combined schedule in section 3.4. In Table 5, the results show that the number of duties required by the plane rotation is 40, and the number of duties required by crew overnights is 39. They are almost equal. We further perform pattern analysis on the combined schedule. An ideal duty pattern for Type I schedule is achieved. Therefore, it is reasonable to expect a more crew-friendly solution by integrating fleet assignment and crew planning.

Table 5 Schedule analysis on the combined schedule

		Fleet I			Fleet II			Combined		
	C B	# ron plane	# ron crews	mid-day	# ron plane	# ron crews	mid-day	# ron plane	# ron crews	mid-day
FAT	--	2	3	1				2	3	1
LAX	Y	1	---	---	2	---	---	3	---	---
MRY	--	2	3	1				2	3	1
PSP	--	0	0	0				0	0	0
SAN	--	2	4	2	1	1	0	3	5	2
SBA	--	2	4	2				2	4	2
SBP	--	2	3	1	1	1	0	3	3	1
SFO	--				1	1	0	1	1	0
SJC	--				3	5	2	3	3	2
SNA	--				1	1	0	1	1	0
Total		11	17	7	9	9	2	20	26	9
Plane ave. flying time		9.382 hrs			8.767 hrs			9.105 hrs		
# duties needed		Max{26,22}=26			Max{17, 14}=17			Max{40, 39} = 40		
Ave duty time		238			278			273		

5.2 Integrated Planning Model

We propose an integrated planning model which integrates FAM and crew in this section. For the FAM part, the robust FAM proposed by Smith (2004) is adopted. For the crew part, the duty partition model introduced in section 4 is adopted. In Smith's robust FAM, station purity constraints were imposed on the traditional FAM. Station purity ensures that the number of fleet types serving a given station does not exceed a specified limit. The benefits of limiting station purity in the crew scheduling problem are that: 1. at the planning stage, the lonely fleet/station combinations are reduced which leads to less crew double overnights; and 2. it can create more opportunity for move-up crew assignments to cover operational disruptions. To improve the computational efficiency, station decomposition was proposed in Smith (2004) to solve this robust FAM. Station decomposition takes advantage of the hub and spoke structure of typical airline flight networks. The problem was decomposed into master problem and subproblems. Subproblems generate plans for spokes; these plans become columns in the master problem. The master problem determines fleeting based on plans. By doing this, the fleet purity constraints are removed from the master problem. The master problem formulation of robust FAM is:

Maximize:

$$\sum_{f\in F}\sum_{h\in H}\sum_{i\in L^h}(R_{f,i}-C_{f,i})x_{f,i}+\sum_{p\in P}(R_p-C_p)x_p \tag{17}$$

Subject to:

$$\sum_{f\in F}x_{f,i}=1, \forall i\in L^h, \forall h\in H \tag{18}$$

$$y_{f,h,t^-}+\sum_{i\in I(f,h,t),i\in L^h}x_{f,i}-y_{f,h,t^+}-\sum_{i\in O(f,h,t),i\in L^h}x_{f,i}+\sum_{p\in P}q_{f,h,t,p}x_p=0, \forall f,h,t \tag{19}$$

$$\sum_{h\in H}y_{f,h,t_m}+\sum_{i\in CL(f),i\in L^h}x_{f,i}+\sum_{p\in P}PC_f^p x_p\le N_f, \forall f\in F \tag{20}$$

$$\sum_{p\in P^s}x_p=1, \forall s\in S \tag{21}$$

$$x_{f,i}\in\{0,1\}, \forall f\in F, \forall i\in L^h \tag{22}$$

$$x_p\in\{0,1\}, \forall p\in P \tag{23}$$

$$y_{f,h,t}\ge 0, \forall f,h,t \tag{24}$$

The definitions of the sets appeared n in the formulation are:

H:	Set of hub airports, indexed by h.
S:	Set of spokes, indexed by s.
P:	Set of spoke plans, indexed by p.
P^s:	Set of plans for spoke s, indexed by p.
F:	Set of fleet types, indexed by f.
L^h:	Set of hub-to-hub flight legs, indexed by i.
L^s:	Set of flight legs in the spoke plans, indexed by i.
CL(f):	Set of flight legs crossing the counting line flow by fleet f.
I(f,a,t):	Set of flight legs inbound to {f,a,t}.
O(f,a,t):	Set of flight legs outbound from {f,a,t}.

The decision variables are:

$x_{f,i}$ $= \begin{cases} 1, \text{if leg i} \in L^h \text{ is assigned fleet f}. \\ 0, \text{otherwise.} \end{cases}$

x_p $= \begin{cases} 1, \text{if plan p is in the solution.} \\ 0, \text{otherwise.} \end{cases}$

y_{f,h,t^-} : The number of aircraft on the ground for fleet type f, at airport a, on the ground arc just prior to time t.

y_{f,h,t^+} : The number of aircraft on the ground for fleet type f, at airport a, on the ground arc just following time t.

The parameters defined in the model include:

$R_{f,i}$: Revenue for flight leg i if it is assigned fleet type f.

$C_{f,i}$: Cost for flight leg i if it is assigned fleet type f.

R_p : Revenue for plan p, $R_p = \sum_{f \in F} \sum_{i \in L} R_{f,i} \alpha_{f,i,p}$.

$\alpha_{f,i,p}$ is 1 if leg i is assigned fleet f in plan p

C_p : Cost for plan p, $C_p = \sum_{f \in F} \sum_{i \in L} C_{f,i} x_{f,i}^p$.

PC_f^p : The number of aircraft from fleet f on the ground or in the air at the counting line in plan p.

N_f : The number of aircraft available of fleet type f.

$q_{f,h,t,p}$ $= \begin{cases} 1, \text{if plan p includes an arrival of aircraft type f at hub h, time t.} \\ -1, \text{if plan p includes a departure of aircraft type f at hub h, time t.} \\ 0, \text{otherwise.} \end{cases}$

Robust FAM maximizes operating profit: revenue minus operating costs, (Eq 17). Constraints (18) – (20) are similar to the Cover, Balance, and Plant Count constraints of the traditional fleet assignment model, with the replacement of some spoke related decision variables and constraints by the addition of plans. Equations (18) are the cover constraints for hub-to-hub legs. Equations (19) are the flow balance constraints at the hubs and its last item at the left hand side provides the incidence of flights to and from each hub for each plan. Equations (20) are the plane count constraints. Equations (21) are the convexity constraints on the plans. Each spoke has one and only one plan assigned.

Using this framework for FAM, to model the crew pairing problem, we have separate duty sets, denoted as D^f ; and separate duty connection networks with node set N^f for each fleet type f. The decision variables $Z_{a,cb,f}$ now are defined on different duty connection networks for different fleets. The integrated FAM and crew planning model has the following objective function:

$$Maximize \sum_{f\in F}\sum_{h\in H}\sum_{i\in L^h}(R_{f,i}-C_{f,i})x_{f,i}+\sum_{p\in P}(R_p-C_p)x_p-\sum_{f\in F}\sum_{d\in D^f}G\cdot(1+P_d)X_d \quad (25)$$

In which cost on crew is added to (Eq. 17). In parallel with trying to reduce the total number of duties, G defined as the average duty guarantee, is multiplied with the original objective function of the duty partition model, to make this cost item more comparable to the real crew cost.

Some constraints are added to relate the FAM model to the duty partition model for crew pairings. Equation (26) provides a duty to cover hub-to-hub legs and (27) assures that legs in the spoke plans are included in a duty for each fleet type. As before, Equation (28) is the flow balance constraints for each duty connection network. Some variables of $Z_{a,cb,f}$ can be fixed if an arc in the network is not possible to be covered by some crew bases. Equation (29) implies that any duty chosen can only appear once in the pairing solution embedded in the networks.

$$\sum_{d:i\in d,d\in D^f}X_d=x_{f,i},\forall i\in L^h,\forall h\in H,\forall f\in F \quad (26)$$

$$\sum_{d:i\in d,d\in D^f}X_d=\sum_{p\in P}\alpha_{f,i,p}x_p,\forall i\in L^s,\forall s\in S,\forall f\in F \quad (27)$$

$$\sum_{a\in O(n)}Z_{a,cb,f}=\sum_{a\in I(n)}Z_{a,cb,f},\quad \forall n\in N^f,\forall cb\in CB,\forall f\in F \quad (28)$$

$$\sum_{cb}\sum_{a:a=d}Z_{a,cb,f}=X_d,\quad \forall d\in D^f,\forall f\in F \quad (29)$$

Computational validation for this integrated model is underway. It is noted that the schedule adjustment options can also be integrated in the model with convexity constraints defined on different options. In fact the integrated FAM and crew model incorporated with schedule adjustments provides a similar formulation for integrated recovery, where flight delay options and flight cancellations need to be modeled. This will be discussed in the next section.

6 Integrated Recovery

Integrated recovery is an appealing answer to deal with schedule interruptions which can capture the availability of all three resources: aircraft, crew and available seats. However, each resource is scheduled separately because of the different sets of rules. This leads to a “snowball effect” when a flight does not operate as scheduled - a small disruption might result in a huge disrupted leg set. The work in this section attempts to address the recovery scope in an integrated recovery framework. Based on the recovery scope, an integrated recovery method is proposed. The method incorporates schedule adjustment into a dated FAM plus crew model. Then a Benders approach with feasibility and optimality cuts for maintenance routing and passenger service is proposed.

It is critical to find a good recovery scope which is able to provide a reasonably good recovery solution, and should assure the formulated problem be computationally tractable in real-time. The difficulty of integrated recovery lies in handling aircraft and crew concurrently, while they have possibly different chained routings (especially when overnight connections are included). The main idea we have is to define different recovery sets for schedule change, aircraft rerouting and crew rerouting, namely L_s, L_a, and L_c. Obviously, $L_a \supseteq L_s$ and $L_c \supseteq L_s$. For the legs in L_a, but not in L_c, the fleet assignment and crew assignment will not change; Vice versa, for the legs in L_c, but not in L_a, the fleet assignment and aircraft assignment will not change. In this way, we can define different recovery time windows for aircraft and crew. For the disrupted and potential swapping crew, all unflown legs in their pairings are considered in L_c. Usually the plane recovery time window is from the start of the disruption until the end of the day.

Next, we emphasize developing the master model which incorporates schedule adjustment into a dated FAM plus crew model. The framework is similar to that of the integrated planning model discussed in the prior section, except for the following three points: 1. The legs being picked up in the recovery set are dated; 2. We need to take into account the various delayed options or even cancellations for the legs, so a more extensive schedule adjustment is necessary to be modeled; 3. Crew pairing recovery is crew-specific. These features lead to two differences in constructing the duty connection network for each fleet type. First, since it is a dated version, we may have 1, 2, or 3 day time horizon for L_c, which means there are different duty sets for different days. It is noted that the first day duty set must include legal partial duties for those disrupted crews. Second, since it is crew-specific, the commodities defined in the duty connection network are no longer crew bases, but specific crews. Assume $D(l)$ is the delayed option set for leg $l \in L_s$. Binary variable κ_l represents whether leg $l \in L_s$ will be cancelled. To simplify, for integrated recovery we adopt the traditional FAM model. For legs in $L_a \setminus L_s$, their fleet types do not change, but they are counted in the balance constraints for the corresponding fleet type. The master problem for integrated recovery is formulated as:

$$Maximize \sum_{f \in F} \sum_{l \in L_s} \sum_{i \in D(l)} (R_{f,i} - C_{f,i}) x_{f,i} - \sum_{f \in F} \sum_{d \in D^f} G \cdot (1 + P_d) X_d \tag{30}$$

Subject to:

$$\sum_{f \in F} \sum_{i \in D(l)} x_{f,i} + \kappa_l = 1, \forall l \in L_s \tag{31}$$

$$y_{f,h,t^-} + \sum_{i \in I(f,h,t), i \in D(l), l \in L_s} x_{f,i} + \sum_{i \in I(f,h,t), i \in L_a \setminus L_s} 1 - y_{f,h,t^+} - \sum_{i \in O(f,h,t), i \in D(l), l \in L_s} x_{f,i} - \sum_{i \in O(f,h,t), i \in L_a \setminus L_s} 1 = N_{f,h,t}, \quad \forall f,h,t \tag{32}$$

$$\sum_{d: i \in d, d \in D^f} X_d = x_{f,i}, \forall i \in D(l), \forall l \in L_s, \forall f \in F \tag{33}$$

$$\sum_{d: j \in d, d \in D^f} X_d = \beta_{f,j}, \forall j \in L_c \setminus L_s, \forall f \in F \tag{34}$$

$$\sum_{a \in O(n)} Z_{a,c,f} = \sum_{a \in I(n)} Z_{a,c,f}, \quad \forall n \in N^f, \forall c \in C, \forall f \in F \tag{35}$$

$$\sum_{c} \sum_{a: a = d} Z_{a,c,f} = X_d, \quad \forall d \in D^f, \forall f \in F \tag{36}$$

The decision variables $x_{f,i}$, y_{f,h,t^-} , y_{f,h,t^+} , X_d are defined as before. Because the commodities on the duty connection network now are specific crews, so $Z_{a,c,f}$ is defined on specific crew instead. Binary variable κ_l represents whether leg $l \in L_s$ will be cancelled. Equation (31) means each leg $l \in L_s$ is either delayed or cancelled. Equation (32) is the balance constraint for each fleet type. At the end of the time window, the right hand side value should be the actual number of planes available at that time. Equation (33) implies if $x_{f,i}$=1, there must be a duty with corresponding fleet type to cover it. Equation (34) implies that for legs in $L_c \setminus L_s$, if the pre-assigned fleet type is f, there must be a duty with corresponding fleet type to cover it. Parameter $\beta_{f,j} = 1, \forall j \in L_c \setminus L_s$, if the pre-assigned fleet type for leg j is f; zero otherwise. Equations (35) and (36) have same meanings as before. They are related to the duty connection networks.

Using Bender's decomposition, aircraft maintenance routing and passenger flow re-accommodation problem can be modeled as subproblems. The aircraft maintenance routing problem is modeled by multi-commodity flow network. If the multi-commodity flow problem is infeasible, a set of feasibility cuts caused by infeasible maintenance are returned to the master model. Passenger re-accommodation can always be feasible by using long delays or appealing to other airlines. Only optimality cuts are generated by solving this subproblem.

7 Summary

In this paper we present schedule analysis methods for daily domestic (short-haul) crew pairing problems. The analysis is used to find the reasons or structure in the schedule that may cause high pay and credit in the crew pairing solution and to explore guidelines for tweaking the schedule in a small range in order to improve the crew pairing solution. The analyses also show the importance of integrating FAM and crew planning. We demonstrate via case studies how schedule analysis methods work and their effects. The resulting schedule changes from schedule analysis can be incorporated into the integrated FAM and crew model as optional legs and retiming of a small number of legs.

Based on the results and methodologies of schedule analysis, we present a new duty partition formulation for the crew pairing problem. The main advantage of this formulation is that it can quickly find a very good legal pairing solution, which makes it promising in an integrated planning or real-time recovery framework. We show how to incorporate this duty partition model for crew pairing into integrated planning as well as integrated recovery problem. Computational validations for these integrated models are underway.

Future work includes studying analysis methods on international long-haul schedule. Due to the structure of long-haul schedules, they are usually solved as weekly, instead of daily problems. The cost structure may be different. However our analysis methods should still be applicable, but the conclusions we can obtain from the schedule analysis may be quite different.

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Chunhua Gao
Ph.D. Candidate
Georgia Institute of Technology
500 Northside Circle NW Apt U1
Atlanta, GA 30309
Email: cgao@isye.gatech.edu
Phone: 404-964-9258

EDUCATION	2002-present Ph.D. student in Operations Research School of Industrial and Systems Engineering Georgia Institute of Technology M.S. Operations Research, Georgia Institute of Technology B.S. Electrical Engineering, Xi'an Jiaotong University, P.R.China
EXPECTED COMPLETION DATE	May 2007
DISSERTATION TOPIC	Airline integrated planning and operations